This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Edu T 119.07.520

HARVARD UNIVERSITY

LIBRARY OF THE

Department of Education

COLLECTION OF TEXT-BOOKS
Contributed by the Publishers

TRANSFERRED
TO
HARVARD COL
LIBRARY

3 2044 097 006 506

ARITHMETIC FOR SCHOOLS

·The MM Co·

ARITHMETIC

FOR SCHOOLS

BY THE

REV. J. B. LOCK, M.A.

FELLOW OF CAIUS COLLEGE, CAMBRIDGE, ENGLAND

AMERICAN EDITION

EDITED AND ARRANGED BY

CHARLOTTE ANGAS SCOTT, D.Sc. (LONDON)

BRYN MAWR COLLEGE, PENNSYLVANIA

New York
THE MACMILLAN COMPANY
LONDON: MACMILLAN & CO., LTD.
1907

~~I 5.5545~~ Educ T 119.07.520

Harvard University,
Dept. of Education Library.
Gift of the Publishers.

July 10, [illegible]

Copyright, 1891,
By GEORGE P. BRETT.

First published elsewhere. Reprinted May, 1906; October, 1907.

HARVARD COLLEGE LIBRARY
TRANSFERRED FROM THE
LIBRARY OF THE
GRADUATE SCHOOL OF EDUCATION

Norwood Press:
Berwick & Smith Co., Norwood, Mass., U.S.A.

PREFACE TO FIRST ENGLISH EDITION

THIS work has been written in the hope of providing a book suitable for class teaching in Schools, which shall be at once simple and scientific.

Novelty in method or in arrangement has been avoided as much as possible; at the same time the book will be found to differ in many ways from any existing text-book.

Rules are nearly always avoided; specimen examples fully worked out are given instead. The theory, concisely stated, is given in large type; the illustrations and explanations are given in smaller type, and are intended as suggestions on which the teacher may enlarge at his discretion. The examples are numerous and for the most part easy; those in the miscellaneous collections at the end of each chapter are somewhat more difficult.

* * * * * *

The book is divided into Chapters each of which is, as far as possible, complete in itself; so that the order in which the Chapters are taken may be varied at the discretion of the teacher. A few articles and exercises are marked with a star; these may sometimes be omitted on a first reading.

The Chapter on Approximation treats of a part of the subject which of late has been rapidly rising in importance.

The so-called Unitary Method of solving problems is most useful for the purposes of explanation and instruction, as it exhibits the idea of number or ratio in a very simple form. It should, however, be used rather as a stepping stone to the more complete understanding of the fundamental idea of ratio, than as a completely satisfactory method for the solution of problems.

* * * * * *

J. B. LOCK.

GONVILLE AND CAIUS COLLEGE, CAMBRIDGE,
March 1886.

PREFACE TO THE AMERICAN EDITION

THE characteristic features of Mr. Lock's Arithmetic, as set forth in the extracts from his preface given above, seem to me also the characteristic excellences of the Book, and it has been my aim to leave them unaltered, while introducing freely such changes and rearrangements as were necessary in order to produce an American Arithmetic.

I trust that this American edition may be found as free from errors, typographical and others, as the English edition.

C. A. SCOTT.

BRYN MAWR, PENNSYLVANIA,
October, 1890.

CONTENTS

PART I

CHAP. PAGE
I. SIMPLE QUANTITIES 1
II. DECIMALS 31
III. PRIME FACTORS; L.C.M.; G.C.M. . . . 42
IV. FRACTIONS 54
V. COMPOUND QUANTITIES; TABLES . . . 80
VI. PRACTICE, SIMPLE AND COMPOUND . . . 119

PART II

VII. RECURRING DECIMALS 129
VIII. APPROXIMATION; THE METRIC SYSTEM . . . 139
IX. SQUARE AND CUBIC MEASURE, DUODECIMALS . . 149
X. PROBLEMS; EXCHANGE 164
XI. PROBLEMS CONCERNING TIME 179
XII. PERCENTAGE, INTEREST, DISCOUNT . . . 184
XIII. PROPORTION; PROPORTIONAL PARTS . . . 208
XIV. PROFIT AND LOSS 216

CHAP. PAGE
XV. AVERAGE VALUE 220
XVI. STOCKS 222
XVII. SQUARE AND CUBE ROOT 229
XVIII. NUMERATION AND NOTATION—SCALES OF NOTATION . 239

EXAMPLES FOR EXERCISE 244

PART I

CHAPTER I

1. *DEFINITION.* A quantity is called a **unit** quantity when it is used for the purpose of comparing the magnitudes of other quantities of the same kind.

Thus, a *foot* is used as the unit length when we speak of a certain length as containing a number of feet. A *shilling* is used as a unit quantity when we speak of a sum of money as containing a number of shillings.

A **unit** is hereafter used as an abbreviation for a *unit quantity.*

A quantity is said to be **measured** by the **number** of units which it contains. Hence,

2. *DEF.* A **number** is that which is *applied to a unit* for the purpose of indicating the magnitude of a quantity of the same kind as the unit.

Thus, when we say that a certain field contains three hundred and fifty acres, the *number* three hundred and fifty applied to the *unit* quantity *acre,* indicates the magnitude of a certain collection of acres when compared with that of the unit quantity *an acre.*

The number and the unit together indicate the *absolute* magnitude of the quantity; the number indicates the *relative* magnitude.

3. One, two, three, four, five, six, seven, eight, nine, etc., are names given to various numbers.

The symbols or **figures** 1, 2, 3, 4, 5, 6, 7, 8, 9 are used as abbreviations of the words one, two, three, etc.

DEF. The number **one** or **1** applied to the unit indicates a quantity which consists of a single unit.

The number **two** or **2** applied to the unit indicates a quantity which consists of one unit and one unit more.

The number **three** or **3** applied to the unit indicates a quantity which consists of two units and one more.

The number **four** or **4** applied to the unit indicates a quantity which consists of three units and one more.

And so on; the next successive numbers being named **five, six, seven, eight, nine,** and represented respectively by the figures **5, 6, 7, 8, 9.**

It may be noticed that the *number* **four** and the words **four times** express the same thing: **four** units is the same as **four times** the unit.

Example. Prove that 5 units *and* 3 units *together are equal to* 8 units.

5 units and 3 units are equal to 5 units and 1 unit and 2 units; which are equal to 6 units and 2 units; which are equal to 6 units and 1 unit and 1 unit; that is, to 7 units and 1 unit; that is, to 8 units.

4. There is no limit to numbers; for if we imagine a number of units we can always imagine one more. Also it would be *possible* for *each* number to have its own peculiar *name* and its own peculiar *figure.*

For example, the next number to nine is named ten, the next eleven, the next twelve, and so on; and these might be indicated respectively by separate figures such as X, Γ, Λ; and so on.

But a multitude of *disconnected* names and figures would be very inconvenient. We therefore proceed to explain a convenient system for arranging the names and the figures which indicate successive numbers.

A little consideration will show (i) that such a method is not *absolutely necessary;* it is useful and convenient, (ii) that such a method does not spring out of the *nature* of numbers. A method which is convenient, but not absolutely necessary, is called a **Convention.**

5. *DEF.* A method by which numbers are systematically expressed in words is called **Numeration.**

NUMERATION.

6. The established method of numeration is based upon the number **ten.**

The following *DEFINITIONS* are necessary :—

the number **ten** applied to the unit indicates a quantity which consists of nine units and one more.

the number **hundred** applied to the unit indicates a quantity which consists of *ten* ten-units.

thousand . . consists of *ten* hundred-units,
ten-thousand . „ *ten* thousand-units,
hundred-thousand „ *ten* ten-thousand-units,
million . . . „ *ten* hundred-thousand-units.

And there are in succession

ten-million . .
hundred-million .
billion . . .
ten-billion . .
hundred-billion .
trillion, and so on

each of which numbers applied to the unit indicates *ten* times as many units as the preceding.

billion is an abbreviation for thousand-million,* **trillion** for thousand-billion, and so on.

Ten is called the **first power** of ten ; hundred is called the **second power** of ten ; thousand, the **third power** of ten, and so on ; million, the **sixth power** of ten ; billion, the **ninth power** of ten, and so on.

A Hundred units are *ten ten*-units,
A Thousand units are *ten ten-ten*-units,
and so on.

The number **one** may conveniently be called the **zero-power** of ten for a reason which will appear later on (Arts. 45, 201).

---

* In England billion is the name for *million*-million, trillion for *million*-billion, and so on. On the continent of Europe the names are used as in the United States.

7. *To show how any given number (of units) may be expressed by means of the above numbers.*

Suppose that the given number of units is represented by that number of counters.

Let all the counters be arranged in groups, each containing *ten* counters, until less than ten counters are left.

Suppose, for example, *three* counters are left.

Next let all these groups of tens be arranged in sets of *ten*, each set containing a *hundred* counters, until the remaining number of groups is less than ten.

Suppose the remainder to be *two* groups (of ten counters).

Next let the hundreds be arranged in heaps of *ten*, each heap containing a *thousand* counters, until the remaining number of sets is less than ten.

Suppose the remainder to be *four* sets (of a hundred counters);

and so on.

In this way, as the number of counters is limited, we shall at last arrive at heaps of counters, of which there are less than ten heaps altogether.

Suppose, for example, we have finally, *seven* thousand counters. We should then in our supposed example have found that the given number of units consisted of seven (thousand) four (hundred) two (ten) and three,

or in the usual way of speaking the *number* of units is seven thousand four hundred and twenty-three.

Thus any given number (of units) may be enumerated in terms of the numbers ten, hundred, thousand, etc., by the aid of the numbers from one to nine inclusive.

This is the **Decimal System** of Numeration.

EXAMPLES.

The pupil should have a large number of counters (a bag of seeds, such as beans, would do), and go through the process of arranging three or four different numbers at the least.

Arrange by means of counters:—**1.** Fifty-three units. **2.** Ninety-four units. **3.** Three hundred and sixty-five units.

8. It should be noticed that any other number than *ten* may be used as the basis of a system of numeration.

EXAMPLES.

Let the pupil take a heap of counters and arrange them in groups each containing any number, say twelve, counters; then arrange the groups in sets of twelve; and so on, as in Art. 7.

Arrange by means of counters:—**1.** 5 gross 7 dozen and 8; **2.** 8 gross 9 dozen and 3; **3.** Ten gross and eight.

The figures of Arts. 143, 169, will be found useful for illustration.

9. It will be seen that the decimal system of numeration has become a part of the English language.

The names of the numbers between ten and a hundred are somewhat irregular; instead of ten and one we have eleven (from the Gothic *ainlif*, *ain* one and *lif* ten); instead of ten and two we have twelve (Gothic *tvalif*, *tva* two and *lif* ten); instead of ten and three we have the *abbreviation* thirteen; and so on; instead of two ten we have twenty; and so on.

It should be noticed also that an expression such as *four hundred million, twenty million, two million, seven hundred thousand, twenty thousand, five thousand,* is abbreviated thus: *four hundred and twenty-two million seven hundred and twenty-five thousand.*

It will often be convenient to use *tens, hundreds, thousands,* etc., as an abbreviation for ten units, hundred units, thousand units, etc.

10. When the figures 1, 2, 3, 4, 5, 6, 7, 8, 9 are used in the numeration of a number greater than ten they are called the **digits** of the number.

Man having *ten* fingers, *ten* came to be used as the basis of numeration, and each of the figures 1, 2, 3, 4, 5, 6, 7, 8, 9 was indicated by its own particular finger (*digitus*, a finger).

Example. Name the powers of ten *and the* digits *employed in*
Four hundred and twenty-one thousand two hundred and four.

The powers of ten used are hundred-thousand, ten-thousand, thousand, hundred, and one; the digits are 4, 2, 1, 2, 4.

EXAMPLES. I.

Name the *powers of ten* and the *digits* employed in the following:—

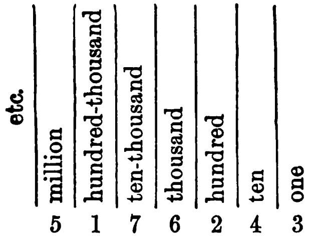
1. Three hundred and forty-seven. **2.** Four thousand two hundred and eighty-one. **3.** Sixty thousand two hundred and twenty. **4.** Four million two hundred and thirty-one thousand five hundred and ninety. **5.** Twenty-three million four thousand and ten. **6.** Four hundred and one million thirty thousand and six. **7.** Two hundred and ten million four hundred and two thousand and thirty. **8.** Ninety million forty thousand and one.

NOTATION.

11. In Art. 7 it has been explained that any given number of units may be expressed by means of certain of the nine digits and the successive powers of ten, viz. one, ten, hundred, thousand, etc. What is required is a convenient method of indicating these powers of ten by symbols; such a method is called **Notation.**

12. *To explain the method of Notation.*

Consider the following diagram:—



It is a method of representing the number five million, one hundred and seventy-six thousand, two hundred and forty-three. The digits expressing the number have been written in a line, and above each is placed the power of ten to which it refers.

In the ordinary method of notation the digits are arranged in the order of the diagram, and the necessity for always writing above each digit the name of its particular power of ten is avoided by the following *convention:*

It is agreed that the **place** or **position** of each digit *shall indicate to which power of ten* it refers; so that the above number is written concisely thus

5176243.

The one-place is the **zero**, or *starting*, place; the ten-place is the **first** place on the left of the one-place; the hundred-place the **second** on the left; the thousand-place the **third**; and so on.

Note.—It is usual to point out that the '**places**' of the digits are arranged by their names in sets of three, the *first* set being one, ten, hundred; the *second* set thousand, ten-thousand, hundred-thousand; and so on.

The student will often find it convenient to count off the places from right to left in threes by commas thus—5,176,243. These commas are merely aids to the eye and have no further meaning.

EXAMPLES. II.

State in words the numbers represented by the following sets of digits:—

1. 4712.
2. 576321.
3. 15216.
4. 8976421.
5. 666667.
6. 87654321.
7. 123,456,789.
8. 12,345,654,321.
9. 11,111,111.
10. 999,999.
11. 41,365,123.
12. 8,614,531,236.

13. Many numbers are such that one or more of the powers of ten are missing in their expression. For example, *four million three thousand and twenty.* In such cases the figure 0 is introduced to keep a place for each of the powers of ten which is not represented by a digit in the expression of the number. Thus the number *four million three thousand and twenty* is written

4003020,

as will be seen from the scheme on the next page.

etc.	million	hundred-thousand	ten-thousand	thousand	hundred	ten	one
	4	0	0	3	0	2	0

Thus, the figure 0, which is called **nought**, or **cypher,** is used to indicate that a certain power of ten is *absent* from the expression of a number; and thus also to indicate for those digits which are used their proper power of ten.

EXAMPLES. III.

Express each of the following numbers in figures :—

1. A thousand and one.
2. Nine hundred.
3. Twenty-one thousand and six.
4. Four thousand.
5. Five hundred thousand.
6. Fifty thousand.
7. Four hundred and five thousand.
8. Five million.
9. Four million five thousand.
10. Twenty million.
11. Seventy million one thousand.
12. Two billion and fifty.
13. Six hundred and nine million one thousand two hundred.
14. Four thousand million nine hundred thousand.
15. A hundred thousand million and ten thousand.
16. Nine hundred thousand and nine hundred million nine hundred thousand and ninety.

Express each of the following numbers in words :—

17. 40.
18. 800.
19. 7000.
20. 601.
21. 70500.
22. 800102.
23. 410000.
24. 7090709.
25. 80080080.
26. 45016000.
27. 800100010.
28. 780907812.
29. 8,110,101,000.
30. 66,600,066.
31. 1,010,101,010.
32. 8,070,060,010.
33. 400,000,000.
34. 4,050,060,000.

ADDITION.

14. The meaning of the word 'add' may be explained thus: To **add** two or more given numbers, say 54, 29, and 108, we take 54 counters, 29 counters, and 108 counters and put them together; the number of the counters taken together is the result of adding the three numbers.

DEF. The result of adding two or more numbers is called the **sum** of those numbers.

DEF. **Addition** is the method or rule by which the sum of two or more given numbers is obtained from those numbers.

It should be noticed that in the definition of successive numbers the idea of addition is assumed; we define *eight* units as the result of *adding* one unit to seven units. [See Art. 25.]

15. *DEF.* The sign + is used to indicate that the two numbers between which it is placed are to be added. It is read '**plus.**'

Example. 3 + 5 means that 5 is to be added to 3. It is read 'three plus five'

Numbers connected by the sign + are called **terms.**

The sign = is used as an abbreviation of the words 'is equal to.' It is read '**equals.**'

NOTE.—The sign = always **makes a statement.**

Example. 3 + 5 = 8, states that the sum of 3 and 5 *is equal to*, that is, *is the same number as*, 8. It is read '3 plus 5 equals 8.'

16. **All accurate facility** in the subsequent processes of Arithmetic depends on a **complete familiarity** with the Addition Table. This Table, which consists of the sums of every pair of numbers less than ten, the pupil should construct for himself. The method of addition is based upon it.

Example. Add 8 + 6 + 9 + 4 + 8 + 7.

Process,

8 + **6** = 14 ; **14** + **9** = 23 ; **23** + **4** = 27 ; **27** + **8** = 35 ; **35** + **7** = 42.

The student must learn to say *mentally* no more than 8, **14**, **23**, 27, **35**, 42.

Mental Arithmetic.

The best possible exercise in the mental process of addition is afforded by the method known as **counting by equal increments**; thus:

Examples. Count aloud by increments of 2 *up to* 20 *starting* (i) *at* 2, (ii) *at* 1.

Answer (i). 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.

Answer (ii). 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

Count aloud by increments of 7 *up to* 100, *starting* (i) *at* 7, (ii) *at* 3.

Answer (i). 7, 14, 21, 28, 35, 42, 49, 56, 63, etc.

Answer (ii). 3, 10, 17, 24, 31, 38, 45, 52, 59, etc.

Each step in the addition must be done mentally by the aid of the **memory** only; the use of fingers or of any other mechanical aid is destructive of all progress.

The rate at which the successive numbers are named should be at first slow; it should be gradually quickened as facility is attained; the increments should be made larger; and the counting taken up to larger numbers.

It may be pointed out that this method provides the most perfectly graduated exercises in mental addition, rendering a multitude of printed examples for practice unnecessary.

It may be also pointed out that this method does not *supersede* the ordinary methods of mental addition; it forms a stepping stone to **facility in mental addition**, the value of which can scarcely be overestimated.

17. Large numbers are added as follows:

Example. Add together 3471, 89054, 406039.

Suppose there are three sets of counters containing 3471, 89054, 406039 respectively, each set being arranged in ones, tens, hundreds, thousands, etc.; we have to find the number in all three sets taken together. We first add the ones; thus, $9+4+1=14=1$ ten $+4$; we next add the tens, **carrying**, that is, *including in the tens*, the 1 ten from the ones; thus $(1+3+5+7)$ tens $=16$ tens $=1$ hundred $+6$ tens. We then add the hundreds, carrying the 1 hundred from the tens; thus $(1+0+0+4)$ hundreds $=5$ hundreds; and so on. It is found convenient to arrange each power of ten in the same vertical column, as on the next page:

Process,	3471 89054 406039 —— 498564 *Ans.*	*Mentally* thus: 9, 13, 14; *carry* 1, 4, 9, 16; *carry* 1, 5; 6, 15, 18; *carry* 1, 9; 4.

EXAMPLES. IV.

[It is recommended that the pupil should work out *with the aid of counters* the first four of the following examples.]

Perform the additions indicated in each of the following:

1. 32 41 70	2. 84 92 127	3. 161 237 219	4. 643 1096 409
5. 76841 39001 967860 4362	6. 79412 8416 32 48699	7. 546711 873461 5400 290036	8. 70776 789654 100700 3456789

Add together each of the six following sets of numbers :—

9. 7942 + 8643 + 4731 + 306.

10. 36910 + 47123 + 96541 + 4806.

11. 579123 + 490 + 4761 + 89742 + 37652.

12. 794316 + 4978 + 324610 + 9876001 + 49136 + 88888.

13. 79105 + 84624 + 888661 + 773770 + 678901 + 45678.

14. 5641 + 7934062 + 8765431 + 976823 + 874 + 29 + 48765.

15. In the Battle of Waterloo it is said that of the soldiers engaged 36273 were British, 7447 were Hanoverians, 8000 were Brunswickers and 21000 were Belgian, whilst there were about 75000 French; how many were there of the four allies, and how many combatants were there altogether?

16. In 1881 the population of England and Wales was 24,974,439, that of Scotland 3,735,573, that of Ireland 5,174,836; what was the total population of the British Isles in 1881?

17. It is stated that the area in square miles of the British Isles is one hundred and twenty-one thousand one hundred and fifteen, of British Possessions in Asia one million five hundred and eighty-eight thousand two hundred and fifty-four, in Africa two hundred and seventy thousand, in America three million seven hundred and thirty-three thousand two hundred and seven, in Australasia three million one hundred and eighty-one thousand three hundred and thirty-four, in other parts ninety-six thousand one hundred and seventy-one; how many square miles are there in the whole of the British Empire?

18. The public revenue in 1882 of New South Wales was seven million four hundred and ten thousand seven hundred and thirty-seven pounds; of Victoria five million nine hundred and thirty-four thousand two hundred and forty-one pounds; of South Australia two million nine thousand four hundred and eighty-four pounds; of Queensland two million four hundred and sixty-six thousand three hundred and fifty-eight pounds; and of Western Australia two hundred and fifty thousand three hundred and seventy-two pounds; what was the total public revenue of Australia?

19. In 1884 the exports of the United States are said to have been worth seven hundred and forty million five hundred and thirteen thousand six hundred and nine dollars and the imports one thousand four hundred and eight million two hundred and eleven thousand three hundred and two dollars; how many dollars were they worth together?

SUBTRACTION.

18. The meaning of the word 'subtract' may be explained thus: To **subtract** one given number, say 395, from a larger given number, say 4791, we find the number which must be **added** to the smaller number that the sum may be the larger number. This number, when found, is the result of subtracting the smaller number from the larger.

DEF. The result of subtracting one given number from a larger number is called the **difference.**

It is also often called the **remainder.**

DEF. **Subtraction** is the method or rule by which the difference between two given numbers may be obtained from those numbers.

19. *DEF.* The sign – is used to indicate that the second of the two numbers between which it is placed must be subtracted from the first. It is read **minus.**

Example. 8 − 3 indicates that 3 is to be subtracted from 8. It is read '8 minus 3.'

The greater of the two given numbers is called the **minuend**, the less is called the **subtrahend**; but these names are not often used.

Numbers connected by the sign + or by the sign − are called **terms**. [Art. 16.]

Example. 8 − 3 = 5. Here 8 is the greater, 3 the less of the two *terms;* 5 is the *difference* between 8 and 3. 8 is also called the minuend, 3 the subtrahend, 5 the remainder.

20. The *process* of Subtraction is the process of finding the number which must be *added* to a given number to make a larger given number; it is sometimes called **complementary addition.**

Example. Thus, 8 − 3 = 5, because 3 + 5 = 8.

Large numbers are subtracted as follows :—

Example. *Subtract* 4837 *from* 8319.

We are to find the number which added to 4837 makes up 8319.

We place the smaller number under the larger, as in Addition; then we see that 7 with **2** added make up 9; put down 2; next, 3 (ten) and **8** (ten) are 11 (ten); put down 8 (ten), and **carry*** **1** (hundred); next, (8 + 1) (hundred) and **4** (hundred) are 13 (hundred); put down **4** (hundred), and **carry 1** (thousand); (4 + 1) (thousand) and **3** (thousand) are 8 (thousand); put down 3 (thousand).

Thus 3482 added to 4837 make up 8319.

Process 8319
4837
————
3482 *Answer.*

Mentally thus: 7 and **2** = 9;
3 and **8** = **11**;
carry 1, 9 and **4** = **13**;
carry 1, 5 and **3** = 8.

---

* Note. **Carry 1.** We are making the lower number up to the higher number; we have added 8 to the 3 and so have made the 3 up to 11; in other words, we have made the *thirty* in the lower number up to *one hundred and ten.* This *one hundred* is accordingly considered as having been already added to the 8 hundred (for our lower number 837, to which 82 has been added, is thereby made up to 919); accordingly in our next step we say 9 hundred and 4 hundred make 13 hundred; and so on. Thus the figure *carried* is added to the next figure on the left in the lower line.

We may notice that the *carrying* is the same process as in addition; we are adding 3482 to 4837 and we *carry* when necessary.

EXAMPLES. V.

Perform the following subtractions :—

1.	85	**2.**	491	**3.**	171	**4.**	1654
	45		62		89		965
5.	291	**6.**	411	**7.**	1000	**8.**	1505
	193		382		645		999
9.	347951	**10.**	796854	**11.**	876123	**12.**	100000
	8067		28999		99999		99999

13. 73461 − 25973.

14. 85736940 − 76846941.

15. 8010203 − 7010204.

16. 999990 − 888889.

17. Find the difference between twenty thousand and two and twelve thousand eight hundred.

18. A million seven hundred − forty-nine thousand and nine.

19. There were 67,374 miles of railway in the United States in 1872 and 71,564 in 1873 ; how many miles were added in one year ?

20. A man has $10,000 ; how much must he borrow that he may be able to pay $13,640 for an estate ?

21. A man sold some land for forty-five thousand pounds, making thereby a profit of seven thousand one hundred and twenty-five pounds ; what did the land cost him ?

22. The population of London in 1881 was 3,831,719, of Paris 1,988,806, of New York and Brooklyn 1,242,533 and 566,689 respectively ; how many more inhabitants has London than all three cities put together ?

23. A man left $176,450 and gave by his will $40,000 to each of two sons and $34,500 to his daughter, the remainder to his wife ; what was the wife's share ?

21. From the meaning given to addition and subtraction it appears that an alteration in the **order** in which numbers are added or subtracted does not alter the result ; provided that it is not at any time required to subtract a larger number from a smaller.

Example. 874 + 39 − 176 − 37 + 495 − 763
is the same as 874 + 39 + 495 − 176 − 37 − 763.

It is generally convenient first to find the sum of the terms to be added, and then of those to be subtracted, and then to find the difference of the results. In this case the sums are 1408 and 976. Therefore the required result is 1408 − 976, that is 432. See also Art. 23.

22. **Brackets** are often useful as a means of abbreviation. When an expression is enclosed in a bracket, the operations of arithmetic indicated inside the bracket are to be performed before any other operation is performed on the numbers in the bracket.

Example (i). $8-(3+4)$ means that $3+4$, *i.e.* 7, is to be subtracted from 8. [NOTE. $8-(3+4)=8-7=8-3-4$.]

Example (ii). $(8+7)-\{5-(2+1)\}$ is to be understood as follows. The *inner* bracket $(2+1)$ indicates that 1 is to be first added to 2 ; the $-$ before the bracket indicates that the result, 3, is to be subtracted from 5. Thus $(8+7)-\{5-(2+1)=15-\{5-3\}\}=15-2=\underline{\underline{13}}$.

23. The method of Subtraction (or complementary Addition) of Art. 20 may be employed very conveniently to abbreviate the work of a question like the following:—

Subtract $4758+39276+47825+9718$ from 167031.

Process

$$\begin{array}{r} 167031 \\ \hline 4758 \\ 39276 \\ 47825 \\ 9718 \\ \hline 65454 \\ \hline \end{array}$$

First add the four digits in a column, and then *make the sum up to the top line*, as in Art. 20.

Mentally thus: 8, 13, 19, 27 and **4**=**31** ;
carry 3, 4, 6, 13, 18 and **5**=**23** ;
carry 2, 9, 17, 19, 26 and **4**=**30** ;
carry 3, 12, 19, 28, 32 and **5**=**37** ;
carry 3, 7, 10, and **6**=**16**.

The first digit in heavy type is the digit in the answer, the second is the figure *carried* [see Note, Art. 20].

EXAMPLES. VI.

Perform the operations indicated below.

1. $8971-432-187$.
2. $439-64-81-43$.
3. $179-36-49-23$.
4. $1476-293-64-491$.
5. $8910-702-361-1421$.
6. $1600-374-400-799$.
7. $479+(3624-1731)$.
8. $74891-(4683+4297)$.
9. $66674+843-\{(9645+8761)+201\}$.
10. $87642+798764-\{379862+4001-(14760-3720)\}$.

11. Find the difference between (9712 − 932) and (954 − 764).

12. What is the remainder when 7481 is subtracted from (39765 − 8462)?

13. What number must be added to the sum of 3691, 4782, 69350 that the sum may be one million?

14. From the sum of 23462 and 9030 subtract the difference between 34000 and 7640.

15. What number subtracted from 2471 + 156 will leave 3754 − 2683?

16. The greater of two numbers is 59253 and their difference is 41684; what is the less?

17. The sum of two numbers is 1363 and one of them is 864; what is the other?

18. A man bought three houses for $7725, $4475, and $10375 respectively, and paid $15900 as a deposit; how much has he still to pay?

19. *A* has $8750; *B* has $1975 less than *A*; and *C*, if he had $735 more than he has, would have as much as *A* and *B* together; how much has *C*?

MULTIPLICATION.

24. The meaning of the word 'multiply' may be explained thus:—To **multiply** a given number, say 34, by a given number, say 23, we take 23 groups of counters, each group containing 34 counters, and **add** them together; the sum is the required result; hence,

DEF. **Multiplication** is a convenient method of finding the **sum** of a given number of repetitions of a certain number.

Thus, when 34 is multiplied by 6, the number required is

$$34 + 34 + 34 + 34 + 34 + 34, \text{ that is, } 204.$$

DEF. The number which is multiplied is called the **multiplicand.**

The number by which it is multiplied is called the **multiplier.**

The resulting number is called the **product.**

The sign of multiplication is ×.

Thus 34 × 6 is an abbreviation for 34 multiplied by 6, which is read six *times* thirty-four.

Since the definition of successive numbers requires the idea of addition, it must involve also the idea of multiplication. Thus, when we multiply a number by twenty-three we perform upon that number the same operation which performed upon *one* gives twenty-three as result; hence the following

25. *GENERAL DEFINITION.*- To **multiply** a number by another number, we must perform on the number multiplied that operation which, when performed upon *one*, gives as result the multiplier.

26. The pupil should make a list of the results of multiplying each number from 1 to 12 inclusive by every number from 1 to 12, and should make himself familiar with it. A Table of these results is given below.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

27. *To prove that the multiplier and multiplicand may be interchanged without altering the product.*

Let the product be the total number of the counters in certain equal groups of counters. Then, the number of the groups is the multiplier; the number of the counters in each group is the multiplicand.

Now rearrange the counters by taking one counter from each group to form a new group, and repeating the process till all the counters have been formed into a new set of equal groups.

The number of the new groups (the new multiplier) is the former multiplicand.

The number of the counters in each of the new groups (the new multiplicand) is the former multiplier.

The total number of the counters (the product) is unaltered and the proposition is proved.

Example. To prove that seven times five = five times seven.

The first product is the number of the counters in seven groups each containing five counters. Representing a counter by a star, they may be arranged thus:

```
* * * * * * *
* * * * * * *
* * * * * * *
* * * * * * *
* * * * * * *
```

Let the number in each column (five) be the multiplicand, then the number of columns (seven) is the multiplier.

Next let the number in each row (seven) be the multiplicand, then the number of rows (five) is the multiplier.

The product, which is the sum of all the counters, is the same in each case.

Since a multiplier and multiplicand are interchangeable, their two distinguishing names are seldom used.

DEF. Two or more numbers connected by the sign of multiplication are called **factors.**

28. The Multiplication of numbers greater than ten is a method of *adding a series of repetitions* of a given number. It is based upon the table of the products of numbers less than ten with which the student is now supposed to be familiar.

Example. Multiply 4793 *by* 785.

That is, find the number of counters in 785 groups each containing 4793 counters; or, find the number in

$$\begin{array}{rl} & 700 \text{ times } 4793 \text{ counters,} \\ + & 80 \text{ times } 4793 \text{ counters,} \\ + & 5 \text{ times } 4793 \text{ counters.} \end{array}$$

We shall consider these three sets separately.

700 times 4793 is equal to 100 times 4793 counters repeated seven times,

and 100 times 4793 is equal to 479300; for each digit in the latter number has a hundred times its value in the former. Hence

$$700 \text{ times } 4793 = 7 \text{ times } 479300.$$

We do not trouble to write down 479300 seven times and add, but we use the following abbreviation:—

$$\left.\begin{array}{r} 479300 \\ 479300 \\ 479300 \\ 479300 \\ 479300 \\ 479300 \\ \underline{479300} \\ \underline{\underline{3355100}} \end{array}\right\} \text{is abbreviated} \left\{\begin{array}{r} 479300 \\ \underline{7} \\ \underline{\underline{3355100}} \end{array}\right.$$

The 7 placed under the number 479300 indicating how many times it is to be repeated.

From the Table of Multiplication we remember that 7 times 3 = 21, that 7 times 9 = 63, and so on; the rest of the process being the same as in addition.

The beginner must learn to multiply a digit and add in the figure carried, in one *mental process;* thus, seven 3's, 21; *carry* 2, seven 9's, 65; *carry* 6, seven 7's, 55; *carry* 5, seven 4's, 33.

Again, 80 times 4793 = 8 times 47930, and we have as above

$$\begin{array}{r} 47930 \\ \underline{8} \\ \underline{\underline{383440}} \end{array}$$

And 5 times 4793 is found in the same way,

$$\begin{array}{r} 4793 \\ \underline{5} \\ \underline{\underline{23965}} \end{array}$$

Hence, 785 times 4793 counters = 3355100 + 383440 + 23965 counters = $\underline{\underline{3762505}}$ counters.

The whole process may be combined as follows:—

$$\left.\begin{array}{r}4793\\785\\\hline 3355100\\383440\\23965\\\hline \underline{3762505}\end{array}\right\}$$

or, omitting the 0's which are not necessary when the place of each digit is evident, (*Product*)

$$\left\{\begin{array}{r}4793\\785\\\hline 33551\\38344\\23965\\\hline \underline{3762505}\end{array}\right\}$$

or, arranging the addition in a different order, (*Product*)

$$\left\{\begin{array}{r}4793\\785\\\hline 23965\\38344\\33551\\\hline \underline{3762505}\end{array}\right.$$

EXAMPLES. VII.

Perform the multiplications indicated:—

1. $\begin{array}{r}21\\3\\\hline\end{array}$ **2.** $\begin{array}{r}36\\4\\\hline\end{array}$ **3.** $\begin{array}{r}81\\9\\\hline\end{array}$ **4.** $\begin{array}{r}45\\7\\\hline\end{array}$

5. $\begin{array}{r}4963\\7\\\hline\end{array}$ **6.** $\begin{array}{r}87931\\8\\\hline\end{array}$ **7.** $\begin{array}{r}49612\\10\\\hline\end{array}$ **8.** $\begin{array}{r}87610\\12\\\hline\end{array}$

9. 47 × 3. **10.** 26 × 8. **11.** 101 × 12.
12. 376 × 9. **13.** 7614 × 5. **14.** 84932 × 11.
15. 67034 × 100. **16.** 365 × 200. **17.** 1571 × 300.
18. 978 × 90. **19.** 8909 × 900. **20.** 4040 × 800.
21. 47 × 63. **22.** 59 × 45. **23.** 73 × 51.

24. $\begin{array}{r}4971\\384\\\hline\end{array}$ **25.** $\begin{array}{r}69830\\476\\\hline\end{array}$ **26.** $\begin{array}{r}81674\\591\\\hline\end{array}$

27. 4912 × 368. **28.** 76841 × 1402. **29.** 397 × 40012.
30. 7901 × 3601. **31.** 40571 × 8070. **32.** 12035 × 61011.
33. 7707 × 6006. **34.** 65432 × 5051. **35.** 23041 × 7676.
36. 98765 × 786. **37.** 89756 × 978. **38.** 76098 × 768.
39. 86405 by 700732. **40.** 76841 by 3760001.
41. 493602 by 8090102. **42.** 709020 by 300700400
43. 277274 by 1728. **44.** 97654 by 78065.
45. 4709123 by 70770. **46.** 837621 by 89795.
47. 4097684 by 49536. **48.** 709377 by 89795.
49. 987761 by 56789. **50.** 394959 by 476787.
51. 314159 by 31416. **52.** 365242 by 45967.

53. How many yards of calico are there in 49 bales, each bale containing 37 pieces and each piece containing 88 yards?

54. 25 masons earn $13 each, and 20 labourers $8 each per week; how many dollars do they all earn in 12 weeks?

55. If 3471000 letters are posted daily, how many are posted (i) in a week, (ii) in a year?

56. If 41320 persons cross London Bridge daily, how many cross (i) in a week, (ii) in a year?

57. Light travels 192500 miles in a second; how many miles does it travel in a day?

29. **Multiplication by factors.** The result of multiplying a number by the product of two or more factors is the same as the result of multiplying the number first by one factor, then multiplying this product by another factor, and so on, till all the factors have been used.

Example. Multiply 364 *by* 48. Here $48 = 8 \times 6$.

By 'multiplying by 48' is meant: 'find the number which results from adding 48 sets, each set containing 364 counters.' These 48 sets may be arranged in 6 groups of 8 sets each. The number of counters in a group is found by multiplying 364 by 8, and since there are 6 groups we multiply this number by 6 and obtain as result the whole number of counters; hence we 'multiply first by 8 and the result by 6.'

Process

$$\begin{array}{rr} & 364 \\ & 8 \\ \hline & 2912 \\ & 6 \\ \hline (\textit{Product}) & \underline{\underline{17472}} \end{array}$$

The **continued product** of three or more numbers is the product of two of the numbers multiplied by the third, the result multiplied by the fourth, and so on.

Example. The *continued product* of 7, 9, and 11 is

$$7 \times 9 \times 11 = (7 \times 9) \times 11 = 63 \times 11 = 693.$$

30. To multiply by any one of the numbers from **11** to **19** the following process is recommended:—

$$\left.\begin{array}{rr} & 76543 \\ & 17 \\ \hline (\textit{Product}) & \underline{\underline{1301231}} \end{array}\right\}$$

mentally thus: seven 3's, **21**;
carry 2, seven 4's, 30 and **3** = **33**;
carry 3, seven 5's, 38 and **4** = **42**;
carry 4, seven 6's, 46 and **5** = **51**;
carry 5, seven 7's, 54 and **6** = **60**;
6 and **7** = 13.

The black figure added is the '10 times,' being the top figure to the right of the one multiplied.

31. The method of Art. 23 may be used with advantage in examples like the following :—

Example. *Subtract* 7 times 7654 *from* 56131.

Process, 56131
———
7654
7
———
(*Result*) 2553

Mentally thus : seven 4's, 28 and **3**=**31** ;
carry 3, seven 5's, 38 and **5**=**43** ;
carry 4, seven 6's, 46 and **5**=**51** ;
carry 5, seven 7's, 54 and **2**=**56**.

The above process is the same as that of Art. 23, except that, since the numbers to be subtracted are seven repetitions of 7654, the multiplication table is used to abbreviate the addition.

EXAMPLES. VIII.

Find the continued products :—

1. $43 \times 5 \times 3$.
2. $111 \times 7 \times 11$.
3. $437 \times 3 \times 5$.
4. $51 \times 9 \times 9 \times 2$.
5. $37 \times 8 \times 5 \times 3$.
6. $99 \times 9 \times 8 \times 7$.

Perform the following multiplications, using factors not greater than 12 :—

7. 479×24.
8. 3684×16.
9. 891×15.
10. 3010×27.
11. 79×35.
12. 891×108.
13. 462×324.
14. 4123×144.
15. 495×81.
16. 967×54.
17. 6012×63.
18. 5210×210.
19. 5001×240.
20. 7012×550.
21. 223×1000.
22. 405×275.
23. 4081×288.
24. 3978×729.

Use the method of Art. 30 in the following :-

25. 45719×13.
26. 34712×13.
27. 894103×14.
28. 775566×15.
29. 3714251×16.
30. 8479123×17.
31. 444671×18.
32. 1701958×19.
33. 564789×16.

Simplify the following by the method of Art. 31 :—

34. $400 - (4 \times 97)$.
35. $843 - (6 \times 131)$.
36. $8654 - (3 \times 2171)$.
37. $5140 + (8 \times 1493)$.
38. $10507 + (4 \times 2401)$.
39. $4372 + (9 \times 7654)$.
40. $100000 - (2 \times 3657)$.
41. $119999 - (7777 \times 7)$.
42. $43012 - (8467 \times 4)$.
43. $41201 - (765 \times 5)$.
44. $81810 - (365 \times 12)$.
45. $41592 - (365 \times 11)$.
46. $730040 - (236421 \times 3)$.
47. $2823012 - (463205 \times 6)$.

DIVISION.

32. The meaning of the word 'divide' may be explained thus:—To **divide** a given number, say 441, by a given number, say 23, we take 441 counters and *subtract* from them 23 counters, then subtract 23 counters from the remainder, and so on, until the remainder is less than 23 counters.

The number of times the subtraction is performed is called the **quotient**; the number of counters left, after the last subtraction, is called the **remainder.**

DEF. The number divided is called the **dividend.** The number by which it is divided is called the **divisor.**

It will be found that 23 counters can be subtracted from 441 counters 19 times, and that then the remainder is 4 counters. Hence when 441 is divided by 23, 441 is the dividend, 23 is the divisor, the quotient is 19, and the remainder is 4.

Division is a method of finding the quotient and remainder when one given number is to be divided by another. The sign of division is $\div$.

Thus $441 \div 23$ is an abbreviation for 441 'divided by' 23.

33. The quotient in the process of division may be regarded in either of **two ways.**

I. It answers the question: If 441 counters are divided into 23 equal groups, how many counters will each group contain? (*Result,* 19 counters and 4 counters remainder.)

II. It also answers the question: If 441 counters are divided into groups of 23 counters each, how many groups will there be? (*Result,* 19 groups and 4 counters remainder.)

Example (i). 441 apples are divided equally among 23 boys; how many does each boy get? (*Res.* 19 apples each, 4 apples remainder.)

Example (ii). 441 apples are divided so that each boy gets 23 apples; how many boys get a share? (*Result,* 19 boys, 4 apples remainder.)

34. It follows from the definition that

divisor × quotient + remainder = dividend. [Art. 27.]

Also that the divisor and quotient are interchangeable.

Division may also be explained as the reverse of Multiplication; thus,

DEF. When there is no remainder the *quotient* is the number whose product by the divisor is the dividend.

35. The division of numbers less than 100 by numbers less than 10 is effected by means of the known results of the Multiplication Table (Art. 26).

Example. *Divide* 53 *by* 6.

It is known that $8 \times 6 = 48$, and that $9 \times 6 = 54$; therefore $53 \div 6$ gives 8 as quotient and 5 remainder.

The division of numbers greater than 100 may be explained by examples as follows:—

Example (i). *Divide* 4976 *by* 8.

Here, 4976 counters are to be arranged in 8 equal groups. The highest power of ten is thousand, and there are 4 of them. Since 4 is less than 8 we pass on to the next lower power of ten, viz. hundred. There are 49 hundreds. $49 \div 8$ gives quotient 6 and remainder 1. That is, 49 hundreds of counters may be arranged in 8 groups of 6 *hundred* each, leaving 1 hundred counters remainder.

This 1 hundred, with the 7 tens, make 17 tens to be divided by 8, giving quotient 20 and remainder 1 ten. This 1 ten with the 6 gives 16; which divided by 8 gives quotient 2.

Thus 4976 counters may be arranged in 8 equal groups, each containing $600 + 20 + 2$, or 622 counters.

The whole process

$$\text{is abbreviated thus: } \left\{ \begin{array}{l} \qquad\quad 8\overline{)4976} \\ \text{quotient} \ \ \underline{\underline{622}}, \text{ remainder } 0. \end{array} \right.$$

EXAMPLES. IX.

Divide—

1. 36 by 6.
2. 48 by 4.
3. 108 by 9.
4. 81 by 3.
5. 144 by 6.
6. 125 by 5.
7. 1000 by 4.
8. 1200 by 8.
9. 153 by 9.
10. 471 by 3.
11. 132 by 11.
12. 240 by 12.
13. 47000 by 6.
14. 381111 by 7.
15. 4123456 by 9.
16. 7000000 by 8.
17. 48120125 by 9.
18. 46700137 by 30.
19. 841267 by 90.
20. 4632195 by 100.
21. 4725111 by 1000.
22. 45678 by 500.
23. 1234567 by 90.
24. 147000 by 8000.

Example (ii). *Divide* 695667 *by* 137.

Here, 695667 counters are to be arranged in 137 equal groups. The highest power of ten, of which there are more than 137, is thousand; there are 695 thousands. 695÷137 gives quotient 5 and remainder 10; that is, 695 thousand counters may be arranged in 137 groups containing 5000 counters each, leaving 10000 counters remainder. These 10000 with the 6 hundred in the dividend make 106 hundreds; 106÷137 gives no quotient, and we proceed to the next lower power of ten. The 106 hundred = 1060 tens, which, with the 6 tens in the dividend, make 1066 tens; 1066÷137 gives 7 as quotient and 107 as remainder; and so on. Process

137)695667(5000
685000
10660(70
9590
1077(7
959
118 (*remainder*)

or, omitting the 0's when the place of each digit is evident,

137)695667(5077
685
1066
959
1077
959
118 (*remainder*).

Thus, 695667 counters can be arranged in 137 equal groups of 5077 counters each, and there are 118 counters over.

36. The multiplication and subtraction in the above process may be *combined* * as in Art. 31.

Example. Divide 561310123 *by* 7654.

Process 7654)561310123(73335 (*Quotient*).
25530
25681
27192
42303
4033 (*remainder*).

First, under the dividend and from the left we draw a line just as far as is necessary to include digits expressing a number *greater* than the divisor. In this case, we draw a line under 56131. Next, we consider what is the first figure in the quotient; in this case it is 7 (representing 70000). Then we say, *mentally*,

seven 4's, 28 and **3** = **3**1, *carry* **3**; seven 5's, 38 and **5** = **4**3, *carry* **4**;
seven 6's, 46 and **5** = **5**1, *carry* **5**; seven 7's, 54 and **2** = **5**6. [Art. 31.]

Thus 2553 added to seven times 7654 makes up 56131.

We take down the next digit in the dividend, and repeat the process for the next digit in the quotient.

---

* This method of abbreviation when rightly understood diminishes the *mental exertion* by at least one-fourth; the reason being that the complete calculation of the separate *products* of the divisor (7654) by each digit (7, 3, 3, etc.) in the quotient is not required.

EXAMPLES. X.

Divide—

1. 4731 by 31.
2. 1000 by 51.
3. 4000 by 23.
4. 1111 by 83.
5. 8975 by 37.
6. 7531 by 61.
7. 41378 by 64.
8. 55555 by 86.
9. 45678 by 78.
10. 379401 by 77.
11. 567890 by 69.
12. 445500 by 97.
13. 100010 by 73.
14. 501030 by 87.
15. 479302 by 49.
16. 478919 by 59.
17. 321098 by 99.
18. 123409 by 89.
19. 8430 by 131.
20. 9012 by 173.
21. 3705 by 237.
22. 45678 by 157.
23. 89012 by 279.
24. 34567 by 371.
25. 36870 by 453.
26. 12012 by 571.
27. 97901 by 693.
28. 98765 by 782.
29. 90123 by 671.
30. 90000 by 457.
31. 79083 by 997.
32. 88771 by 773.
33. 89999 by 679.
34. 60907 by 837.
35. 97901 by 557.
36. 69103 by 973.
37. 865432 by 2171.
38. 796121 by 3331.
39. 600000 by 1085.
40. 888888 by 2133.
41. 100000 by 3657.
42. 1000000 by 2401.
43. 4763210 by 4913.
44. 866555 by 5843.
45. 7663210 by 7613.
46. 8795412 by 3762.
47. 4967123 by 1061.
48. 999999 by 7777.
49. 7663210 by 7612.
50. 36942536 by 4204.
51. 57300652 by 5129.
52. 629348206 by 52312.
53. 730500429 by 61073.
54. 7300400029 by 236421.
55. 8230124037 by 463205.
56. 12345678901 by 456789.

57. A garrison had a million rations, how many days will these last 12500 soldiers?

58. 18144 eggs are to be packed in boxes, each holding 144 eggs; how many boxes are required?

59. What is the smallest number which subtracted from 47630 will make the result divisible by 871 without remainder?

60. What is the smallest number which added to 10,000 will make the result exactly divisible by 888?

61. What number besides 137 will exactly divide 11371?

62. I subtract 1061 from 4967123, then subtract 1061 from the result, and so on; how often can I do this?

37. Division by factors. The result of dividing a number by the product of two or more factors, is the same as the result of dividing the number first by one of the factors, then dividing the quotient by another factor, and so on, till all the factors have been used. [See Art. 29.]

Example. *Divide* 17472 *by* 48.

$$\text{Process,}\quad \begin{array}{r} 8\overline{)17472} \\ 6\overline{)2184} \\ (\textit{Quotient})\ \underline{364} \end{array}$$

Since $48 = 8 \times 6$, we see that if 17472 counters be divided into groups of 8 counters each, and then into sets of groups each containing 6 groups, the sets will each contain 48 counters. That is, the number of the sets will be the result of dividing by 48.

38. The remainder, when one exists, is found thus:

Example. *Divide* 4793 *by* 48.

8)4793 counters
6)599 groups of 8 counters + 1 counter (*remainder*),
99 sets of 48 counters + 5 groups of 8 counters (*remainder*).

$$\text{Process,}\quad \left.\begin{array}{r} 8\overline{)4793} \\ 6\overline{)599},\ \textit{rem.}\ 1 \\ (\textit{Quotient})\ \underline{99},\ \textit{rem.}\ 5 \end{array}\right\}\ \textit{rem.} = 5 \times 8 + 1 = 41\ (\textit{remainder}).$$

EXAMPLES. XI.

In the following examples, use factors not greater than 12 :—

1. $33008 \div 16$.
2. $12300 \div 25$.
3. $5439 \div 49$.
4. $12771 \div 27$.
5. $25830 \div 35$.
6. $34104 \div 56$.
7. $452160 \div 64$.
8. $388575 \div 55$.
9. $2953216 \div 128$.
10. $12150 \div 450$.
11. $443520 \div 840$.
12. $1554768 \div 216$.

Find the quotient and remainder, as in Art. 38,—

13. $159990 \div 144$.
14. $33788 \div 168$.
15. $58420 \div 135$.
16. $49102 \div 81$.
17. $687310 \div 132$.
18. $47631 \div 144$.
19. $876312 \div 240$.
20. $76543 \div 108$.
21. $43210 \div 99$.
22. $46100 \div 49$.
23. $100000 \div 168$.
24. $63844 \div 135$.
25. $135791 \div 125$.
26. $479102 \div 165$.
27. $975319 \div 616$.
28. $34567 \div 847$.
29. $109723 \div 729$.
30. $469750 \div 256$.
31. $97496 \div 343$.
32. $40975 \div 875$.
33. $818191 \div 462$.
34. $703707 \div 315$.
35. $45678 \div 1728$.
36. $98765 \div 1331$.

*MISCELLANEOUS EXAMPLES. XII.

1. What number must be added to 1368 to make 3145?

2. What number subtracted from 7000 leaves 4512?

3. What number multiplied by 85 will produce 7905?

4. What number must be divided by 63 that the quotient may be 99?

5. If the divisor is 84 and the quotient 111, what is the dividend?

6. If the quotient is 101, the divisor 36, and the remainder 27, what is the dividend?

7. If the dividend is 441, the remainder 4, and the quotient 23, what is the divisor?

8. How many times can 461 be subtracted from 57820? and what is the remainder?

9. What is the least number which must be subtracted from 4762 that the remainder may be exactly divisible by 93?

10. Find the least number which added to 5760 will make the sum exactly divisible by 101.

11. The sum of three numbers is 4761; the first is 1792, the second is 367 less than the first; what is the third?

12. A war-ship has provisions for 645 men for 80 days. How long would they last one man?

13. A farmer sold 32 sheep at $15 a head and 9 cows for $55 a head; how much more did the cows cost than the sheep?

14. What is the value of 1267 acres of land at $255 an acre?

15. A man having 75 lambs sold 50 of them at $8 apiece and the rest at $9 apiece; how many dollars did he get for them all?

16. A farmer sent 12 loads of wheat to market, each load contained 12 sacks and each sack 3 bushels, and each bushel sold for $5; how many dollars did they fetch?

17. A man owns 7 orchards, each containing 7 rows of trees, and each row containing 7 trees; how many trees has he?

18. A farmer sold 6 cows at $125, 150 bushels of wheat at $10, and 75 barrels of apples at $20, and laid out his money in land at $250 an acre; how many acres did he buy?

19. How many sheep at $15 a head must be given for 37 cows at $105 each?

20. A man bought two farms, one containing 175 acres at $140 per acre, and the other containing 320 acres at $185 per acre; what was the cost of the two together?

21. A man receives $1500 a year salary, and pays $455 a year for board, $345 a year for clothing, $185 a year for books, and $235 a year for other expenses; what can he save in 5 years?

22. There are in the United States 1,922,890,880 acres of land, and of this in 1850 there were 1,449,075 farms each containing, on an average, 203 acres; how many acres were left uncultivated?

23. Each of the 1,449,075 farms in question 22 was valued at $2258, and upon each farm the implements were valued at $105; how many dollars were all the farms and implements worth taken together?

24. A man bought 127 cows at $17 each, and 97 at $23 each, and sold the whole at $21 each; how many dollars did he gain or lose?

25. The earnings of a man and his two sons amount to $18300 a year, and their expenses to $7950; if the balance be divided equally what will each receive?

26. How many pounds of sugar at 8 cents a pound must be given for 488 lbs. of coffee at 22 cents a pound?

27. A farmer gave 40 loads of hay for 120 tons of coal worth $5 per ton; what did he get for a load of hay?

28. A man bought 96 apples at the rate of 4 for 6 cents, and exchanged them for pears at 8 cents apiece; how many pears did he receive?

29. A number of cows at $190 each and twice the same number of sheep at $25 each are sold for $24000; find the number.

30. Find the value of 20 dozen of wine at $3 per bottle; and if 8 bottles are spoiled, what will be gained by selling the remainder at $4 per bottle?

31. A father left his eldest son $5000 more than his second son; and he left his second son $3000 more than his third; his property was $20000; what did each son receive?

32. A man buys 2800 fives-balls at the rate of 28 for $1, and sells them at the rate of 48 for $3; how many dollars did he gain?

33. A man living at the rate of $3500 a year for 6 years finds that he is exceeding his income, and reduces his expenditure to $2500 a year; at the end of 4 years he finds that he is just out of debt; what is his income?

34. In a game of cricket *A*, *B*, and *C* together score 197 runs, *B* and *C* together score 90 runs, and *A* and *C* together score 120 runs; find the number of runs scored by each.

35. A grocer wishes to put 240 pounds of tea into three kinds of boxes, containing respectively 5, 10, and 15 pounds, and wishes to use the same number of boxes of each size; how many boxes will be required?

36. A man sold a number of chests of tea for $2800 which cost him $2100, gaining thereby $10 per chest; what did each chest cost him?

37. Divide 900 dollars among three persons so that for every three dollars given to the first, the second may get 5 dollars and the third 7 dollars.

38. Prove that if we multiply a number by 4 and divide the result by 100, we obtain the result of dividing by 25.

39. Prove that if we multiply a number by 8 and divide the result by 1000, we obtain the result of dividing by 125.

40. By what number must we divide a given number to obtain the same result as when the given number is multiplied by 2 and divided by 70?

41. Prove that if we multiply a given number by 4 and divide the result by 900, we obtain the result of dividing the given number by 225.

42. Prove that if we multiply a given number by 100 and divide the product by 4, we obtain the product of the given number by 25.

43. Prove that if we multiply a given number by 1000 and divide the product by 8, we obtain the result of multiplying by 125.

44. Prove that if we multiply a given number by 12, the product by 1200 and the last product by 1200 and add the three results, we obtain the result of multiplying by 17294412.

45. Obtain the product of 412345 by 729819 by three lines of multiplication.

46. Obtain the product of 123123 by 125255 by three lines of multiplication.

47. Prove that 13 times 11 = 11 times 13.

CHAPTER II

THE DECIMAL SYSTEM OF NUMERATION AND NOTATION (*continued*)

39. Suppose the number of inches in the length of a given piece of wood to be required; it is very unlikely that a *whole number* of inches will measure the length *exactly.*

40. Suppose we wish to record as exactly as possible the length of a line and to express it in inches; in such a case it is clearly necessary to have some method of conveniently expressing any part of an inch we wish.

41. Such a method is conveniently based on the number **ten**, because the new system then forms a **continuous** system with the already established decimal notation, as will presently appear.

NOTE.—The student is advised to compare Arts. 1 to 13 with what follows.

42. The following *DEFINITIONS* are necessary to complete those of Art. 6.

The number **tenth,** applied to the unit, indicates that quantity of which *ten* make up the unit.

Thus *ten* of (the **tenth** of the unit) = the unit.

The number **hundredth,** applied to the unit, indicates that quantity of which *ten* make up the tenth of the unit.

The number **thousandth,** applied to the unit, indicates that quantity of which *ten* make up the hundredth of the unit,

and there are in succession

34. In a game of cricket *A*, *B*, and *C* together score 197 runs, *B* and *C* together score 90 runs, and *A* and *C* together score 120 runs; find the number of runs scored by each.

35. A grocer wishes to put 240 pounds of tea into three kinds of boxes, containing respectively 5, 10, and 15 pounds, and wishes to use the same number of boxes of each size; how many boxes will be required?

36. A man sold a number of chests of tea for $2800 which cost him $2100, gaining thereby $10 per chest; what did each chest cost him?

37. Divide 900 dollars among three persons so that for every three dollars given to the first, the second may get 5 dollars and the third 7 dollars.

38. Prove that if we multiply a number by 4 and divide the result by 100, we obtain the result of dividing by 25.

39. Prove that if we multiply a number by 8 and divide the result by 1000, we obtain the result of dividing by 125.

40. By what number must we divide a given number to obtain the same result as when the given number is multiplied by 2 and divided by 70?

41. Prove that if we multiply a given number by 4 and divide the result by 900, we obtain the result of dividing the given number by 225.

42. Prove that if we multiply a given number by 100 and divide the product by 4, we obtain the product of the given number by 25.

43. Prove that if we multiply a given number by 1000 and divide the product by 8, we obtain the result of multiplying by 125.

44. Prove that if we multiply a given number by 12, the product by 1200 and the last product by 1200 and add the three results, we obtain the result of multiplying by 17294412.

45. Obtain the product of 412345 by 729819 by three lines of multiplication.

46. Obtain the product of 123123 by 125255 by three lines of multiplication.

47. Prove that 13 times 11 = 11 times 13.

CHAPTER II

THE DECIMAL SYSTEM OF NUMERATION AND NOTATION (*continued*)

39. Suppose the number of inches in the length of a given piece of wood to be required; it is very unlikely that a *whole number* of inches will measure the length *exactly*.

40. Suppose we wish to record as exactly as possible the length of a line and to express it in inches; in such a case it is clearly necessary to have some method of conveniently expressing any part of an inch we wish.

41. Such a method is conveniently based on the number **ten**, because the new system then forms a **continuous** system with the already established decimal notation, as will presently appear.

NOTE.—The student is advised to compare Arts. 1 to 13 with what follows.

42. The following *DEFINITIONS* are necessary to complete those of Art. 6.

The number **tenth**, applied to the unit, indicates that quantity of which *ten* make up the unit.

Thus *ten* of (the **tenth** of the unit) = the unit.

The number **hundredth**, applied to the unit, indicates that quantity of which *ten* make up the tenth of the unit.

The number **thousandth**, applied to the unit, indicates that quantity of which *ten* make up the hundredth of the unit,

and there are in succession

34. In a game of cricket *A*, *B*, and *C* together score 197 runs, *B* and *C* together score 90 runs, and *A* and *C* together score 120 runs; find the number of runs scored by each.

35. A grocer wishes to put 240 pounds of tea into three kinds of boxes, containing respectively 5, 10, and 15 pounds, and wishes to use the same number of boxes of each size; how many boxes will be required?

36. A man sold a number of chests of tea for $2800 which cost him $2100, gaining thereby $10 per chest; what did each chest cost him?

37. Divide 900 dollars among three persons so that for every three dollars given to the first, the second may get 5 dollars and the third 7 dollars.

38. Prove that if we multiply a number by 4 and divide the result by 100, we obtain the result of dividing by 25.

39. Prove that if we multiply a number by 8 and divide the result by 1000, we obtain the result of dividing by 125.

40. By what number must we divide a given number to obtain the same result as when the given number is multiplied by 2 and divided by 70?

41. Prove that if we multiply a given number by 4 and divide the result by 900, we obtain the result of dividing the given number by 225.

42. Prove that if we multiply a given number by 100 and divide the product by 4, we obtain the product of the given number by 25.

43. Prove that if we multiply a given number by 1000 and divide the product by 8, we obtain the result of multiplying by 125.

44. Prove that if we multiply a given number by 12, the product by 1200 and the last product by 1200 and add the three results, we obtain the result of multiplying by 17294412.

45. Obtain the product of 412345 by 729819 by three lines of multiplication.

46. Obtain the product of 123123 by 125255 by three lines of multiplication.

47. Prove that 13 times 11 = 11 times 13.

CHAPTER II

THE DECIMAL SYSTEM OF NUMERATION AND NOTATION (*continued*)

39. Suppose the number of inches in the length of a given piece of wood to be required; it is very unlikely that a *whole number* of inches will measure the length *exactly.*

40. Suppose we wish to record as exactly as possible the length of a line and to express it in inches; in such a case it is clearly necessary to have some method of conveniently expressing any part of an inch we wish.

41. Such a method is conveniently based on the number **ten**, because the new system then forms a **continuous** system with the already established decimal notation, as will presently appear.

NOTE.—The student is advised to compare Arts. 1 to 13 with what follows.

42. The following *DEFINITIONS* are necessary to complete those of Art. 6.

The number **tenth,** applied to the unit, indicates that quantity of which *ten* make up the unit.

Thus *ten* of (the **tenth** of the unit) = the unit.

The number **hundredth,** applied to the unit, indicates that quantity of which *ten* make up the tenth of the unit.

The number **thousandth,** applied to the unit, indicates that quantity of which *ten* make up the hundredth of the unit,

and there are in succession

34. In a game of cricket *A*, *B*, and *C* together score 197 runs, *B* and *C* together score 90 runs, and *A* and *C* together score 120 runs; find the number of runs scored by each.

35. A grocer wishes to put 240 pounds of tea into three kinds of boxes, containing respectively 5, 10, and 15 pounds, and wishes to use the same number of boxes of each size; how many boxes will be required?

36. A man sold a number of chests of tea for $2800 which cost him $2100, gaining thereby $10 per chest; what did each chest cost him?

37. Divide 900 dollars among three persons so that for every three dollars given to the first, the second may get 5 dollars and the third 7 dollars.

38. Prove that if we multiply a number by 4 and divide the result by 100, we obtain the result of dividing by 25.

39. Prove that if we multiply a number by 8 and divide the result by 1000, we obtain the result of dividing by 125.

40. By what number must we divide a given number to obtain the same result as when the given number is multiplied by 2 and divided by 70?

41. Prove that if we multiply a given number by 4 and divide the result by 900, we obtain the result of dividing the given number by 225.

42. Prove that if we multiply a given number by 100 and divide the product by 4, we obtain the product of the given number by 25.

43. Prove that if we multiply a given number by 1000 and divide the product by 8, we obtain the result of multiplying by 125.

44. Prove that if we multiply a given number by 12, the product by 1200 and the last product by 1200 and add the three results, we obtain the result of multiplying by 17294412.

45. Obtain the product of 412345 by 729819 by three lines of multiplication.

46. Obtain the product of 123123 by 125255 by three lines of multiplication.

47. Prove that 13 times 11 = 11 times 13.

CHAPTER II

THE DECIMAL SYSTEM OF NUMERATION AND NOTATION (*continued*)

39. Suppose the number of inches in the length of a given piece of wood to be required; it is very unlikely that a *whole number* of inches will measure the length *exactly.*

40. Suppose we wish to record as exactly as possible the length of a line and to express it in inches; in such a case it is clearly necessary to have some method of conveniently expressing any part of an inch we wish.

41. Such a method is conveniently based on the number **ten**, because the new system then forms a **continuous** system with the already established decimal notation, as will presently appear.

NOTE.—The student is advised to compare Arts. 1 to 13 with what follows.

42. The following *DEFINITIONS* are necessary to complete those of Art. 6.

The number **tenth,** applied to the unit, indicates that quantity of which *ten* make up the unit.

Thus *ten* of (the **tenth** of the unit) = the unit.

The number **hundredth,** applied to the unit, indicates that quantity of which *ten* make up the tenth of the unit.

The number **thousandth,** applied to the unit, indicates that quantity of which *ten* make up the hundredth of the unit,

and there are in succession

34. In a game of cricket A, B, and C together score 197 runs, B and C together score 90 runs, and A and C together score 120 runs; find the number of runs scored by each.

35. A grocer wishes to put 240 pounds of tea into three kinds of boxes, containing respectively 5, 10, and 15 pounds, and wishes to use the same number of boxes of each size; how many boxes will be required?

36. A man sold a number of chests of tea for $2800 which cost him $2100, gaining thereby $10 per chest; what did each chest cost him?

37. Divide 900 dollars among three persons so that for every three dollars given to the first, the second may get 5 dollars and the third 7 dollars.

38. Prove that if we multiply a number by 4 and divide the result by 100, we obtain the result of dividing by 25.

39. Prove that if we multiply a number by 8 and divide the result by 1000, we obtain the result of dividing by 125.

40. By what number must we divide a given number to obtain the same result as when the given number is multiplied by 2 and divided by 70?

41. Prove that if we multiply a given number by 4 and divide the result by 900, we obtain the result of dividing the given number by 225.

42. Prove that if we multiply a given number by 100 and divide the product by 4, we obtain the product of the given number by 25.

43. Prove that if we multiply a given number by 1000 and divide the product by 8, we obtain the result of multiplying by 125.

44. Prove that if we multiply a given number by 12, the product by 1200 and the last product by 1200 and add the three results, we obtain the result of multiplying by 17294412.

45. Obtain the product of 412345 by 729819 by three lines of multiplication.

46. Obtain the product of 123123 by 125255 by three lines of multiplication.

47. Prove that 13 times $11 = 11$ times 13.

CHAPTER II

THE DECIMAL SYSTEM OF NUMERATION AND NOTATION (*continued*)

39. Suppose the number of inches in the length of a given piece of wood to be required; it is very unlikely that a *whole number* of inches will measure the length *exactly*.

40. Suppose we wish to record as exactly as possible the length of a line and to express it in inches; in such a case it is clearly necessary to have some method of conveniently expressing any part of an inch we wish.

41. Such a method is conveniently based on the number **ten**, because the new system then forms a **continuous** system with the already established decimal notation, as will presently appear.

NOTE.—The student is advised to compare Arts. 1 to 13 with what follows.

42. The following *DEFINITIONS* are necessary to complete those of Art. 6.

The number **tenth,** applied to the unit, indicates that quantity of which *ten* make up the unit.

Thus *ten* of (the **tenth** of the unit) = the unit.

The number **hundredth,** applied to the unit, indicates that quantity of which *ten* make up the tenth of the unit.

The number **thousandth,** applied to the unit, indicates that quantity of which *ten* make up the hundredth of the unit,

and there are in succession

ten-thousandth;
hundred-thousandth;
millionth;
ten-millionth.
} each of which numbers, applied to the unit, indicates the *tenth* part of that which the preceding number indicates,

and so on as in Art. 6.

Tenth is called the **first negative power of ten,**
hundredth.................**second**..............................
thousandth...............**third**...............................
and so on.

a **hundredth** of the unit = a **tenth** of a **tenth** of the unit;
a **thousandth** of the unit = a **tenth** of a **tenth** of a **tenth** of the unit;
and so on.

NOTE.—Tenths, hundredths, etc., are often used as abbreviations for 'tenth-parts of the unit quantity,' 'hundredth-parts of the unit,' etc.

43. *To explain a complete method of Numeration.*

That is, *to show how any given quantity may be completely described as regards magnitude in terms of a given unit quantity of the same kind.*

Let the given quantity be the length of a given line; and the given unit an inch.

The length being given, it is known how many complete inches it contains; these can be enumerated by Art. 7.

Unless the given length contains this number of inches *exactly*, there must be a *part* of an inch, in the given length, in addition to the number of whole inches.

Suppose for example the line contains thirty-nine inches + some part of an inch.

To enumerate this *part of an inch*, we proceed as follows:—

We first find what number of **tenth** parts of an inch this part contains.

The number will be less than ten, and the remaining portion of the line (if any) will be less than the tenth part of an inch.

Suppose we find that there is one **tenth** of an inch.

We next find what number of **hundredth** parts of an inch the new remaining part contains.

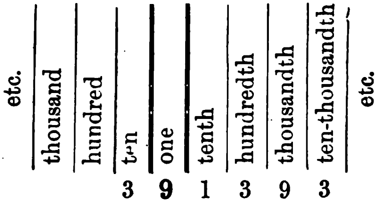
The number will be less than ten, and the remainder (if any) will be less than the hundredth of an inch.

Suppose that there are three **hundredths** of an inch.

And so on, until either there is no remainder, or the remainder is so small that it need not be recorded.

Suppose that there are 9 thousandths of an inch and, finally, 3 ten-thousandths of an inch. The given length is thus recorded as containing 39 inches, and 1 tenth part, 3 hundredth parts, 9 thousandth parts, and 3 ten thousandth parts of an inch.

44. The method of **notation** adopted will be clear from the following diagram :—



The *number* indicated is thirty-nine and one-tenth, three hundredth, nine thousandth, three ten-thousandth.

Since in the above number the last digit on the right hand is not in the one-place, it is necessary to indicate quite clearly where the one-place is.

The method (suggested in the diagram) of indicating the one-place by printing the digit in that place in deeper type—thus: 3**9**1393—would be sufficiently clear when a number is *printed*, but would not be so clear when a number is *written*.

The notation adopted universally is to put a **point** between the one-place and that of the tenth, thus

39·1393.

This point is called the **decimal point.**

EXAMPLES. XIII.

Read and express in words the following numbers :—

1. 1·5. **2.** 36·75. **3.** 100·001. **4.** ·0006.
5. 1·1007. **6.** 404·0404. **7.** 100·0001. **8.** 5·00505.

Express in figures :—

9. One tenth four hundredths. **10.** One millionth.
11. Fifty, and one thousandth. **12.** 9 ten-thousandths.
13. Fifty-one, and one hundredth.
14. Ten thousand, and one thousandth.
15. One hundred and fifty-four, four tenths and six thousandths.
16. One thousand and seventy-nine, nine tenths six hundredths.
17. Six million and twenty, one tenth nine hundredths.

45. In a number expressed in the above notation, the digit in the 'one-place' is multiplied by 1, that is, by 'the zero power of ten.' [Art. 6.]

Every other digit in the expression is understood to be multiplied by some power of ten, 'positive' or 'negative,' according to its **locality**.

Example. In the expression 4103·7506 the 3 indicates three; the 4 indicates 4 times a thousand; the 6 indicates 6 times a ten-thousandth; and so on.

46. The student should notice the arrangement of the places of the **powers of ten** in the diagram of Art. 44, thus [see also Art. 201] :—

etc.	4th...	3rd...	2nd...	1st positive	zero power	1st negative	2nd...	3rd...	4th...	etc.

We may notice that the **negative** powers of ten are so called because they are represented in the notation of vulgar fractions, thus :—

The zero **power of ten** $=\frac{1}{1}$.

the 1st positive power $=10$	1st negative power $=\frac{1}{10}$
2nd ,, ,, $=10\times10$	2nd ,, ,, $=\frac{1}{10\times10}$
3rd ,, ,, $=10\times10\times10$	3rd ,, ,, $=\frac{1}{10\times10\times10}$
and so on.	and so on.

47. The **continuity** of this system of notation is most important.

It will be observed, that **the moving a digit from any place whatever to the next place on its left multiplies its value by 10.**

Also, that **the moving a digit from any place whatever to the next place on its right divides its value by 10.**

Example i. ·3 = ten times ·03.
30· = ten times 3·
Example ii. *Multiply* 39·1393 *by* 10.
Result, 391·393.
Example iii. *Divide* 39·1393 *by* 10.
Result, 3·91393.
Example iv. A line which contains
39·1393 inches may be expressed
as containing 391·393 tenths-of-an-inch,
or as containing 3·91393 tens-of-inches,
or as containing ·0391393 thousands-of-inches,
or as containing 39139·3 thousandths-of-an-inch,
and so on.

EXAMPLES. XIV.

1. Write down numbers which are ten times the following numbers respectively :—

(i) 3. (ii) 30. (iii) ·3. (iv) ·003.
(v) ·016. (vi) 50·1. (vii) 1·3. (viii) ·000039.
(ix) 31·042. (x) 400·01. (xi) 30·303. (xii) 410·02.

2. Write down numbers which are a tenth of the above numbers respectively.

3. Write down numbers which are a thousand times the above numbers respectively.

4. Write down numbers which are a thousandth part of the above numbers respectively.

5. How many hundredth parts of an inch are there in the following ?—

(i) 20 in. (ii) 3·1 in. (iii) ·0412 in. (iv) ·00016 in.

6. How many thousand-inches are there in the following ?—

(i) 4312·1 in. (ii) 1·012 in. (iii) 34·12 tens-of-inches.
(iv) 50000 tenths-of-an-inch. (v) 5·0125 millions-of-inches.

7. How many tens-of-inches are there in the following ?—

(i) 4510 in. (ii) 37·56 in. (iii) 467·5 tenths-of-an-inch.
(iv) 456·25 thousands-of-inches. (v) 567·5 thousandths-of-an-inch.

ADDITION OF DECIMALS.

48. The process of **Addition**, explained in Art. 17, applies to any numbers, whole or fractional, expressed in the Decimal notation; the process only requires, that the moving a digit one place to the left shall imply a tenfold increase in its value. [Art. 47.]

Example. *Add together* 49·375, 1001·25, 407·7125.

Process, 49·375
1001·25
407·7125
Result, 1458·3375

The different powers of ten are placed in the same vertical line, and the addition is performed as on page 11.

EXAMPLES. XV.

Add together—

1. 3·406, 45·372, 300, 4·7561.
2. 4·01, ·003, 29, 1·2345.
3. 44·04, 3650, 4·12305, ·000375.
4. ·012, 32·1, 4·12, ·00036.
5. 671·75, ·00003675, 4120, 8·125.
6. ·00023, ·00967, ·00476.
7. 376·15, 40000, 800·01725, ·0125.
8. 39·102, ·046, 471·2.
9. 43·702, 30102·5, ·46125, 3·67003.
10. 8, 70·007, 3·295, ·014.
11. 4·00102 + 370·2 + 4·6003 + ·007945.
12. 376·01 + ·4135 + 6·00795 + 365·141.
13. 7·7171 in. + 3460·1 in. + 4·0106 in. + ·05679 in.
14. ·00412 in. + 3·7056 in. + 64·035 in. + 379·4 in.

49. Since the process of Addition is the same as before, it follows that the processes of **Subtraction,** or **Multiplication by whole numbers,** and of **Division by whole numbers,** are the same. [Arts. 18, 24, 32.]

For, **subtraction** asks a question which is answered by addition.

Multiplication by a whole number is the repetition of addition a certain number of times.

Division by a whole number asks a question which is answered by multiplication.

Example i. *Subtract* 4·12575 *from* 21.

Process, 21·
4·12575
Result, 16·87425

The mental process is as follows:—5 (hundred-thousandths) and 5 (hundred-thousandths) = 10 (hundred-thousandths); put down 5 (hundred-thousandths and carry 1 (ten-thousandth), and so on.

EXAMPLES. XVI.

Subtract—

1. 432·125 from 678·427.
2. 412·865 from 503·125.
3. 36·75 from 40·1.
4. 4·38675 from 5.
5. 100·376 from 451·86975.
6. ·000625 from 30.
7. 3·07625 from 500·8.
8. 473·5 from 1000.
9. ·00061765 from ·001.
10. ·00384752 from ·012345.

Simplify—

11. 300 – 25·475 – ·3625 – 1·75.
12. 4·835 – 2·00125 + 67·4375 – 39·80625.
13. 4500 + 3·125 – 7·63475 – ·0016875 – 1·4005.
14. 436·025 – (7·36125 + 36·0075 + ·0001645).
15. 3000 – (8·462 + 30·0125 – 5·71875).
16. 300 + 20·471 – 3·6742 – 127·36.
17. 5713 – 37·0706 – 3·47777.
18. 1·643 – ·94361 – (30·243 – 30·20975).
19. 3·14159 – ·314159 – ·0314159.
20. 7·1475 – 6·2586 + 1·4321.

Example ii. *Multiply* 41·376 *by* 871.

Process,	41·376 871·		41·376 871·
	33100·8 2896·32 41·376	or, arranging the addition in a different order, see p. 20.	41·376 2896·32 33100·8
Result,	36038·496		36038·496

When any digit is multiplied by the 8, that is by 800, the resulting digit must be placed *two* places to the left of its former place; when any digit is multiplied by the 7, that is by 70, the resulting digit must be placed *one* place to the left of its former; and so on.

NOTE.—The multiplier 871 may be placed anywhere under the multiplicand.

EXAMPLES. XVII.

Multiply—

1. 4·3125 by 5.
2. 36·4875 by 20.
3. 36·5483 by 100.
4. 463·125 by 1000.
5. ·06175 by 3.
6. ·001623 by 8.
7. 34·1275 by 32.
8. 46·0125 by 64.
9. ·001382 by 153.
10. ·01864 by 233.
11. ·01375 by 415000.
12. 4·0123 by 60103.
13. ·0010101 by 1101010.
14. 10·0267 by 301000.

Example iii. *Divide* 41·375 by 8.

Process, 8)41·375
Quotient 5·171875

Dividing as usual, when we come to the thousandths there is a remainder 7, *i.e.* 70 ten-thousandths; so that the process can be continued to any number of decimal places, or until there is no remainder.

Example iv. *Divide* 945·081 *by* 78300.

We first divide 945·081 by 100; the quotient is 9·45081; and then divide 9·45081 by 783, as follows:—

Process, 783)9·45081(·01207
783
1620
1566
5481
5481

or, by the method of Art. 36,
783)9·45081(·01207 *Quotient.*
1620
5481

[The *mental* process is similar to that of Art. 36. We draw a line under 945 *hundredths;* so that the 1, the first digit* obtained for the quotient, indicates 1 *hundredth, i.e.* ·01.]

Division by factors, when possible [Art. 37], is the best process.

Example v. *Divide* 10·0653 *by* 1680.

Process, 3)1·00653
7)·33551
8)·04793
Quotient ·00599125

First divide by 10 and notice that 168 = 3 × 7 × 8.

EXAMPLES. XVIII.

Divide—

1. 3·175 by 2.
2. 41·325 by 8.
3. ·04321 by 80.
4. 341 by 2.
5. 1410 by 8.
6. 463 by 100.
7. 3670 by 16.
8. 1·69 by 13.
9. ·00169 by 130.
10. ·022015 by 350.
11. 457·71 by 240.
12. 8·547 by 132.
13. ·06227 by 3200.
14. ·79434 by 225.
15. 95·018 by 385.
16. ·0009197 by 1360.
17. 49985 by 104.
18. 71·71 by 284.
19. ·06001 by 425.
20. ·04009 by 1520.
21. ·001007 by 47500

Divide, giving the quotient to the sixth negative power of ten,—

22. 360·1 by 3.
23. 4·21 by 3.
24. 5·94 by 6.
25. 13·75 by 12.
26. 40·125 by 21.
27. 4000 by 113.
28. 100 by 9.
29. 41·362451 by 115.
30. ·041326 by 101.

---

* This first digit is always in the same **place** as the *last digit on the right,* of those used in the dividend in obtaining this first digit.

50. To **multiply** a number, say 41·376, **by a decimal fraction,** say 871·47, we must perform on 41·376 that operation which performed on *one* gives 871·47 as result. [Art. 25.]

Example. Multiply 41·376 *by* 871·47.

The process is similar to that of Ex. ii. p. 37.

41·376		41·376
871·47		871·47
33100·8	or, arranging	2·89632
2896·32	the addition	16·5504
41·376	in a different	41·376
16·5504	order,	2896·32
2·89632		33100·8
36057·94272		36057·94272

To multiply by ·4, we multiply by 4 in the ordinary way and move each digit *one* place to the *right*, that is, we divide the product by 10; for ·4 = 4 tenths and the product of 41·376 by 4 is 10 times too great. So to multiply by ·07, we multiply by 7, moving each digit *two* places to the right; and so on* [see Art. 47]. [Of the two arrangements above the one on the left is preferable. See the chapter on Approximation.]

EXAMPLES. XIX.

Multiply—

1. 41·2 by 1·2. **2.** 307·05 by 11·1. **3.** 59·04 by ·002.
4. 3·0501 by ·001. **5.** ·0123 by ·0123. **6.** 30·0065 by 3·0303.
7. ·000115 by ·00032. **8.** 46000 by ·000123. **9.** 4070·3 by 3·0105.
10. 45·675 by ·00128. **11.** 3·6125 by ·0512. **12.** 4225000 by ·00512.
13. ·000625 by 12·8. **14.** 6400 by ·000125. **15.** ·00064 by ·125.
16. ·128 by ·036125. **17.** ·000123 by ·123. **18.** ·043 × ·0043 × 43000.
19. 1·21 × ·121 × ·0121. **20.** ·0003 × ·003 × ·03. **21.** 40·4 × 3·03 × ·202.
22. 300 × ·0031 × ·21. **23.** 1000 × ·0006 × 9. **24.** 4·44 × ·031 × ·0001.

---

* The *process* is the same as the multiplying 41376 by 87147; this product is in this case 1000 times 100 times too great; the required result is therefore obtained by dividing 41376 × 87147 by 100000. Hence the usual **RULE. To multiply two decimals, multiply the numbers as if there were no decimal point and in the product mark off as many decimal places as there are in the two decimals together.**

51. To **divide** a number, say 3·762505, **by a decimal fraction,** say ·0785, we must find a number which, when multiplied by ·0785, gives 3·762505 as product. [Art. 34.]

Example. Divide 3·762505 *by* ·0785.

That is, find a number which when multiplied by 785 ten-thousandths, will give as product 3·762505.

Therefore we may multiply 3·762505 by 10000 and divide the product by 785 ; that is, we may divide 37625·05 by 785 ; thus

```
Process,  785)37625·05(47·93   |  or, by the method of Art. 36,
              3140             |       785)37625·05(47·93
              ----             |           6225
               6225            |           ----
               5495            |            7300
               ----            |            ----
                7300           |             2355
                7065           |
                ----           |
                 2355          |
                 2355          |
```

Hence the usual **RULE. To divide by a decimal, multiply both divisor and dividend by that** power of ten **which will make the divisor a whole number; then divide the new dividend by this whole number.** [N.B.—Divide by factors when possible, see Ex. v, p. 38.]

EXAMPLES. XX.

Divide—

1. 3·461 by ·02.
2. 4·6125 by ·004.
3. 450 by ·0012.
4. 8·454 by 240000.
5. 3·625 by ·001.
6. 41·5 by ·0001.
7. ·3125 by ·01.
8. ·000079 by ·00001.
9. 3·672 by ·016.

Divide, finding the quotient as far as the sixth decimal place,—

10. 4·375 by 1·26.
11. ·012345 by 1·001.
12. 40 by ·03125.
13. 4000 by 401·02.
14. ·003 by 32·5.
15. 100 by ·0006.
16. 3·413682 by 49·1.
17. 411·0125 by 1·023.
18. ·456789 by ·0123.
19. 456·8 by 3060·125.
20. ·001 by ·0000431.
21. 4·04 by ·00071.
22. 30 by ·00000421.
23. 4444·4 by 36500.
24. 156·25÷·25.
25. 15÷6·25.
26. ·0345÷·543.
27. 4·116÷·00075.
28. 40001÷·0003.
29. 1234·5678÷·031.
30. ·333333÷·004.
31. ·000144÷·012.
32. 1·0665÷·00135.
33. 418·25÷·128.
34. 3·84006÷1230.
35. 61·450824÷102·01.
36. ·37625÷·0785.
37. ·00015129÷·0123.
38. ·18496÷·0512.
39. 216·32÷·00512.
40. ·8÷·000125.
41. ·008÷12·8.
42. 7·9507÷·0043.
43. ·003624÷·036125.
44. 1953÷·0031.
45. 216·32÷·4225.
46. 5658÷·00123.
47. ·11808÷·111.
48. 49440÷·0131.

MISCELLANEOUS EXAMPLES. XXI.

1. Find the difference between the product and the quotient of 3·125 by ·64.

2. Multiply 72·9 by itself in two lines.

3. Multiply 4156371 by 216·728 in 3 lines.

4. Multiply 345·21 by 12·1911, using only 3 lines of multiplication.

5. Multiply ·047321 by 1728·14412, using only 3 lines of multiplication.

6. How many times can ·013 be subtracted from 125·78, and what is the remainder?

7. What must 1562·5 be divided by to give 6250000 as quotient?

8. Find the rent of 42·75 acres at $6.75 per acre.

9. A roller 4·12 ft. in circumference makes 35·2 revolutions in passing from one end of a lawn to another; what is the length of the lawn?

10. *A* is ·875 times as old as *C*, and *C* is 1·08 times as old as *B*; *B* is 25; how old is *A*?

11. A man gave his daughter $5.26, and to each of his two sons five times as much. How much did he give in all?

12. A man divides $15.78 evenly among his three children; how much does each receive?

13. I buy 72 yards of carpet, for which I pay $1.25 a yard. How much does my carpet cost?

14. A workman earns $2.50 a day; how much does he earn in a week (*i.e.* in six working days)?

15. A boy's father gives him $1 a week; the boy divides this into four equal parts; one part he saves, one he gives away, two he spends. How much does he save in a year (*i.e.* in 52 weeks)? how much does he spend? and how much does he give away?

16. On Monday I receive $31, on Tuesday I spend $6.52, on Wednesday $3.27, on Thursday $1.16, on Friday $13.15, and on Saturday $3. How much have I left?

CHAPTER III

PRIME FACTORS

52. It is often important to know what numbers, if any, will exactly divide a given number; in other words, to know what are the **factors** of a given number [Art. 27.] [In the present chapter *divide* is used in the sense of *exactly divide*, that is *divide without a remainder.*]

Example. 143 is divisible by 11 and by 13; hence 11 and 13 are **factors** of 143.

In what follows, when we speak of *factors*, the number *one* (which may be said to be a factor of every number) is excluded.

53. *DEF.* A **prime number** is a number which has *no* factors (except *itself* and *one*). A **prime** is often used as an abbreviation for a *prime number.*

2, 3, 5, 7, 11, 13, 17, 19, 23, etc., are *prime* numbers.

Every number which is not a prime number, is called a **composite** number.

Example. 6, 8, 12 are *composite* numbers.

For their factors are 2 × 3; 2 × 2 × 2; 2 × 2 × 3 respectively.

Example. Is 433 *a prime or a composite number?*

That is: *Is* 433 *divisible by any number greater than* 1 *and less than* 433?

We obtain the answer to this question by *trial.*

It is not necessary to try any but *prime* numbers; for when a number is divisible by 6 it is divisible by 2, and also by 3.

By dividing 433 by 2, 3, 5, 7, 11, 13, 17, 19 in succession we find that there is a remainder in each case.

It is not necessary to try any higher number than 19; for the quotient in the division by 19 is less than 23; and therefore if 433 were divisible by 23 or by any number greater than 23, the quotient would be less

than 19, and 433 would be divisible by a number less than 19; which is not the case.

Hence, 433 has no exact divisor, and is therefore a *prime*.

54. The **prime factors** of a composite number are the prime numbers whose product is that number.

Example i. 4 and 3 are *factors* of 12; so also is 6; for 4 × 3 = 12 and 2 × 6 = 12; they are not the prime factors.

The *prime* factors of 12 are 2, 2, 3;
for 2 and 3 are each a prime, and 2 × 2 × 3 = 12.

Example ii. *Is* 473 *a prime?* If not, *find its prime factors.*

By trial we find that 473 is not divisible by 2, 3, 5, or 7; but that it is divisible by 11, the quotient being 43. By trial we find that 43 is a prime. Hence the prime factors of 473 are 11 and 43.

55. A number can be expressed as the product of different sets of factors when the factors are not *all* prime.

Example. 12 is 2 × 6, or 3 × 4. Here 6 is not a prime, neither is 4.

A number cannot be expressed as the product of different sets of factors which are all prime.

This is a very important proposition; its truth may easily be verified by trial in any particular case.

Example i. 12 cannot be expressed as the product of any other *prime* factors except 2 × 2 × 3.

Example ii. *Find the prime factors of* 4095.

4095 is divisible by 5, 5)4095
and by 3, 3)819
and by 3, 3)273
and by 7, 7)91
and by 13, 13

Therefore the prime factors of 4095 are 5 × 3 × 3 × 7 × 13.

EXAMPLES. XXII.

Find the prime factors of each of the following numbers:—

1. 6.
2. 10.
3. 14.
4. 35.
5. 9.
6. 8.
7. 12.
8. 24.
9. 27.
10. 36.
11. 90.
12. 84.
13. 100.
14. 121.
15. 144.
16. 132.
17. 112.
18. 51.
19. 117.
20. 495.
21. 1050.
22. 2520.
23. 6480.
24. 42600.
25. 1760.
26. 3650.
27. 1017.
28. 3432.
29. 3575.
30. 1690.
31. 114400.
32. 100100.

*EXAMPLES. XXIII.

Determine which of the following numbers are prime, and find the prime factors of those which are composite :—

1. 37. **2.** 93. **3.** 101. **4.** 111. **5.** 103.
6. 107. **7.** 113. **8.** 119. **9.** 207. **10.** 227.
11. 239. **12.** 349. **13.** 299. **14.** 169. **15.** 269.
16. 469. **17.** 499. **18.** 503. **19.** 507. **20.** 509.
21. 2197. **22.** 2327. **23.** 4573. **24.** 3029. **25.** 619.
26. 4393. **27.** 7111. **28.** 7771. **29.** 997. **30.** 713.
31. 1363. **32.** 9959. **33.** 6539. **34.** 1793. **35.** 581.

56. One number is said to be a **multiple** of another, when the first is some number of times the second. A multiple of a number is exactly divisible by that number.

*57. The following **principles** are of great importance.

The student must carefully bear in mind what is meant by **division** [Art. 31]. A number, say 630, is *divisible* exactly by another number, say 7, when 630 counters can be arranged in groups of 7.

I. When each of two numbers is divisible by a third number, then the sum (and the difference) of the two numbers is also divisible by the third.

For, the sum (and the difference) of two sets of counters, each set arranged in groups of 7, can also be arranged in groups of 7.

Example. $7 \times 6 + 7 \times 5 = 7 \times 11$.

II. When a number is exactly divisible by another number, then any multiple of the first number is also divisible by the second.

For, any number of times a set of groups of 7 counters can also be arranged in groups of 7. *Example.* 3 times 42 is divisible by 7.

III. When each of two numbers is divisible by a third, then the sum (and the difference) of any multiple of the first and any multiple of the second is also divisible by the third number.

For the two multiples are *each* divisible by the number [by II.]; therefore their sum is also divisible by the same number [by I.].

Example. (3 times 42 + 4 times 35) is divisible by 7.

*58. *DEF.* An **even** number is a number divisible by 2. An **odd** number is not divisible by 2.

*59. Every number whose last digit is divisible by 2 is itself divisible by 2.

For it consists of (some multiple of 10 + some multiple of 2), *i.e.* it is the sum of two numbers each divisible by 2 [Art. 57, I.]

Every number whose last digit is divisible by 5 is itself divisible by 5.

Example. 75 is divisible by 5.

For it consists of (some multiple of 10 + some multiple of 5).

Every number whose last two digits represent a number divisible by 4 (or by any other factor of 100) is itself divisible by that factor.

For it consists of (some multiple of 100 + some multiple of 4), *i.e.* it is the sum of two numbers each of which is divisible by 4 [Art. 57, I.] *Example.* 532 is divisible by 4, since 32 is divisible by 4.

Every number whose last three digits represent a number divisible by 8 (or by any other factor of 1000) is itself divisible by that factor.

Example. 1160 is divisible by 8, since 160 is divisible by 8.

*60. A number is divisible by 9 when the sum of its digits is divisible by 9, and not otherwise.

For let the number be 875426. Then, since

100000 − 1, or 99999, is divisible by 9, therefore 800000 − 8 is so too.
10000 − 1, or 9999, ... 9, ... 70000 − 7 ....
1000 − 1, or 999, ... 9, ... 5000 − 5 ....
100 − 1, or 99, ... 9, ... 400 − 4 ....
10 − 1, or 9, ... 9, ... 20 − 2 ....

Hence by addition [Art. 57, I.] $875426-(8+7+5+4+2+6)$ is divisible by 9.

Hence, 875426 when divided by 9 gives the same remainder, as $8+7+5+4+2+6$ when divided by 9.

Thus $8+7+5+4+2+6=32=3\times 9+5$; therefore 875426 divided by 9 also gives remainder 5.

In the same way it can be shown that the difference between *any* number and the sum of its digits, is always divisible by 9.

Hence, when the sum of the digits is divisible by 9, the number must be so too, and not otherwise [Art. 57, I.]

***61. Similarly it may be shown that a number is divisible by 3 when the sum of its digits is divisible by 3, and not otherwise.**

When the sum of the digits of an *even* number is divisible by 3, the number itself is divisible by 6.

When the sum of the digits of a number is divisible by 3, and its last two digits are divisible by 4, the number itself is divisible by 12.

***62. A number is divisible by 11 when the difference between the sum of its first, third, fifth, etc., digits and the sum of its second, fourth, sixth, etc., digits is divisible by 11, and not otherwise.**

For we can prove that any number + the above-described difference is always divisible by 11. Take any number 875426. Then, since

$100000 + 1$, or $99990 + 11$, is divisible by 11, therefore $800000 + 8$ is so too.
$10000 - 1$, or 9999, ... 11, ... $70000 - 7$ ....
$1000 + 1$, or $990 + 11$, ... 11, ... $5000 + 5$ ....
$100 - 1$, or 99, ... 11, ... $400 - 4$ ....
$10 + 1$, or 11, ... 11, ... $20 + 2$ ....

Hence by addition [Art. 57, I.] $875426 + 8 - 7 + 5 - 4 + 2 - 6$ is divisible by 11.

Hence, 875426 when divided by 11 gives the same remainder as $7 + 4 + 6 - (8 + 5 + 2)$ when divided by 11; that is, they both give remainder 2.

*EXAMPLES. XXIV.

State which of the following numbers are divisible by 5, or by 4, or by 8, or by 9, or by 11.

1. 7160.
2. 8190.
3. 745327.
4. 7776.
5. 57145.
6. 123456.
7. 4,732,101.
8. 111,111,111.

9. Prove that when a given number is divisible by 9, any other number expressed by the *same digits* is divisible by 9.

10. What is the least number which added to 77893 will make the sum divisible (i) by 8, (ii) by 9, (iii) by 11?

11. Prove that when the sum or the difference of two numbers is a multiple of a third number, and one of them is divisible by that number, the other is so too.

12. When the sum of the digits of two numbers is divisible by 9, then if one of the numbers is divisible by 9, the other is so too.

HIGHEST COMMON FACTOR.

63. A Factor which exactly divides each of two or more numbers is called a **Common** Factor of those numbers.

Example. 3 is a common factor of 6, 12, and 18.

The **Highest Common Factor** of two or more numbers is the highest number which exactly divides each of those numbers. The three letters **H.C.F.** stand for Highest Common Factor.

Example. 6 is the H.C.F. of 6, 12, and 18.

64. When numbers have been expressed as the product of their *prime factors*, their H.C.F. can be obtained by inspection.

Example. *Find the* H.C.F. *of* 36, 42, *and* 84.

Process

$$\left.\begin{aligned}36&=2\times2\times3\times3\\42&=2\times3\times7\\84&=2\times2\times3\times7\end{aligned}\right\}\text{ H.C.F. is }2\times3\text{, that is, }\underline{6}.$$

The H.C.F. required is 6; for each number is divisible by 2 and also by 3, and there is no other prime factor common to all three.

In finding the H.C.F. it is not necessary to find *all* the prime factors of more than one of the numbers.

Example. *Find the* H.C.F. *of* 4095, 3003, and 1755.

The prime factors of 4095 are $5\times3\times3\times7\times13$; of these factors those which divide both 3003 and 1755 are 3 and 13; also 3×3 does not divide both; therefore the required H.C.F. is $\underline{3\times13}$.

[It does not concern us what other factors 3003 and 1755 may have, since *every* factor of the H.C.F. must divide *each* of the given numbers.]

EXAMPLES. XXV.

Find, by the method of factors, the H.C.F. of the following:—

1. 16 and 24.
2. 30 and 75.
3. 80 and 140.
4. 80 and 112.
5. 81 and 108.
6. 66 and 121.
7. 272 and 425.
8. 390 and 672.
9. 728 and 455.
10. 15, 45, 225.
11. 13, 52, 91.
12. 34, 51, 85.
13. 176, 1100, 4444.
14. 1680, 1920.
15. 192, 576, 1760.
16. 182, 221, 299.
17. 808, 568, 1112.
18. 639, 873, 747.
19. 3090, 1515, 1065.
20. 868, 3164, 4228.
21. 1435, 3535, 7385.
22. 671, 781, 1441.
23. 455, 403, 481.
24. 833, 323, 3107.

65. By the following method the highest common factor of two numbers can always be found.

Example. Find the H.C.F. *of* 2021 *and* 6407.

Divide 6407 by 2021 ; divide 2021 by the remainder ; divide this divisor by the new remainder ; and so on ; when there is no remainder the last divisor is the required H.C.F. Thus :

2021)6407(3
6063
344)2021(5
1720
301)344(1
301
43)301(7
301

or, by the abbreviated method of Art. 36,

5)2021 | 6407(3
7)301 | 344(1
| 43

Therefore the H.C.F. required is 43.

66. The reason why the required factor is retained throughout the above process may be illustrated thus :

2021 = 47 times 43 ; 6407 = 149 times 43.

47 times 43)149 times 43(3
141 times 43
8 times 43)47 times 43(5
40 times 43
7 times 43)8 times 43(1
7 times 43
43)7 times 43(7
7 times 43

* The general **proof** is as follows :—

The remainder (in the process of dividing one number by another) = the larger number − some number of times of the smaller. Hence, by Art. 57, every factor common to the two numbers is a factor of the remainder ; also, every factor common to the remainder and the smaller number is a factor of the larger.

That is, the *common* factors of the smaller number and the remainder (in the process of Arts. 65, 66) are the *same* as the *common* factors of the two given numbers.

In other words, the H.C.F. of the smaller number and the remainder is the same as that of the two numbers.

Proceeding in this way, it follows that the H.C.F. of **any** two remainders is the H.C.F. of the given numbers.

The H.C.F. of the last two remainders is the last remainder.

Therefore the H.C.F. of the given numbers is the last remainder.

EXAMPLES. XXVI.

Find the H.C.F. of

1. 169 and 299.
2. 391 and 629.
3. 437 and 1691.
4. 841 and 1247.
5. 1219 and 667.
6. 2542 and 5487.
7. 2809 and 6731.
8. 4738 and 8234.
9. 4559 and 7003.
10. 15987, 30295.
11. 7319 and 9971.
12. 39455, 36556.
13. 2949 and 1563.
14. 13618 and 38830.
15. 15602 and 28797.
16. 23673 and 60203.
17. 218707 and 526769.
18. 2698703 and 54987261.
19. 45862, 29026, and 7153.
20. 720100, 913330, and 15409.

67. Numbers are said to be **prime to each other** when they have no *common* factor.

EXAMPLES. XXVII.

Prove that the following are prime to each other respectively—

1. 223 and 409.
2. 617 and 733.
3. 401 and 527.
4. 2951 and 3317.
5. 4427 and 7219.
6. 1726 and 1623.

Are the following prime to each other?

7. 3370 and 2703.
8. 4735 and 6017.
9. 73030 and 29991.
10. 97990 and 814629.
11. 88981 and 100091.
12. 72700 and 10901.

13. What is the largest sum of money which will divide both $71.61, and $84.63 exactly?

* Note.—The process of Art. 65 may often be shortened by using the method of Art. 64 as far as possible.

Example. Find the H.C.F. *of* 472700 *and* 17516. We notice that these numbers are 25 × 4 × 4727 and 4 × 4379; then we find the H.C.F. of 4727 and 4379 by Art. 69 to be 29; hence the required H.C.F. is 4 × 29 = 116. [The advanced student will observe that it follows from the proof on page 48, that this process may be applied to find the H.C.F. of *any* two remainders.]

Lowest Common Multiple.

68. **A number has one set of prime factors and no more.** [See Art. 55.]

Example. 35 has for its prime factors 5 and 7. It can easily be proved by trial that 35 is exactly divisible by no other prime.

This proposition is very important. It asserts that when we know the prime factors of a number we know all there is to know as to the divisors of a number.

69. It follows from Art. 68 that a *multiple* of a given number must contain among its prime factors every prime factor of the given number.

Example. Every multiple of 35 must contain each of the factors 5 and 7.

Also, when a prime factor is repeated in the given number, it must be repeated as least as many times in the multiple.

Example. No number can be a multiple of 72, that is, of $2\times2\times2\times3\times3$, which does not contain *three* factors 2 and *two* factors 3.

70. *DEF.* The **Lowest Common Multiple** (the **L.C.M.**) of two or more numbers is the smallest number which is exactly divisible by each of those numbers.

The L.C.M. must therefore contain all the *different* prime factors in the given numbers; and if a prime factor is *repeated* in any of them, it must be repeated in the L.C.M. as many times as it is repeated in that one of them in which it is most often repeated, and no more.

Example (i). *Find the* L.C.M. *of* 18, 24, 27, 45.

Here
$$\left.\begin{aligned}18&=2\times3\times3\\24&=2\times2\times2\times3\\27&=3\times3\times3\\45&=3\times3\times5\end{aligned}\right\}\begin{array}{l}\text{Therefore the required}\\ \text{L.C.M. is } 2\times2\times2\times3\times3\times3\times5\,^{*}\\ \text{i.e. } \underline{1080}.\end{array}$$

For the L.C.M. must contain the factors 2, 3, and 5; there must be *three* factors 2, since they occur in 24; there must be *three* factors 3, since they occur in 27. 1080 contains these factors, and no more, therefore it is the required L.C.M.

The process may be arranged as follows:—

$$\begin{array}{r|rrrr} 2 & 18, & 24, & 27, & 45\\ \hline 2 & 9, & 12, & 27, & 45\\ \hline 2 & 9, & 6, & 27, & 45\\ \hline 3 & 9, & 3, & 27, & 45\\ \hline 3 & 3, & 1, & 9, & 15\\ \hline & 1, & 1, & 3, & 5\end{array}$$

Hence the
$$\begin{aligned}\text{L.C.M.}&=2\times2\times2\times3\times3\times3\times5\\&=10\times12\times9\\&=\underline{1080}\end{aligned}$$

---

* The index notation of Art. 201 will be found convenient.

[Here, after dividing by 2, the numbers not exactly divisible by 2 are written in the same line with the quotients; the process is repeated until only prime numbers are left. The L.C.M. is the product of all the divisors and of the primes in the last line.]

With experience the work can often be shortened—

(i) by erasing at any stage of the work a number which exactly divides one of the others.

(ii) by dividing by a *composite* number; in which case we must take care to divide out by any factor of the divisor which is a factor of any one of the numbers of the dividend.

Example (ii). *Find the* L.C.M. *of* 15, 18, 20, 28, 45, 54.

Process, $10\underline{)\cancel{15}, \cancel{18}, 20, 28, 45, 54}$ L.C.M. $= 10 \times 14 \times 27$

$\cancel{2}, 14, \cancel{9}, 27$ $= 420 \times 9 = \underline{3780}$.

[NOTICE (i) that 15, which divides 45, is erased (since any multiple of 45 must also be a multiple of 15); 18 which divides 54 is erased; 2 divides 14; 9 divides 27.

(ii) that since 10 is 2 × 5, 28 is divided by 2, 45 by 5, 54 by 2.

EXAMPLES. XXVIII.

Find the L.C.M. of the following:—

1. 12, 18, 20.
2. 16, 28, 84, 112.
3. 15, 45, 125, 225.
4. 18, 27, 36, 42.
5. 20, 22, 25, 30, 33.
6. 14, 18, 20, 21.
7. 30, 32, 36, 40, 48.
8. 45, 50, 60, 63, 84.
9. 8, 9, 10, 12, 18, 20.
10. 7, 8, 9, 10, 14, 15, 18.
11. 6, 7, 8, 9, 10, 12, 14.
12. 8, 20, 28, 32, 56, 70.
13. 5, 11, 22, 55, 110.
14. 11, 121, 132, 144.
15. 26, 39, 169.
16. 22, 88, 132, 198.
17. 21, 32, 44, 63.
18. 21, 22, 24, 26, 28, 30.
19. 51, 187, 153, 165.
20. 3, 6, 9, 12, 15, 18, 20, 21.
21. 7, 8, 9, 10, 11, 12, 18.
22. 30, 32, 34, 35, 36, 37, 42.
23. The first 10 numbers.
24. The first 10 even numbers.

71. When the factors of the numbers cannot be seen by inspection we must proceed as follows:—

I. When there are two numbers—

The **Common** factors of the two can be found by Art. 65 (since the product of the common factors is the H.C.F.

of the two numbers); the factors **peculiar** to each can be found by dividing by their H.C.F.

The required L.C.M. is the product of the factors peculiar to each and of the common factors.

Example. Find the L.C.M. *of* 2021 *and* 6407.

The H.C.F. by Art. 65 is 43. By division we find that

$$2021 = 43 \times 47, \text{ and that } 6407 = 43 \times 149.$$

Hence the required L.C.M. is $\underline{43 \times 47 \times 149}$.

II. When there are more than two numbers—

The L.C.M. of three numbers is the product of the L.C.M. of two of the numbers by the factors peculiar to the third; and so on.

Example. Find the L.C.M. *of* 2021, 6407, *and* 3619.

The L.C.M. of 2021 and 6407 is $43 \times 47 \times 149$.

By division we find that the H.C.F. of $43 \times 47 \times 149$ and 3619 is 47; so that the factors peculiar to 3619, that is, to 47×77, are contained in 77.

Hence the required L.C.M. is $\underline{43 \times 47 \times 149 \times 77}$.

*EXAMPLES. XXIX.

Find the L.C.M. of the following—

1. 1955 and 2001.
2. 2041 and 8476.
3. 2501 and 8651.
4. 7409 and 4619.
5. 83390 and 74028.
6. 9997 and 7670.
7. 1121, 2831, and 6821.
8. 3161, 36830, and 13514.
9. 65270, 45445, and 5885.
10. 6241, 71416, and 12543.

Find the H.C.F. and the L.C.M. of

11. 4165 and 686.
12. 5325 and 8307.
13. 471, 1256, and 3454.
14. 504, 5292, and 4560.

*MISCELLANEOUS EXAMPLES. XXX.

1. Find the L.C.M. of the numbers from 20 to 30 inclusive.

2. What is the smallest sum of money which can be paid either in £5 notes, or in £8 notes, or in £25 notes, or in £30 notes, using only notes of one value?

3. What is the least number which when divided by 8, by 9, by 10, and by 12 gives in each case remainder 5?

4. One man can walk 50 miles a day, a second 35, a third 32, a fourth 36; find the least distance which would occupy each of them a complete number of days.

5. Four bells toll at intervals of 32, 35, 45, and 60 seconds respectively, if they all commence together after what interval will they again toll together?

6. Three men are walking together in step, their strides measure respectively 27 in., 33 in., and 36 in.; how many inches must they walk together before they are again in step?

7. The circumference of the driving wheel of an engine is 24 ft., that of the small wheel 10 ft., and that of a carriage wheel 9 ft.; how many feet must the train travel from rest before the same point on each of the wheels is again in contact with the rails?

8. What is the largest number of men amongst whom 437 apples and 1691 oranges can be distributed so that every man gets as many apples and as many oranges as any other man?

9. Two cylinders measure 391 and 629 inches respectively in circumference; find the shortest length of wire that can be wrapped round each an exact number of times.

10. Three men run round a circular track 1980 yds. in circumference, the first runs 330 yds. a minute, the second 264 yds., and the third 198 yds.; if they start together how long will it be before they are together again?

11. The sum of the digits of each of two numbers having been divided by 9, the remainders are multiplied together and their product divided by 9; prove that the remainder in this last division is equal to the remainder in the division by 9 of the sum of the digits in the product of the two original numbers.

12. Find the quotient obtained by dividing the product of seven whole numbers next in order after 40 by the product of the first seven whole numbers.

13. Prove that of any four consecutive numbers one is divisible by 2, another by 4, and that one of them is divisible by 3.

14. Prove that the product of any four consecutive numbers is divisible by $1 \times 2 \times 3 \times 4$.

15. Prove that ($23 \times 24 \times 25 \times \ldots$ any number, say n, of factors $- 22 \times 23 \times 24 \times \ldots n$ factors) is divisible by n; hence prove that the continued product of any n consecutive numbers is divisible by n.

16. Five bells which commence tolling together, toll at intervals of 1·2, 1·5, 1·75, 1·8, 2·1 seconds respectively; at what intervals will they toll together?

17. From a heap of shot weighing 54345 grains, a heap weighing 12321 grains is taken; show that a single shot cannot weigh more than 3 grains.

18. Two masses of silver weighing 1379 oz. and 2401 oz. respectively are each to be made into coins of the same weight; what is the weight of the largest possible coin?

CHAPTER IV

FRACTIONS

72. A **whole** number applied to the unit indicates a quantity composed solely of *complete* units.

The numbers defined on pages 2 and 3 are *whole* numbers.

DEF. A **Fraction** is a number which when applied to the unit indicates a quantity consisting of one or more equal *parts* of the unit.

Example. *Three-fourths* is a fraction ; for three-fourths of the unit is a quantity which consists of *three* such equal parts as the unit contains *four*.

73. The usual notation of Fractions is as follows :—The number of the equal parts into which the unit is divided is placed under a line, above which is placed the number of these parts taken to compose the quantity.

NOTE.—The fraction $\frac{3}{10}$ is read 'three-tenths.'

A fraction expressed in this notation is called a **Vulgar** Fraction.

Examples. $\frac{3}{4}$ of a dollar contains *three* such equal parts of which *four* make one dollar.

$\frac{5}{12}$ of a foot contains 5 such equal parts of which 12 make one foot, *i.e.* 5 inches.

NOTE.—$\$\frac{3}{10}$ is used as an abbreviation for $\frac{3}{10}$ of \$1.

The number above the line is called the **numerator**, the number below the line is called the **denominator**.

EXAMPLES. XXXI.

How many cents are there in each of the following :—

1. $\frac{9}{20}$ of \$1.
2. $\frac{3}{16}$ of \$4.
3. $\frac{5}{20}$ of \$2.
4. $\frac{9}{112}$ of \$28.
5. $\frac{5}{36}$ of \$9.
6. $\frac{1}{9}$ of \$27.
7. $\frac{3}{12}$ of \$6.
8. $\frac{3}{240}$ of \$6.
9. $\frac{1}{4}$ of \$1.
10. $\frac{1}{8}$ of \$10.
11. $\frac{1}{40}$ of \$20.
12. $\frac{1}{8}$ of \$16.
13. $\frac{1}{14}$ of \$21.
14. $\frac{3}{140}$ of \$7.
15. $\frac{10}{88}$ of \$22.
16. $\frac{7}{30}$ of \$12.
17. $\frac{5}{18}$ of \$9.
18. $\frac{5}{7}$ of \$91.

74. When the **numerator and the denominator of a fraction are each multiplied** (or are each divided) **by the same number the fraction is unaltered in value.**

On this proposition the whole theory of Vulgar Fractions depends. It may be proved as follows :—

Let the unit be divided into a number, say 5, of equal parts, of which some number, say 3, are taken to form the fraction of the unit.

Now let *each* of these 5 equal parts be subdivided into a number, say 4, of smaller equal parts.

There will be 20 in all of these smaller equal parts, and these 20 make up the unit ; *i.e.* each of them is $\frac{1}{20}$ of the unit.

But one of the first equal parts $=4$ of the smaller equal parts ;

that is, $\frac{1}{5}$ of the unit $=\frac{4}{20}$ of the unit ;

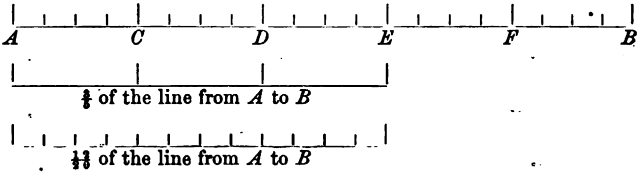
and three of the first equal parts $=12$ of the smaller equal parts ;

that is, $\frac{3}{5}$ of the unit $=\frac{12}{20}$ of the unit.

This is what is meant by the statement

$$\frac{3}{5}=\frac{12}{20}.$$

Example. Let the unit be the length of the line from A to B in the figure


A
C
D
E
F
B
$\frac{3}{5}$ of the line from A to B
$\frac{12}{20}$ of the line from A to B

Let the line be divided into 5 equal parts at the points C, D, E, F, and let each of these parts be subdivided into four equal parts as in the figure. Then, by the definition [Art. 73],

$$\text{the line } AC=\frac{1}{5} \text{ of line } AB, \text{ and } AC \text{ also } =\frac{4}{20} \text{ of line } AB.$$

Again

$$\text{the line } AE=\frac{3}{5} \text{ of line } AB, \text{ and } AE \text{ also } =\frac{12}{20} \text{ of line } AB.$$

Therefore
$$\frac{3}{5}=\frac{12}{20}.$$

75. A fraction is said to be in its **lowest terms** when the numerator and denominator have no common factor.

Example i. *Reduce* $\frac{204}{228}$ *to its lowest terms.*

By trial, or, by employing the tests of divisibility on pp. 46, 47, we have

$$\frac{204}{228}=\frac{3\times 68}{3\times 76}$$
$$=\frac{4\times 17}{4\times 19}=\frac{17}{19}.$$

Example ii. *Simplify* $\frac{591}{2167}$.

Testing for divisors of numerator and denominator, we have

$$\frac{591}{2167}=\frac{3\times 197}{11\times 197}=\frac{3}{11}.$$

Example iii. *Reduce* $\frac{1690}{2990}$ *to its lowest terms.*

It is clear that 1690 and 2990 are each divisible by 10. Hence

$$\frac{1690}{2990}=\frac{169\times 10}{299\times 10}=\frac{169}{299}.$$

Next, we find by Art. 65 that the H.C.F. of 169 and 299 is 13; then, dividing *both* numerator and denominator by 13, we have

$$\frac{169}{299}=\frac{13\times 13}{23\times 13}=\frac{13}{23}.$$

Hence, $\frac{13}{23}$ is the value of $\frac{1690}{2990}$ in its **lowest** terms.

EXAMPLES. XXXII.

Express in their lowest terms—

1. $\frac{24}{56}$. **2.** $\frac{39}{69}$. **3.** $\frac{125}{425}$. **4.** $\frac{500}{3125}$. **5.** $\frac{132}{4158}$.

6. $\frac{333}{405}$. **7.** $\frac{875}{1125}$. **8.** $\frac{350}{725}$. **9.** $\frac{363}{616}$. **10.** $\frac{1428}{1530}$.

11. $\frac{388}{404}$. **12.** $\frac{852}{1164}$. **13.** $\frac{781}{1331}$. **14.** $\frac{3100}{10925}$. **15.** $\frac{1144}{9900}$.

16. $\frac{156}{208}$. **17.** $\frac{630}{9999}$. **18.** $\frac{324}{1092}$. **19.** $\frac{3861}{5094}$. **20.** $\frac{2640}{2970}$.

21. $\frac{3944}{4812}$. **22.** $\frac{1991}{13200}$. **23.** $\frac{1670}{2105}$. **24.** $\frac{2424}{2712}$. **25.** $\frac{365}{438}$.

26. $\frac{169}{221}$. **27.** $\frac{361}{437}$. **28.** $\frac{527}{629}$. **29.** $\frac{1209}{1599}$. **30.** $\frac{2755}{3515}$.

31. $\frac{2103}{2253}$. **32.** $\frac{4055}{4985}$. **33.** $\frac{2651}{7271}$. **34.** $\frac{9110}{9910}$. **35.** $\frac{6539}{12963}$.

76. A fraction whose numerator is equal to or greater than its denominator is called an **improper** fraction. A fraction whose numerator is less than its denominator is called a **proper** fraction.

An improper fraction is equal either to a whole number, or to a whole number + a fraction.

Example. $\frac{7}{3}=\frac{6}{3}+\frac{1}{3}=2+\frac{1}{3}$;

for $\frac{3}{3}$ of the unit $=1$ of the unit; hence $\frac{3}{3}=1$, and therefore $\frac{6}{3}=2$.

77. A number which consists of the sum of a whole number and a fraction is called a **mixed number.**

Example. $(2+\frac{1}{3})$ *is a mixed number.*

The Notation adopted must be noticed.

$(2+\frac{1}{3})$ is abbreviated thus, $2\frac{1}{3}$.

Example i. *Express* $\frac{21}{5}$ *as a mixed number.*

$\frac{21}{5}=\frac{20}{5}+\frac{1}{5}=4+\frac{1}{5}$, that is, $4\frac{1}{5}$.

Hence an improper fraction may be expressed as a mixed number by dividing the numerator by the denominator; thus

$$\left.\begin{array}{l} 5)\underline{21} \\ \quad 4 + \text{remainder } 1. \end{array}\right\} \quad \text{Hence, } \frac{21}{5}=4\frac{1}{5}.$$

Example ii. *Express* $5\frac{4}{9}$ *as an improper fraction.*

$$5\tfrac{4}{9} \text{ of a unit} = 5 \text{ units} + \tfrac{4}{9} \text{ of a unit} = \left(\frac{5\times 9}{9}+\frac{4}{9}\right) \text{ of a unit}$$

$$= \tfrac{49}{9} \text{ of a unit; hence,}$$

Process, $5\frac{4}{9}=\frac{5\times 9+4}{9}=\frac{49}{9}$.

EXAMPLES. XXXIII.

Express as whole or mixed numbers—

1. $\frac{31}{7}$. **2.** $\frac{37}{10}$. **3.** $\frac{84}{11}$. **4.** $\frac{107}{12}$. **5.** $\frac{99}{11}$.

6. $\frac{365}{30}$. **7.** $\frac{310}{12}$. **8.** $\frac{450}{30}$. **9.** $\frac{171}{31}$. **10.** $\frac{105}{21}$.

11. $\frac{59}{49}$. **12.** $\frac{600}{70}$. **13.** $\frac{119}{41}$. **14.** $\frac{625}{125}$. **15.** $\frac{370}{50}$.

16. $\frac{390}{41}$. **17.** $\frac{8201}{365}$. **18.** $\frac{1885}{70}$. **19.** $\frac{4919}{240}$. **20.** $\frac{8000}{1760}$.

Reduce to improper fractions—

21. $3\frac{1}{7}$. **22.** $4\frac{5}{9}$. **23.** $18\frac{1}{10}$. **24.** $19\frac{3}{11}$. **25.** $11\frac{11}{31}$.

26. $11\frac{24}{31}$. **27.** $12\frac{17}{25}$. **28.** $21\frac{7}{100}$. **29.** $3\frac{1}{1000}$. **30.** $1\frac{121}{1000}$.

31. $3\frac{7}{99}$. **32.** $41\frac{28}{99}$. **33.** $50\frac{7}{50}$. **34.** $11\frac{11}{101}$. **35.** $31\frac{12}{201}$.

78. A whole number can be expressed as a fraction with any given denominator.

Example. Express 5 *as a fraction with denominator* 7.

$$5=\frac{5}{1}=\frac{5\times 7}{1\times 7}=\frac{35}{7}.$$

79. A given fraction can always be transformed to another fraction whose denominator is any multiple of the given denominator.

Example. Express $\frac{3}{5}$ *as a fraction with denominators* (i) 15, (ii) 20, (iii) 100, *each number being a* **multiple** *of* 5.

$$\text{Since } 15\div 5=3, \text{ therefore } \frac{3}{5}=\frac{3\times 3}{5\times 3}=\frac{9}{15}\text{..................(i)},$$

$$\text{since } 20\div 5=4, \text{ therefore } \frac{3}{5}=\frac{3\times 4}{5\times 4}=\frac{12}{20}\text{..................(ii)},$$

$$\text{since } 100\div 5=20, \text{ therefore } \frac{3}{5}=\frac{3\times 20}{5\times 20}=\frac{60}{100}\text{...............(iii)}.$$

EXAMPLES. XXXIV.

1. Express 5, 7, 21, 85 each as fractions with denominator 15.
2. Express 11, 30, 51, 101 each with denominator (i) 11, (ii) 7.
3. Express 12, 7 each with denominator (i) 7, (ii) 30, (iii) 100.
4. Express 33, 10 each with denominator (i) 21, (ii) 59, (iii) 10.
5. Express $\frac{7}{9}$ and $\frac{5}{9}$ each as a fraction with denominator (i) 18, (ii) 27, (iii) 81, (iv) 54, (v) 900.
6. Express $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{7}{15}$, $\frac{20}{21}$, $\frac{8}{35}$ each with denominator 105.
7. Can $\frac{2}{9}$, $\frac{4}{15}$, $\frac{4}{7}$, $\frac{4}{21}$, $\frac{11}{63}$ each be expressed with denominator 63? Mention a denominator with which they can each be expressed.
8. Express 5, $\frac{3}{5}$, $\frac{2}{15}$, $\frac{3}{10}$ each with denominator 30.
9. What fractions with denominator 12 are equivalent to 3, $\frac{18}{27}$, $\frac{75}{90}$, $\frac{36}{48}$ respectively?
10. Mention the least denominator which can be *common* to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{5}{6}$, $\frac{5}{12}$, $\frac{7}{15}$.

80. Fractions with different denominators can always be expressed as fractions with a common denominator.

It is generally convenient that this new **common denominator** should be as small as possible.

By Art. 79 the **lowest** common denominator is the lowest common multiple of the denominators.

Example i. *Express* $\frac{4}{9}, \frac{5}{12}, \frac{7}{15}$ *as fractions with equal denominators.*

When the numerator and the denominator of a fraction are each multiplied by the same number the fraction is not altered in value; accordingly, we must find some number which is a **multiple** of each denominator; for then each denominator can be transformed into that number by multiplication.

Therefore we proceed to find the L.C.M. of the denominators, thus:

$$\left.\begin{array}{r|l} 3 & 9,\ 12,\ 15 \\ \hline & 3,\ \ 4,\ \ 5 \end{array}\right\}\ \text{L.C.M.} = 3 \times 3 \times 2 \times 2 \times 5 = 9 \times 20 = 180.$$

Consider what factor is required in each case to convert the denominator into the L.C.M. $3 \times 3 \times 4 \times 5$; then

$$\text{since } 180 \div 9 = 20, \text{ therefore } \frac{4}{9} = \frac{4 \times 20}{9 \times 20} = \frac{80}{180},$$

$$\text{since } 180 \div 12 = 15, \text{ therefore } \frac{5}{12} = \frac{5 \times 15}{12 \times 15} = \frac{75}{185},$$

$$\text{since } 180 \div 15 = 12, \text{ therefore } \frac{7}{15} = \frac{7 \times 12}{15 \times 12} = \frac{84}{180}.$$

Thus $\frac{4}{9}, \frac{5}{12}, \frac{7}{15}$ are equal to $\frac{80}{180}, \frac{75}{180}, \frac{84}{180}$ respectively.

Example. Express $\frac{3}{8}, \frac{4}{9}, \frac{11}{12}$ *with a common denominator.*

The L.C.M. of 8, 9, 12 is $2 \times 2 \times 2 \times 3 \times 3 = 72$; hence

$$\frac{3}{8} = \frac{3 \times 9}{72} = \frac{27}{72};\ \frac{4}{9} = \frac{4 \times 8}{72} = \frac{32}{72};\ \frac{11}{12} = \frac{11 \times 6}{72} = \frac{66}{72}.$$

81. The example on the next page illustrates the reduction of two fractions to their L.C.D.:—

Example. Reduce $\frac{3}{10}$ and $\frac{3}{4}$ to their L.C.D.

FIG. I.

Let the unit be the length of the above line *AB*.

FIG. II.

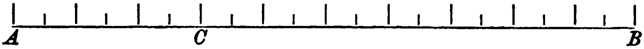
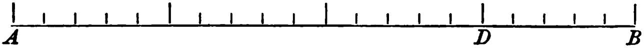


FIG. III.



In Fig. II. let the line *AB* be divided into ten equal parts by the longer lines of division.

In Fig. III. let the line *AB* be divided into four equal parts by the longer lines of division.

Then in Fig. II. $AC=\frac{3}{10}$ of the line, in Fig. III. $AD=\frac{3}{4}$ of the line.

The L.C.M. of 10 and 4 is 20; and $10\times2=20$, $4\times5=20$.

Hence in Fig. II. each tenth is subdivided into 2 twentieths,
in Fig. III. each fourth ....5 twentieths,

and $AC=\frac{6}{20}$ of *AB*, $AD=\frac{15}{20}$ of *AB*.

EXAMPLES. XXXV.

Express as fractions having a common denominator—

1. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$.

2. $\frac{1}{5}, \frac{1}{3}, \frac{2}{9}$.

3. $\frac{2}{7}, \frac{3}{5}, \frac{8}{21}$.

4. $\frac{3}{10}, \frac{7}{20}, \frac{11}{30}$.

5. $\frac{4}{1}, \frac{25}{7}, \frac{36}{11}$.

6. $3, 5, \frac{4}{9}$.

7. $\frac{3}{7}, \frac{9}{5}, 2$.

8. $\frac{2}{18}, \frac{3}{8}, \frac{5}{36}$.

9. $\frac{5}{49}, \frac{8}{63}, \frac{11}{147}$.

10. $\frac{2}{15}, \frac{3}{10}, \frac{7}{100}$.

11. $\frac{1}{24}, \frac{7}{27}, \frac{11}{72}$.

12. $\frac{33}{100}, \frac{13}{125}, \frac{7}{25}$.

13. $\frac{1}{6}, \frac{1}{12}, \frac{1}{18}, \frac{1}{8}$.

14. $\frac{1}{9}, \frac{3}{18}, \frac{5}{54}, \frac{11}{36}$.

15. $\frac{2}{7}, \frac{3}{11}, \frac{5}{9}$.

16. $\frac{2}{9}, \frac{5}{11}, \frac{37}{33}, 5$.

17. $\frac{85}{11}, \frac{40}{7}, \frac{61}{21}, \frac{100}{77}$.

18. $\frac{2}{2}, \frac{3}{11}, \frac{4}{13}$.

82. Of two fractions having a common denominator **the greater** is that which has the greater numerator.

For $\frac{11}{36}$ of the unit is greater than $\frac{10}{36}$ of the unit.

Example. Which is the greatest and which is the least of

$$\frac{7}{9}, \frac{4}{5}, \frac{17}{20}?$$

The L.C.M. of the denominators is $9 \times 5 \times 4 = 180$; hence

$$\frac{7}{9} = \frac{7 \times 20}{180} = \frac{140}{180}; \qquad \frac{4}{5} = \frac{4 \times 36}{180} = \frac{144}{180}; \qquad \frac{17}{20} = \frac{17 \times 9}{180} = \frac{153}{180}.$$

Thus $\frac{153}{180}$, that is, $\frac{17}{20}$ is the greatest; $\frac{140}{180}$, *i.e.* $\frac{7}{9}$ is the least.

Of two fractions having a common numerator the greater is that which has the smaller denominator.

They each represent the same number of '*parts*,' but the parts of the one are greater than the parts of the other.

Example. Which is the greater $\frac{12}{151}$ or $\frac{15}{193}$?

The L.C.M. of the numerators is 60, hence

$$\text{since } 60 \div 12 = 5, \text{ therefore } \frac{12}{151} = \frac{12 \times 5}{151 \times 5} = \frac{60}{755};$$

$$\text{since } 60 \div 15 = 4, \text{ therefore } \frac{15}{193} = \frac{15 \times 4}{193 \times 4} = \frac{60}{772}.$$

Thus, $\frac{60}{755}$, that is $\frac{12}{151}$, is greater than $\frac{60}{772}$, that is, than $\frac{15}{193}$.

EXAMPLES. XXXVI.

Which is the greatest and which the least of the following fractions?

1. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.
2. $\frac{5}{6}, \frac{6}{7}, \frac{17}{21}$.
3. $\frac{3}{5}, \frac{2}{3}, \frac{7}{12}$.
4. $\frac{8}{11}, \frac{41}{55}, \frac{3}{4}$.
5. $\frac{9}{13}, \frac{9}{11}, \frac{9}{10}$.
6. $\frac{12}{21}, \frac{13}{24}, \frac{15}{28}, \frac{8}{15}$.
7. $\frac{11}{30}, \frac{12}{35}, \frac{12}{36}, \frac{7}{20}$.
8. $\frac{11}{15}, \frac{17}{20}, \frac{19}{25}, \frac{23}{30}$.
9. $\frac{1}{3}, \frac{3}{8}, \frac{4}{11}$.
10. $\frac{4}{7}, \frac{4}{9}, \frac{8}{15}, \frac{8}{17}$.
11. $\frac{5}{16}, \frac{7}{24}, \frac{11}{36}, \frac{9}{28}$.
12. $\frac{21}{25}, \frac{26}{30}, \frac{31}{36}$.

Arrange in order of magnitude—

13. $\frac{5}{12}, \frac{7}{18}, \frac{11}{25}, \frac{13}{30}$.

14. $\frac{4}{9}, \frac{7}{12}, \frac{11}{20}, \frac{13}{24}$.

15. $\frac{19}{24}, \frac{23}{30}, \frac{29}{36}$.

16. $\frac{45}{23}, \frac{60}{31}, \frac{100}{51}, \frac{25}{13}$.

17. $\frac{7}{24}, \frac{13}{48}, \frac{17}{64}, \frac{4}{15}$.

18. $\frac{7}{13}, \frac{7}{9}, \frac{14}{17}$.

19. $\frac{7}{20}, \frac{5}{16}, \frac{13}{40}, \frac{19}{60}$.

20. $\frac{7}{22}, \frac{18}{55}, \frac{53}{160}$.

21. $\frac{5}{24}, \frac{7}{36}, \frac{11}{54}, \frac{13}{64}$.

ADDITION OF FRACTIONS.

83. The **addition of fractions** with a common denominator is effected by adding their numerators.

Example. Add $\frac{4}{7}, \frac{5}{7}$, and $\frac{6}{7}$.

According to the definition of addition [Art. 14] we must take $\frac{4}{7}$ of the unit, $\frac{5}{7}$ of the unit, and $\frac{6}{7}$ of the unit, and put them together, the number which represents these three taken together is the sum of the three numbers.

That is, we must take 4 sevenths of the unit, 5 sevenths of the unit, and 6 sevenths of the unit, and put them together: these together make $(4+5+6)$ sevenths, *i.e.* $\frac{15}{7}$ of the unit.

Thus, $\frac{4+5+6}{7}$, that is, $\frac{15}{7}$ is the result of adding $\frac{4}{7}, \frac{5}{7}$, and $\frac{6}{7}$.

EXAMPLES. XXXVII.

Write down the sum of

1. $\frac{3}{5}, \frac{7}{5}, \frac{2}{5}$.

2. $\frac{4}{11}, \frac{1}{11}, \frac{7}{11}$.

3. $\frac{1}{3}, \frac{5}{3}, \frac{7}{3}$.

4. $\frac{3}{10}+\frac{7}{10}+\frac{8}{10}+\frac{11}{10}$.

5. $\frac{6}{101}+\frac{20}{101}+\frac{39}{101}$.

6. $\frac{5}{9}, \frac{10}{9}, \frac{11}{9}$.

7. $\frac{7}{20}$ of \$1 + $\frac{5}{20}$ of \$1.

8. $\frac{7}{112}+\frac{14}{112}$.

9. $\frac{5}{12}$ of 1 ft. + $\frac{7}{12}$ of 1 ft.

10. $\frac{7}{60}$ of an hour + $\frac{33}{60}$ of an hour.

84. The **addition of fractions** with different denominators may be effected by reducing each of the fractions to their least common denominator.

Example. *Add* $\frac{1}{3}+\frac{2}{5}+\frac{4}{15}+\frac{11}{12}$.

The least common denominator is the L.C.M. of the denominators

$$\left.\begin{array}{r|l} 3 & \cancel{3},\ \cancel{5},\ 15,\ 12 \\ \hline & \qquad\quad 5,\ \ 4 \end{array}\right\} \begin{aligned} \text{L.C.M.} &= 3\times5\times4=3\times20 \\ &=60 \end{aligned}$$

$$\text{then } \frac{1}{3}=\frac{1\times20}{60}=\frac{20}{60}$$

$$\frac{2}{5}=\frac{2\times12}{60}=\frac{24}{60}$$

$$\frac{4}{15}=\frac{4\times4}{60}=\frac{16}{60}$$

$$\frac{11}{12}=\frac{11\times5}{60}=\frac{55}{60}$$

$$\text{Hence } \frac{1}{3}+\frac{2}{5}+\frac{4}{15}+\frac{11}{12}$$

$$=\frac{20+24+16+55}{60}$$

$$=\frac{115}{60}=1\tfrac{55}{60}$$

$$=1\tfrac{11}{12}.$$

N.B.—The work on the left may be omitted after a little practice.

NOTE.—It will be found by experience that the addition of fractions may often be shortened by first adding two of the fractions together, as in the Example of Art. 85.

EXAMPLES. XXXVIII.

Simplify—

1. $\frac{1}{2}+\frac{2}{3}$.

2. $\frac{2}{3}+\frac{8}{15}$.

3. $\frac{1}{8}+\frac{5}{12}$.

4. $\frac{3}{5}+\frac{1}{25}+\frac{7}{75}$.

5. $\frac{3}{8}+\frac{7}{24}+\frac{5}{32}$.

6. $\frac{1}{7}+\frac{3}{14}+\frac{11}{28}$.

7. $\frac{1}{2}+\frac{2}{3}+\frac{5}{6}+\frac{7}{12}$.

8. $\frac{1}{5}+\frac{1}{3}+\frac{1}{15}+\frac{1}{45}$.

9. $\frac{3}{10}+\frac{7}{20}+\frac{9}{40}+\frac{7}{8}$.

10. $\frac{8}{9}+\frac{25}{27}+\frac{7}{18}+\frac{5}{36}$.

11. $\frac{5}{11}+\frac{5}{33}+\frac{5}{22}+\frac{5}{6}$.

12. $\frac{3}{7}+\frac{5}{9}+\frac{5}{8}+\frac{5}{72}$.

13. $\frac{8}{23}+\frac{8}{69}+\frac{8}{115}+\frac{8}{15}$.

14. $\frac{4}{19}+\frac{9}{57}+\frac{5}{38}+\frac{15}{16}$.

15. $\frac{5}{8}+\frac{5}{9}+\frac{5}{21}$.

16. $\frac{3}{7}+\frac{3}{8}+\frac{3}{28}+\frac{3}{56}$.

17. $\frac{1}{7}+\frac{4}{35}+\frac{3}{70}+\frac{7}{10}$.

18. $\frac{8}{63}+\frac{13}{21}+\frac{15}{42}$.

19. $\frac{3}{7}+\frac{4}{21}+\frac{5}{56}+\frac{7}{24}$.

20. $\frac{1}{17}+\frac{32}{34}+\frac{3}{8}+\frac{3}{56}$.

21. $\frac{4}{13}+\frac{2}{5}+\frac{11}{65}+\frac{9}{26}$.

85. In the **addition of mixed numbers** it is convenient to add the whole numbers by themselves and the fractions by themselves, and then add the two results.

It is also convenient to reduce improper fractions to mixed numbers.

Example. Add $2\frac{1}{3}+3\frac{2}{5}+\frac{4}{15}+5\frac{11}{12}$.

This expression is equal to

$$2+3+5+\tfrac{1}{3}+\tfrac{2}{5}+\tfrac{4}{15}+\tfrac{11}{12},$$
$$=10+\tfrac{11}{15}+\tfrac{4}{15}+\tfrac{11}{12}=10+\tfrac{15}{15}+\tfrac{11}{12}=11\tfrac{11}{12}.$$

EXAMPLES. XXXIX.

Simplify—

1. $3\frac{1}{5}+2\frac{4}{5}$.
2. $5\frac{1}{9}+6\frac{8}{9}+7$.
3. $2\frac{2}{3}+4\frac{5}{6}$.
4. $4\frac{1}{3}+5\frac{2}{5}+\frac{4}{15}$.
5. $6\frac{1}{4}+5\frac{3}{8}+7\frac{7}{12}+11$.
6. $2\frac{1}{5}+\frac{21}{15}+\frac{1}{9}+4$.
7. $5\frac{5}{6}+4\frac{5}{9}+3\frac{5}{18}+\frac{5}{12}$.
8. $1\frac{1}{2}+2\frac{3}{4}+3\frac{7}{8}+4\frac{7}{16}$.
9. $\frac{19}{3}+3\frac{1}{5}+\frac{5}{12}+\frac{7}{20}$.
10. $12\frac{1}{11}+1\frac{3}{22}+7\frac{5}{33}$.
11. $11\frac{1}{18}+2\frac{2}{27}+\frac{1}{54}+\frac{2}{9}$.
12. $1\frac{3}{14}+\frac{60}{49}+\frac{11}{7}+2$.
13. $4\frac{1}{7}+4\frac{1}{14}+4\frac{1}{28}+\frac{1}{49}$.
14. $3\frac{2}{7}+5\frac{8}{63}+3\frac{11}{21}+2$.
15. $\frac{41}{22}+1\frac{5}{33}+1\frac{5}{6}$.
16. $\frac{27}{5}+\frac{48}{25}+\frac{101}{15}$.
17. $\frac{1000}{24}+\frac{100}{8}+\frac{10}{3}$.
18. $\frac{99}{40}+7\frac{9}{25}+8\frac{7}{50}$.
19. $\frac{63}{7}+\frac{45}{14}+\frac{100}{28}$.
20. $\frac{101}{100}+\frac{203}{25}+\frac{301}{120}+\frac{21}{4}$.
21. $\frac{300}{21}+7+\frac{400}{49}+\frac{5}{3}$.

Add together—

22. $2\frac{3}{5}$, $5\frac{1}{10}$, $2\frac{11}{20}$.
23. $1\frac{1}{4}$, $4\frac{7}{12}$, $3\frac{1}{16}$, $\frac{27}{24}$.
24. $4\frac{4}{13}$, $4\frac{4}{39}$, $4\frac{4}{91}$.
25. $2\frac{5}{11}$, $4\frac{3}{7}$, $\frac{100}{21}$, $\frac{671}{33}$.
26. 2, $3\frac{4}{15}$, $\frac{231}{55}$, $\frac{491}{33}$.
27. $1\frac{1}{9}$, $1\frac{1}{30}$, $1\frac{1}{15}$, $1\frac{1}{45}$.
28. 4, $5\frac{1}{9}$, $\frac{271}{12}$, $\frac{88}{45}$.
29. 1, $7\frac{3}{23}$, $5\frac{6}{13}$, $\frac{1901}{65}$.
30. $\frac{1000}{77}$, $\frac{1000}{11}$, $\frac{1000}{33}$, 10.
31. $21\frac{3}{56}$, $43\frac{11}{112}$, $51\frac{25}{84}$.
32. $91\frac{1}{144}$, $90\frac{37}{864}$, $9\frac{451}{576}$.
33. $9\frac{37}{176}$, $8\frac{41}{88}$, $6\frac{1}{110}$.
34. $\frac{13}{27}$, $3\frac{40}{135}$, $\frac{173}{15}$, $\frac{7}{9}$.
35. $1\frac{1}{19}$, $2\frac{7}{38}$, $3\frac{4}{57}$, $\frac{5}{6}$.
36. $\frac{5}{52}$, 100, $\frac{7}{4}$, $\frac{9}{26}$.
37. $1\frac{1}{8}$, $1\frac{1}{9}$, $1\frac{1}{10}$, $1\frac{1}{11}$.
38. $3\frac{1}{6}$, $4\frac{2}{7}$, $5\frac{3}{8}$, $6\frac{4}{9}$.
39. $\frac{21}{8}$, $\frac{22}{9}$, $\frac{23}{10}$, $\frac{24}{11}$, $\frac{25}{12}$.

86. The **subtraction of fractions** is effected by the same method as the addition of fractions.

Example. Subtract $\frac{3}{14}$ *from* $\frac{17}{35}$.

$$7\underline{)14,\ 35}\qquad \text{L.C.M.} = 7 \times 2 \times 5$$
$$2,\ 5.\qquad = 70.$$

Hence $\frac{3}{14} = \frac{3 \times 5}{70} = \frac{15}{70}$ | $\frac{17}{35} - \frac{3}{14} = \frac{34 - 15}{70}$

$\frac{17}{35} = \frac{17 \times 2}{70} = \frac{34}{70}$ | $= \frac{19}{70}$.

NOTE.—The work on the left may be omitted in practice.

EXAMPLES. XL.

Simplify—

1. $\frac{1}{2} - \frac{1}{3}$.
2. $\frac{3}{4} - \frac{2}{3}$.
3. $\frac{5}{6} - \frac{4}{9}$.
4. $\frac{5}{12} - \frac{2}{15}$.
5. $\frac{11}{18} - \frac{11}{24}$.
6. $\frac{7}{25} - \frac{2}{15}$.
7. $\frac{3}{20} - \frac{1}{40}$.
8. $\frac{21}{25} - \frac{7}{45}$.
9. $\frac{5}{21} - \frac{3}{28}$.
10. $\frac{18}{52} - \frac{7}{39}$.
11. $\frac{5}{121} - \frac{1}{33}$.
12. $\frac{21}{64} - \frac{7}{48}$.

Find the difference between

13. $\frac{5}{9}$ and $\frac{2}{9}$.
14. $\frac{5}{12}$ and $\frac{5}{18}$.
15. $\frac{7}{12}$ and $\frac{7}{10}$.
16. $\frac{13}{15}$ and $\frac{13}{12}$.
17. $1\frac{3}{4}$ and $1\frac{3}{8}$.
18. $5\frac{1}{2}$ and $4\frac{3}{7}$.
19. $7\frac{5}{9}$ and $7\frac{7}{16}$.
20. $\frac{5}{14}$ and $\frac{11}{18}$.
21. $\frac{7}{9}$ and $\frac{14}{17}$.

87. The following examples of the subtraction of mixed numbers should be noticed.

NOTE.—It was stated in Art. 77 that the expression $2\frac{3}{7}$ stands for $2 + \frac{3}{7}$; it must be noticed that $2\frac{3}{7}$ is not *fully* expressed by $2 + \frac{3}{7}$; it needs a bracket $(2 + \frac{3}{7})$ [Art. 22], to express it completely.

Example i. *Simplify* $6\frac{5}{9} - 2\frac{3}{7}$.

That is, *simplify* $(6 + \frac{5}{9}) - (2 + \frac{3}{7})$. This expression

$$= 6 + \tfrac{5}{9} - 2 - \tfrac{3}{7} = 6 - 2 + \tfrac{5}{9} - \tfrac{3}{7} = 4 + \tfrac{5}{9} - \tfrac{3}{7}$$

$$= 4 + \frac{35 - 27}{63} = 4 + \tfrac{8}{63} = 4\tfrac{8}{63}.$$

Example ii. *Simplify* $6\frac{5}{9}-2\frac{6}{7}$.

$$6\tfrac{5}{9}-2\tfrac{6}{7}=6+\tfrac{5}{9}-2-\tfrac{6}{7}=4+\tfrac{5}{9}-\tfrac{6}{7}=4+\tfrac{35}{63}-\tfrac{54}{63}.$$

Now $\frac{54}{63}$ cannot be subtracted from $\frac{35}{63}$, and we proceed thus:

$$4+\tfrac{35}{63}-\tfrac{54}{63}=3+(1+\tfrac{35}{63})-\tfrac{54}{63}=3+\tfrac{98}{63}-\tfrac{54}{63}=3\tfrac{44}{63}.$$

Process, $6\frac{5}{9}-2\frac{6}{7}=4\frac{5}{9}-\frac{6}{7}$

$$=4+\frac{35-54}{63}=3+\frac{63+35-54}{63}$$

$$=3\tfrac{44}{63}.$$

EXAMPLES. XLI.

Subtract—

1. $1\frac{5}{8}$ from $2\frac{3}{8}$. **2.** $21\frac{5}{9}$ from $22\frac{1}{3}$. **3.** $7\frac{11}{12}$ from $8\frac{7}{24}$.
4. $3\frac{6}{7}$ from $5\frac{3}{14}$. **5.** $4\frac{11}{21}$ from $20\frac{11}{28}$. **6.** $\frac{21}{22}$ from $11\frac{7}{33}$.
7. $3\frac{8}{13}$ from 5. **8.** $4\frac{7}{18}$ from 5. **9.** $201\frac{55}{64}-176\frac{47}{48}$.
10. $200-1\frac{7}{9}$. **11.** $27\frac{1}{3}-26\frac{26}{27}$. **12.** $45\frac{27}{28}-35\frac{34}{35}$.

Find the difference between

13. 4 and $1\frac{11}{19}$. **14.** $8\frac{4}{9}$ and $9\frac{1}{9}$. **15.** $2\frac{3}{11}$ and 3.
16. $1\frac{11}{20}$ and $2\frac{11}{30}$. **17.** $5\frac{12}{77}$ and $6\frac{12}{49}$. **18.** $4\frac{13}{21}$ and $4\frac{13}{28}$.

Write down the results of subtraction in the following :—

19. $\begin{array}{r}14\frac{10}{17}\\ 4\frac{5}{9}\\ \hline\end{array}$ **20.** $\begin{array}{r}9\frac{23}{24}\\ 8\frac{21}{22}\\ \hline\end{array}$ **21.** $\begin{array}{r}5\frac{25}{32}\\ 4\frac{25}{36}\\ \hline\end{array}$ **22.** $\begin{array}{r}4\frac{15}{16}\\ 4\frac{30}{32}\\ \hline\end{array}$ **23.** $\begin{array}{r}22\frac{21}{22}\\ 11\frac{11}{21}\\ \hline\end{array}$

Simplify—

24. $3\frac{1}{3}+4\frac{1}{9}-6\frac{5}{6}$. **25.** $8\frac{1}{4}-7\frac{5}{8}+\frac{11}{16}$. **26.** $5\frac{7}{8}-2\frac{1}{4}-\frac{1}{2}$.
27. $3\frac{6}{7}-1\frac{5}{3}-\frac{4}{21}$. **28.** $7-2\frac{3}{10}-1\frac{4}{15}$. **29.** $20-12\frac{12}{35}-\frac{4}{5}$.
30. $4-2\frac{7}{8}-\frac{5}{6}$. **31.** $8-7\frac{7}{12}+2\frac{7}{18}$. **32.** $3\frac{2}{3}-1\frac{11}{12}-\frac{5}{16}$.
33. $8-\frac{5}{6}+3\frac{2}{9}-1\frac{3}{4}$. **34.** $9-\frac{3}{8}-2\frac{5}{12}+\frac{7}{15}$. **35.** $1\frac{3}{11}-\frac{5}{22}-\frac{7}{55}$.
36. $4\frac{7}{8}-3\frac{9}{10}-\frac{11}{12}$. **37.** $5\frac{7}{11}+\frac{3}{22}-\frac{21}{44}$. **38.** $\frac{29}{30}+5\frac{1}{5}-\frac{37}{18}$.
39. $\frac{3}{8}+\frac{5}{20}-\frac{3}{12}$. **40.** $\frac{3}{8}+4\frac{5}{14}-2\frac{11}{28}$.
41. $\frac{1}{2}-\frac{1}{7}+\frac{7}{8}$. **42.** $\frac{11}{70}+\frac{5}{21}+\frac{1}{5}-\frac{17}{42}$.
43. $2\frac{2}{3}-\frac{3}{5}+4+5\frac{5}{6}$. **44.** $2\frac{2}{3}-5\frac{7}{12}+\frac{3}{10}$ of 15.
45. $4\frac{1}{2}-2\frac{2}{3}+27$. **46.** $3\frac{5}{11}+37\frac{1}{3}-27\frac{1}{2}$.
47. $21\frac{1}{4}+4\frac{11}{20}-7\frac{3}{10}$. **48.** $\frac{73}{100}-\frac{37}{7000}$.

88. To **multiply a fraction by a whole number** we multiply the numerator by the number.

Example i. *Multiply* $\frac{6}{7}$ *by* 3,

i.e. add together $\frac{6}{7}+\frac{6}{7}+\frac{6}{7}$. Hence $\frac{6}{7}\times 3=\frac{6\times 3}{7}=\frac{18}{7}=2\frac{4}{7}$.

Example ii. $191\frac{5}{8}\times 5=(191+\frac{5}{8})\times 5=955+\frac{25}{8}=955+3\frac{1}{8}=958\frac{1}{8}$.

Example iii. $4\frac{3}{4}\times 165=(4+\frac{3}{4})\times 165=4\times 165+\frac{3}{4}\times 165$
$=660+41\frac{1}{4}\times 3=660+123\frac{3}{4}=783\frac{3}{4}$.

EXAMPLES. XLII.

Multiply—

1. $\frac{3}{4}$ by 5. **2.** $\frac{5}{8}$ by 4. **3.** $\frac{7}{9}$ by 3. **4.** $10\frac{2}{3}$ by 6.
5. $5\frac{1}{4}$ by 6. **6.** $\frac{25}{36}$ by 12. **7.** $\frac{41}{50}$ by 3. **8.** $25\frac{5}{11}$ by 7.
9. $3\frac{1}{21}\times 7$. **10.** $5\frac{5}{42}\times 14$. **11.** $3\frac{8}{33}\times 11$. **12.** $4\frac{7}{56}\times 28$.
13. $5\frac{1}{4}\times 365$. **14.** $473\frac{5}{9}\times 5$. **15.** $791\frac{11}{12}\times 8$. **16.** $8\frac{1}{9}\times 4375$.

89. To **multiply** a number, say $\frac{5}{9}$, **by a fraction,** say $\frac{6}{7}$, we take $\frac{5}{9}$ of the unit and divide it into 7 equal parts, of which equal parts we take 6, the resulting fraction of the unit is said to be the result of multiplying $\frac{5}{9}$ by $\frac{6}{7}$; for, by Art. 25, we must perform upon $\frac{5}{9}$ that operation which when performed upon *one* gives $\frac{6}{7}$ as result.

Example. Multiply $\frac{5}{9}$ *by* $\frac{6}{7}$. We take $\frac{5}{9}$ of the unit, and divide it by 7. This gives us $\frac{5}{9\times 7}$ of the unit.

6 of these parts $=\frac{5\times 6}{9\times 7}$ of the unit. Therefore $\frac{5}{9}\times\frac{6}{7}=\frac{5\times 6}{9\times 7}=\frac{10}{21}$.

Hence, **to multiply one fraction by another, we multiply the numerators together for the new numerator and the denominators together for the new denominator.**

It follows from Art. 27 that when the multiplier and multiplicand are interchanged the quotient is not altered.

Example. $\frac{4}{5}$ of $\frac{3}{7}=\frac{4\times3}{5\times7}=\frac{3\times4}{7\times5}=\frac{3}{7}\times\frac{4}{5}$.

The word **of** is often used instead of the sign ×.

For, by 3 **of** *a quantity* we mean 3 **times** the quantity. [Art. 25; see also Art. 95.]

A *fraction of a fraction* is called a **compound fraction.**
$\frac{6}{7}$ of $\frac{5}{9}$ is a compound fraction. It is equal to $\frac{6}{7}\times\frac{5}{9}$.

Example i. $4\frac{3}{4}\times\frac{5}{38}=\frac{19}{4}\times\frac{5}{2\times19}=\frac{5}{8}$.

Example ii. $1\frac{8}{27}\times2\frac{19}{40}\times\frac{16}{33}=\frac{\overset{5\times7}{\cancel{35}}}{\underset{9\times3}{\cancel{27}}}\times\frac{\overset{9\times11}{\cancel{99}}}{\underset{8\times5}{\cancel{40}}}\times\frac{\overset{2\times8}{\cancel{16}}}{\underset{3\times11}{\cancel{33}}}$

$=\frac{5\times9\times11\times8\times2\times7}{5\times9\times11\times8\times3\times3}=\frac{14}{9}=1\frac{5}{9}$. [Art. 78.]

NOTE I. $\frac{5}{6}$ of $\frac{6}{5}=\frac{5\times6}{6\times5}=\frac{5\times6}{5\times6}=\frac{1}{1}=1$.

NOTE II. $\frac{3+4}{3\times4}=\frac{7}{12}$; 3 + 4 is **not divisible** by 3, nor by 4.

EXAMPLES. XLIII.

Multiply—

1. $\frac{3}{4}$ by $\frac{4}{3}$. **2.** $\frac{5}{8}$ by $\frac{3}{25}$. **3.** $\frac{25}{26}$ by $\frac{27}{35}$. **4.** $\frac{54}{125}$ by $\frac{64}{105}$.
5. $1\frac{1}{4}\times3\frac{1}{5}$. **6.** $2\frac{1}{7}$ by $4\frac{1}{5}$. **7.** $5\frac{2}{3}$ by $\frac{11}{51}$. **8.** $3\frac{5}{11}$ by $1\frac{9}{57}$.
9. $3\frac{1}{5}$ of $4\frac{2}{7}$ of $5\frac{5}{13}$. **10.** $4\frac{1}{3}\times\frac{27}{32}\times4\frac{7}{26}$.
11. $4\frac{1}{4}\times5\frac{1}{5}$ of $1\frac{3}{17}$. **12.** $\frac{6}{7}$ of $\frac{11}{12}\times9$ of $6\frac{1}{8}$.
13. $2\frac{1}{9}\times\frac{3}{7}$ of $\frac{5}{38}\times2\frac{1}{5}$. **14.** $3\frac{2}{3}$ of $5\frac{1}{9}\times\frac{27}{46}$.
15. $4\frac{5}{8}$ of $\frac{27}{35}\times7\times1\frac{3}{37}$. **16.** $\frac{43}{126}\times\frac{7}{860}\times\frac{540}{53}$.
17. $(\frac{1}{3}+\frac{1}{2})$ of $\frac{6}{11}$. **18.** $\frac{7}{8}\times\frac{15}{28}\times\frac{4}{11}\times\frac{44}{75}$.
19. $\frac{7}{33}$ of $(\frac{2}{7}+3\frac{1}{21})$. **20.** $(2\frac{17}{20}-2\frac{4}{5})\times3\frac{4}{7}$.
21. $(3\frac{7}{25}-\frac{7}{35})\times1\frac{3}{77}$. **22.** $(4\frac{3}{7}-2\frac{4}{5})\times2\frac{2}{19}$.
23. $1\frac{1}{23}\times(8\frac{1}{2}-4\frac{11}{12})$. **24.** $(3\frac{1}{2}+2\frac{1}{3})$ of $(1\frac{2}{5}-\frac{4}{5})$.
25. $(5\frac{3}{5}-4\frac{7}{8})\times(4\frac{5}{29}+1\frac{5}{58})$.

Add together—

26. $\frac{1}{5}$, $\frac{3}{2}$ of $\frac{1}{5}$, $\frac{1}{2}$ of $\frac{7}{9}$, $\frac{3}{2}$ of $\frac{1}{5}$. **27.** $\frac{3}{4}\times\frac{4}{7}$, $\frac{3}{8}\times1\frac{1}{2}$, $\frac{3}{7}$ of $1\frac{1}{4}$.
28. $1\frac{1}{2}$ of $\frac{2}{9}$, $4\frac{1}{5}$ of $1\frac{1}{7}$, $2\frac{2}{3}$ of $2\frac{2}{3}$. **29.** $\frac{2}{3}$ of $\frac{2}{3}$, $2\frac{1}{2}$ of $\frac{1}{3}$, $3\frac{1}{2}\times\frac{1}{4}$.
30. $2\frac{1}{6}$, $2\frac{1}{12}\times1\frac{1}{5}$, $2\frac{7}{10}\times6\frac{1}{4}$ of $\frac{1}{9}$. **31.** $4\frac{1}{3}$ of $1\frac{4}{5}$, $\frac{3}{8}$ of 10, $(\frac{1}{2}-\frac{1}{3})$.
32. $3\frac{1}{3}$ of $5\frac{1}{5}\times7\frac{1}{7}$, 2, $3\frac{2}{5}\times\frac{8}{34}$. **33.** $(\frac{5}{4}-3\frac{1}{3}\times\frac{3}{8}$ of $\frac{2}{5})$, $1\frac{1}{9}\times4\frac{1}{2}$.

90. **Division of Fractions.** By the aid of fractions we are enabled, in the division of whole numbers by whole numbers, to obtain a complete quotient in every case; for we are able to express by a fraction the result of dividing the remainder.

Example. Divide 300 *by* 7.

$$\begin{array}{r} 7\underline{)300} \\ 42 \text{ remainder } 6. \end{array}$$

Now, 6 divided by $7=\frac{6}{7}$; for $\frac{6}{7}$ of a unit is a part of a unit such that seven such parts added together make up 6 units;

hence $300\div 7=42\frac{6}{7}$ (*Complete Quotient*).

91. A fraction is divided by a whole number when its denominator is multiplied by that number.

Example i. *Divide* $\frac{6}{7}$ *by* 5. Consider $\frac{6}{7\times 5}$ *i.e.* $\frac{6}{35}$ of a unit. It is a part of a unit such that 5 such parts added together make up $\frac{6}{7}$ of a unit. Thus $\frac{6}{7}=5\times\frac{6}{7\times 5}$; so that $\frac{6}{7}\div 5=\frac{6}{7\times 5}=\frac{6}{35}$.

Example ii. *Divide* $102\frac{2}{7}$ *by* 9. $9\underline{)102\frac{2}{7}}$

Quotient 11, *remainder* $3\frac{2}{7}$.

Now $3\frac{2}{7}=\frac{23}{7}$, hence $3\frac{2}{7}\div 9=\frac{23}{7}\div 9=\frac{23}{7\times 9}=\frac{23}{63}$.

Therefore $102\frac{2}{7}\div 9=11\frac{23}{63}$ (*Complete Quotient*).

Example iii. *Divide* $4731\frac{5}{11}$ *by* 35.

Process,

$$\begin{array}{rr} & 5\underline{)4731\frac{5}{11}} \\ & 7\underline{)946\frac{16}{55}} \\ \textit{Quotient} & 135\frac{71}{385}. \end{array}$$

EXAMPLES. XLIV.

Divide—

1. 100 by 3. **2.** 21 by 5. **3.** 3000 by 9. **4.** 237 by 16.

5. 3000 by 21. **6.** 4701 by 35. **7.** $\frac{2}{3}$ by 3. **8.** $\frac{5}{7}$ by 6.

9. $1\frac{5}{6}$ by 8. **10.** $\frac{1}{9}$ by 5. **11.** $3\frac{1}{3}$ by 5. **12.** $4\frac{1}{4}$ by 2.

13. $2\frac{3}{9}$ by 7. **14.** $5\frac{1}{3}$ by 4. **15.** $401\frac{1}{4}$ by 16.

16. $543\frac{1}{5}$ by 21. **17.** $147\frac{3}{8}$ by 33. **18.** $477\frac{5}{11}$ by 45.

19. $1011\frac{11}{12}$ by 99. **20.** $555\frac{5}{7}$ by 25.

92. To **divide** a given number, say $\frac{5}{9}$, **by a fraction,** say $\frac{4}{7}$, we must find that number which when multiplied by $\frac{4}{7}$ gives $\frac{5}{9}$ as product; for, by Art. 34,

$$\textit{divisor} \times \textit{quotient} = \textit{dividend}.$$

[There is no remainder; see Art. 90.]

Example i. *Divide* $\frac{5}{9}$ *by* $\frac{4}{7}$.

Here, the *required quotient* × divisor = dividend. But $\frac{5\times 7}{9\times 4}$ is such a number that when multiplied by $\frac{4}{7}$ the product is $\frac{5}{9}$.

Therefore $\frac{5\times 7}{9\times 4}$ is the required quotient.

Hence to divide $\frac{5}{9}$ by $\frac{4}{7}$, we must multiply $\frac{5}{9}$ by $\frac{7}{4}$; whence the usual Rule, **to divide by a fraction invert the divisor and multiply.**

Process, $\frac{5}{9} \div \frac{4}{7} = \frac{5}{9} \times \frac{7}{4} = \frac{35}{36}$.

Example ii. *Divide* $2\frac{4}{9}$ *by* $1\frac{4}{7}$.

Process, $2\frac{4}{9} \div 1\frac{4}{7} = \frac{22}{9} \div \frac{11}{7} = \frac{22}{9} \times \frac{7}{11} = \frac{14}{9} = 1\frac{5}{9}$.

NOTE.—**The divisor**, when a mixed number, **must first be reduced to an improper fraction.**

EXAMPLES. XLV.

Divide—

1. $\frac{2}{3}$ by $\frac{1}{5}$. **2.** $\frac{3}{4}$ by $\frac{5}{7}$. **3.** $\frac{5}{9}$ by $\frac{7}{8}$. **4.** $\frac{11}{12}$ by $\frac{5}{7}$.
5. $\frac{3}{5} \div \frac{3}{10}$. **6.** $\frac{4}{7} \div \frac{2}{21}$. **7.** $\frac{8}{11} \div \frac{12}{13}$. **8.** $\frac{10}{11} \div \frac{5}{22}$.
9. $\frac{2}{5}$ by $1\frac{3}{5}$. **10.** $2\frac{7}{10}$ by $1\frac{1}{8}$. **11.** $3\frac{1}{3}$ by $\frac{10}{27}$. **12.** $1\frac{1}{4}$ by $\frac{5}{8}$.
13. $1\frac{4}{5}$ by $4\frac{1}{2}$. **14.** $2\frac{3}{7}$ by $3\frac{2}{5}$. **15.** $11\frac{1}{11}$ by $12\frac{1}{5}$.
16. $8\frac{1}{9}$ by $12\frac{1}{6}$. **17.** $(3 - \frac{1}{5})$ by $4\frac{1}{2}$. **18.** $(2\frac{1}{3} + 3\frac{1}{4})$ by $\frac{7}{12}$.
19. $(6\frac{1}{4} - 5\frac{3}{4})$ by $\frac{5}{16}$. **20.** $(6\frac{1}{6} - 4\frac{3}{4})$ by $2\frac{1}{3}$. **21.** $(5\frac{3}{5} - 4\frac{7}{8})$ by $\frac{1}{20}$.
22. $(3\frac{5}{7} - \frac{20}{21})$ by $(\frac{7}{8} - \frac{1}{7})$. **23.** $(3\frac{6}{25} - 2\frac{3}{50})$ by $(4 - \frac{7}{8} - 2\frac{13}{100})$.
24. $(7\frac{3}{11} + 2\frac{1}{4} - 8\frac{7}{22})$ by $(4\frac{1}{4} - 3\frac{10}{11})$. **25.** $3\frac{3}{4}$ of $6\frac{2}{3}$ of $\frac{1}{4}$ by $18\frac{3}{4}$.
26. $4\frac{1}{56}$ by $3\frac{1}{7}$ of $2\frac{1}{2}$. **27.** $3\frac{1}{5}$ of $11\frac{1}{4} \times 7\frac{1}{9}$ by $\frac{8}{33}$ of $7\frac{1}{3}$.
28. $(14\frac{3}{8} - 1\frac{3}{4})$ by $14\frac{3}{7}$ of $5\frac{3}{5}$. **29.** $\frac{5}{6}$ by $\{2\frac{1}{8} - (\frac{7}{8} + \frac{5}{12})\}$.

93. **Complex fractions** may be simplified as in the following examples.

Example i. *Simplify* $\frac{\frac{4}{15}}{\frac{7}{66}}$. The fraction $\frac{\frac{4}{15}}{\frac{7}{66}}$ applied to a unit indicates [Art. 72] $\frac{4}{15}$ of that quantity which is such that $\frac{7}{66}$ of it makes up the unit; this quantity therefore must be ($\frac{66}{7}$ of the unit).

Hence $\frac{\frac{4}{15}}{\frac{7}{66}}$ of a unit is $\frac{4}{15}$ of ($\frac{66}{7}$ of the unit).

Or thus: Since $\frac{3}{7}$ of a unit is the quotient when 3 units are divided by 7; so, $\frac{\frac{4}{15}}{\frac{7}{66}}$ of a unit is the quotient when $\frac{4}{15}$ units are divided by $\frac{7}{66}$; hence

METHOD I. $\frac{\frac{4}{15}}{\frac{7}{66}}=\frac{4}{\underset{3\times 5}{\cancel{15}}}\times\frac{\overset{3\times 22}{\cancel{66}}}{7}=\frac{3\times 4\times 22}{3\times 5\times 7}=\frac{88}{35}=2\frac{18}{35}.$

METHOD II. We may use the principle of Art. 74.

Find the L.C.M. of 15 and 66; it is $3\times 5\times 22$;

then, $$\frac{\frac{4}{15}}{\frac{7}{66}}=\frac{\frac{4}{15}\times 3\times 5\times 22}{\frac{7}{66}\times 3\times 5\times 22}=\frac{4\times 22}{7\times 5}=\frac{88}{35}=2\frac{18}{35}.$$

Example ii. $$\frac{\frac{3}{4}+\frac{5}{8}}{1\frac{5}{6}-1\frac{9}{13}}=\frac{\frac{11}{8}}{\frac{11}{78}}=\frac{11}{8}\times\frac{78}{11}=\frac{11\times 2\times 39}{11\times 2\times 4}=\frac{39}{4}=9\frac{3}{4}.$$

EXAMPLES. XLVI.

Simplify—

1. $\frac{\frac{3}{4}}{\frac{7}{8}}$.
2. $\frac{1\frac{5}{6}}{\frac{11}{13}}$.
3. $\frac{\frac{5}{8}}{3\frac{3}{4}}$.
4. $\frac{4\frac{1}{3}}{5\frac{2}{3}}$.
5. $\frac{3\frac{1}{7}}{2\frac{1}{5}}$.
6. $\frac{100\frac{1}{10}}{30\frac{1}{3}}$.
7. $\frac{99\frac{1}{10}}{24\frac{7}{41}}$.
8. $\frac{33\frac{1}{3}}{10\frac{10}{11}}$.
9. $\frac{2-1\frac{1}{5}}{\frac{4}{5}}$.
10. $\frac{3\frac{1}{3}-1\frac{1}{4}}{9\frac{1}{4}-8\frac{4}{7}}$.
11. $\frac{7\frac{1}{7}-6\frac{1}{4}}{4\frac{2}{7}-3\frac{3}{4}}$.
12. $\frac{5\frac{1}{3}+3\frac{3}{7}}{9\frac{5}{7}-5\frac{1}{3}}$.
13. $\frac{4-\frac{1}{3}+\frac{1}{2}}{4+\frac{1}{3}-\frac{1}{2}}$.
14. $\frac{3\frac{1}{5}-2\frac{3}{4}-\frac{1}{8}}{5\frac{1}{8}-2\frac{3}{5}-\frac{1}{4}}$.
15. $\frac{8\frac{1}{9}+9\frac{1}{8}-7\frac{1}{72}}{8\frac{8}{9}+7\frac{5}{72}-\frac{7}{8}}$.
16. $\frac{6\frac{2}{7}-1\frac{3}{4}}{17\frac{1}{5}\text{ of }17\frac{1}{2}}$.
17. $\frac{4\frac{1}{4}\text{ of }3\frac{1}{8}\times 1\frac{3}{5}}{4\frac{1}{4}-2\frac{1}{8}}$.
18. $\frac{3\frac{5}{8}+2\frac{5}{12}-1\frac{5}{24}}{5\frac{4}{5}+2\frac{9}{10}-1\frac{9}{20}}$.
19. $\frac{5\frac{1}{5}\text{ of }3\frac{1}{8}\div 2\frac{3}{5}}{3\frac{1}{8}+5\frac{1}{5}-2\frac{3}{5}}$.
20. $\frac{\frac{2}{5}-\frac{8}{25}+\frac{12}{125}}{\frac{1}{3}-\frac{1}{9}+\frac{26}{81}}$.
21. $\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}}$.

94. A fraction in the following form is called a **continued fraction.**

Example. Simplify $3+\cfrac{1}{10+\cfrac{1}{12+\cfrac{1}{3+\frac{1}{2}}}}$.

Such fractions are simplified by first simplifying by Art. 93 the part enclosed in the inner bracket; thus,

$$3+\cfrac{1}{10+\left\{\cfrac{1}{12+\left(\cfrac{1}{3+\frac{1}{2}}\right)}\right\}}=3+\cfrac{1}{10+\left\{\cfrac{1}{12+\frac{2}{7}}\right\}}=3+\cfrac{1}{10+\frac{7}{86}}=3\tfrac{86}{867}.$$

NOTE.—After a little practice, the brackets may be omitted.

EXAMPLES. XLVII.

Simplify—

1. $\cfrac{1}{2+\frac{1}{3}}$.

2. $\cfrac{2}{5-\cfrac{5}{6+\frac{1}{5}}}$.

3. $1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{3}}}$.

4. $1+\cfrac{1}{1-\cfrac{1}{1+\frac{3}{5}}}$.

5. $3+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}$.

6. $\cfrac{1}{5+\cfrac{1}{6+\cfrac{1}{7+\frac{1}{2}}}}$.

7. $3+\cfrac{2}{6+\cfrac{1}{5+\cfrac{1}{4+\frac{1}{3}}}}$.

8. $4+\cfrac{6}{7+\cfrac{1}{5+\cfrac{1}{4+\frac{1}{4}}}}$.

9. $8\frac{1}{2}+\cfrac{2}{3+\cfrac{2}{5+\cfrac{5}{6+\frac{1}{9}}}}$

10. $\cfrac{2}{3+\cfrac{4}{5+\cfrac{6}{7+\frac{8}{9}}}}$.

11. $\cfrac{5}{4+\cfrac{3}{3+\cfrac{2}{2+\cfrac{1}{1+\frac{1}{5}}}}}$.

12. $\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\frac{1}{5}}}}}$

95. In an expression in which there is addition or subtraction combined with multiplication or division, **the**

multiplication (or division) is to be worked before the addition and subtraction.

For [Art. 19] numbers connected by the signs + or − are *terms*, and as their name implies are considered as *separate* numbers.

Example. Simplify $3\frac{1}{7}+4\frac{1}{4}\times1\frac{1}{17}-3\frac{1}{3}$ *of* $1\frac{1}{5}+2\div3\frac{1}{5}$.

This expression is equal to

$$3\tfrac{1}{7}+(\tfrac{17}{4}\times\tfrac{18}{17})-(\tfrac{10}{3}\times\tfrac{6}{5})+(2\times\tfrac{5}{16})$$
$$=3+\tfrac{1}{7}+\tfrac{9}{2}-\tfrac{4}{1}+\tfrac{5}{8}=3+\tfrac{1}{7}+4\tfrac{1}{2}-4+\tfrac{5}{8}$$
$$=3+\tfrac{1}{7}+\tfrac{1}{2}+\tfrac{5}{8}=3+\tfrac{1}{7}+\tfrac{9}{8}=4+\tfrac{1}{7}+\tfrac{1}{8}=4\tfrac{15}{56}.$$

In an expression such as $\frac{2}{3}\times\frac{5}{7}\div\frac{5}{4}\times\frac{3}{8}$ each of the symbols × or ÷ **applies only to the number immediately following it.**

Example i. $\frac{2}{3}\div\frac{4}{5}\times\frac{7}{9}=\frac{2}{3}\times\frac{5}{4}\times\frac{7}{9}$.

Example ii. $\frac{5}{2}\div(\frac{3}{4}\times\frac{6}{5})\times\frac{2}{3}=\frac{5}{2}\times\frac{4}{3}\times\frac{5}{6}\times\frac{2}{3}$.

NOTE.—It is usual to consider $\frac{4}{5}$ of $\frac{7}{9}$ as one number.

Example. $\frac{2}{3}\div\frac{4}{5}$ of $\frac{7}{9}=\frac{2}{3}\div(\frac{4}{5}$ of $\frac{7}{9})=\frac{2}{3}\times\frac{5}{4}\times\frac{9}{7}$.

EXAMPLES. XLVIII.

Simplify—

1. $4\frac{1}{4}-3\frac{1}{5}\times\frac{5}{8}+3\frac{5}{16}$.
2. $5\frac{1}{2}+3\frac{1}{3}\times4\frac{1}{5}-7$ of $1\frac{1}{14}$.
3. $3\frac{1}{12}-5\frac{1}{2}\div4\frac{1}{3}\times1\frac{2}{11}-\frac{5}{24}$.
4. $7\frac{3}{7}$ of $4\frac{11}{13}\div5-1\frac{1}{2}\times3\frac{1}{3}$.
5. $11\frac{2}{7}+5\frac{3}{14}-1\frac{1}{5}-4\frac{1}{5}$ of $\frac{3}{49}$.
6. $3\frac{1}{3}$ of $5\frac{1}{5}-\frac{3}{4}\div\frac{7}{8}+1\frac{1}{3}\div3\frac{1}{9}$.
7. $4\frac{3}{4}$ of $1\frac{3}{5}+2\frac{3}{7}-4\frac{4}{21}$ of $\frac{3}{4}$.
8. $2\frac{3}{4}+1\frac{2}{5}$ of $5\frac{2}{77}\div\frac{2}{5}$ of $\frac{7}{11}-4\frac{1}{7}$
9. 8 of $\dfrac{1}{4\frac{4}{5}}-2\frac{1}{2}$ of $\frac{1}{15}-\frac{1}{9}\div\frac{7}{3}$.
10. $3\frac{3}{7}\div5\frac{5}{7}$ of $\frac{7}{32}-2\frac{2}{7}$.
11. $3\frac{1}{2}$ of $5\frac{1}{6}+\frac{2}{3}$ of $(6\frac{1}{4}-2\frac{1}{2})$.
12. $10\frac{5}{6}\times1\frac{12}{13}-\frac{19}{35}+\frac{3}{7}$ of $3\frac{3}{4}$.
13. $2\frac{1}{8}\div4\frac{1}{4}-\frac{23}{24}+5\frac{1}{4}\times\frac{7}{9}$.
14. $12-1\frac{1}{8}\div3\frac{3}{5}+4\frac{5}{9}\div4\frac{1}{10}$.
15. $5\frac{7}{8}\times1\frac{3}{47}-3\frac{3}{8}\div2\frac{4}{7}-\frac{13}{16}$.
16. $8-20\frac{1}{4}\times3\frac{1}{9}$ of $\frac{1}{28}-\frac{3}{16}$.
17. $\dfrac{3\frac{8}{9}}{5\frac{5}{11}}$ of $3\frac{3}{8}+2-1\frac{7}{8}\div5\frac{5}{7}$.
18. $8\frac{8}{9}$ of $\dfrac{5\frac{1}{4}}{3\frac{2}{3}}-\frac{5}{11}+1\frac{5}{6}$ of $\frac{3}{121}$.

19. $1\frac{1}{8}$ of $\dfrac{21\frac{1}{2}-9\frac{5}{6}}{8\frac{2}{3}+5\frac{3}{16}}$ of $\dfrac{6\frac{10}{11}}{4\frac{1}{5}\text{ of }9\frac{1}{11}}$. **20.** $\dfrac{\frac{1}{3}\text{ of }2\frac{1}{7}+\frac{4}{5}\text{ of }3\frac{3}{4}}{\frac{6}{7}\times 3\frac{1}{9}-2\frac{1}{5}\div 1\frac{4}{7}}$.

21. $\left(2\frac{3}{4}+7\text{ of }\dfrac{2\frac{1}{2}}{3\frac{4}{5}}-\dfrac{1\frac{2}{3}}{2\frac{1}{2}}\right)\div\left(2-\frac{151}{228}\right)$. **22.** $\dfrac{\frac{1}{2}+\frac{2}{5}\times 1\frac{1}{4}-\frac{1}{7}}{\left(\frac{3}{4}\text{ of }1\frac{7}{9}\right)\div\frac{2}{3}}$.

23. $\dfrac{5\frac{2}{5}\text{ of }7\frac{2}{9}}{8\frac{7}{24}-3\frac{5}{12}}$. **24.** $\dfrac{\frac{2}{9}\text{ of }1\frac{2}{3}\text{ of }2\frac{2}{5}}{\frac{7}{9}+\frac{5}{12}-\frac{3}{4}}$. **25.** $\dfrac{11\frac{5}{7}-7\frac{5}{11}}{11\frac{5}{7}+7\frac{5}{11}}$.

26. $\dfrac{5\frac{1}{4}}{1\frac{1}{3}}\times 2\frac{1}{8}-4\frac{1}{8}\times 3\frac{3}{11}\times\dfrac{2\frac{1}{4}}{3\frac{3}{7}}$. **27.** $\left(4\frac{3}{4}-2\frac{5}{7}\right)\div 3\frac{4}{5}+8-5\frac{1}{11}\times\frac{11}{21}$.

28. $\frac{1}{7}\times\dfrac{1}{\frac{37}{49}}-\frac{2}{13}$ of $1\frac{2}{37}+\dfrac{\frac{2}{3}}{\frac{37}{17}}$. **29.** $\frac{15}{16}\div\dfrac{1}{7\frac{3}{5}}+\frac{3}{7}$ of $10\frac{1}{5}-\frac{5}{7}$ of $2\frac{9}{10}$.

30. $14-2\frac{1}{2}\div\left\{\frac{3}{2}\text{ of }6\frac{1}{7}+\frac{3}{7}\text{ of }9\right\}$. **31.** $\frac{1}{26}\times 3\frac{1}{4}+2\frac{2}{3}-\left(\frac{7}{8}-\frac{4}{9}\right)\times 1\frac{7}{9}$.

32. $\left(2\frac{3}{4}+2\frac{1}{2}\text{ of }\dfrac{7}{3\frac{4}{5}}-\dfrac{1\frac{2}{3}}{2\frac{1}{2}}\right)\div\left(2-\frac{151}{228}\right)$. **33.** $3\frac{1}{4}\div\dfrac{1\frac{1}{2}-1\frac{1}{13}}{3\frac{1}{2}+1\frac{1}{4}}$.

34. $5+\dfrac{3\frac{1}{7}-2\frac{1}{5}}{4\frac{5}{7}-\dfrac{11}{3\frac{1}{2}}}$. **35.** $\dfrac{3\frac{3}{4}+2\frac{5}{8}-4\frac{1}{16}}{3\frac{1}{12}\times 2\frac{2}{5}\div 2\frac{1}{2}}$. **36.** $2\frac{1}{2}\times\dfrac{1}{3\frac{1}{3}+\dfrac{1}{4\frac{1}{2}}}$.

37. $\left(2\frac{1}{2}+1\frac{3}{4}-3\frac{2}{3}\right)\times\left(\frac{1}{2}\text{ of }\dfrac{2\frac{1}{4}}{4\frac{1}{2}}\right)$. **38.** $\dfrac{1\frac{7}{8}-3\frac{1}{4}\text{ of }\frac{4}{13}-\frac{3}{16}}{\frac{4}{9}+\frac{1}{2}-\frac{13}{14}}$.

39. $\dfrac{2\frac{1}{5}\times\frac{13}{33}}{\frac{3}{7}\times 1\frac{2}{17}}\div\dfrac{1\frac{2}{5}\times\frac{11}{19}}{\frac{3}{13}\times 1\frac{6}{17}}$. **40.** $\dfrac{1\frac{2}{7}+2\frac{2}{3}}{6\frac{1}{2}+3\frac{1}{3}}\times\dfrac{2\frac{1}{2}+6\frac{1}{3}}{3\frac{4}{5}-2\frac{2}{7}}$.

41. $21\frac{2}{9}-6\frac{1}{4}\times 1\frac{1}{35}-1\frac{7}{9}\times\frac{1}{14}+6-3\frac{5}{9}\div 5\frac{1}{3}-3\frac{1}{8}\div\frac{7}{9}$ of $2\frac{1}{4}\times\frac{2}{25}\div\frac{1}{42}$.

42. $\left(11\frac{11}{12}\times 1\frac{3}{13}\right)\div 2\frac{1}{4}+7-3\frac{1}{9}\times 2\frac{1}{7}+\frac{1}{4}$ of $\frac{2}{3}\div 2\frac{2}{3}\times\frac{5}{6}$.

43. $\dfrac{2\frac{7}{8}+\frac{3}{8}\text{ of }9\frac{3}{5}+3\frac{11}{12}}{117\frac{13}{40}-106\frac{14}{15}}\times\dfrac{1\frac{2}{3}\text{ of }\frac{3}{5}\text{ of }21}{23}$ of $\dfrac{3\frac{5}{6}}{5\frac{3}{12}}$.

44. $\dfrac{8\frac{3}{5}+7\frac{3}{4}+5\frac{2}{3}-4\frac{1}{2}}{13-11\frac{9}{10}+10\frac{7}{9}-9\frac{17}{20}}$ of $\frac{2}{11}$ of 365.

45. $\left(\dfrac{3\frac{1}{3}}{7}+\dfrac{2}{10\frac{1}{2}}-\frac{15}{16}\times\frac{4}{21}\right)\times 1\frac{3}{4}$. **46.** $\dfrac{5\frac{3}{4}+2\frac{2}{35}\div 1\frac{11}{25}-\frac{3}{7}\text{ of }15\frac{3}{4}}{\frac{3}{4}\times 7\frac{3}{7}-5\frac{3}{5}\div 3\frac{4}{15}}$.

47. $\frac{4}{15}$ of $\frac{39}{32}-3\frac{1}{3}\div 13\frac{1}{3}$ of $1\frac{1}{19}+\frac{6}{7}$ of $\dfrac{6\frac{5}{12}}{1\frac{5}{6}}$.

48. $\dfrac{1\frac{2}{3}\text{ of }1\frac{1}{4}}{3\frac{2}{5}+\dfrac{5}{1\frac{19}{21}}}+\dfrac{4}{5+\dfrac{6}{7+\frac{8}{9}}}$. **49.** $4+\dfrac{1}{2-\dfrac{3}{4-\frac{5}{6}}}-\dfrac{10\frac{5}{9}}{2+\dfrac{3}{4\frac{5}{6}}}$.

50. $1\frac{1}{11}-\dfrac{1-\frac{7}{22}}{2-\frac{1}{3}}+\dfrac{1\frac{2}{5}}{3\frac{1}{2}}\div\dfrac{5\frac{5}{8}}{6\frac{1}{4}}$ of $\left(\frac{1}{5}-\dfrac{\frac{1}{2}-\frac{1}{3}}{4\frac{3}{4}-3\frac{2}{9}}\right)$.

96. A decimal fraction is expressed in the form of a vulgar fraction as follows:—

Example. Express 31·135 *as a vulgar fraction.*

·135 = 135 thousandths $= \frac{135}{1000} = \frac{27}{200}$; hence 31·135 $= 31\frac{27}{200}$.

Hence, a *decimal* fraction may be expressed as a *vulgar* fraction by taking for the *numerator* the digits of the decimal, for the *denominator*, the same power of ten as the number of the digits.

The following results are often useful:—

$\frac{1}{2} = \cdot 5$; $\frac{1}{4} = \cdot 25$; $\frac{3}{4} = \cdot 75$; $\frac{1}{8} = \cdot 125$; $\frac{5}{8} = \cdot 625$.

Example i. ·365625 $[= \cdot 365625 \times \frac{4}{4}] = \frac{1}{4}$ of 1·4625 $[= \frac{1}{4}$ of $1\cdot 4625 \times \frac{4}{4}] = \frac{1}{16}$ of 5·85 $[= \frac{1}{16}$ of $5\cdot 85 \times \frac{2}{2} = \frac{1}{32}$ of $11\cdot 7] = \frac{117}{320}$.

Example ii. *Simplify* $\frac{3\cdot 4375}{53\cdot 125}$.

$$\frac{3\cdot 4375}{53\cdot 125} \times \frac{8}{8} = \frac{27\cdot 5}{425} \times \frac{4}{4} = \frac{110}{1700} = \frac{11}{170}.$$

EXAMPLES. XLIX.

Express as vulgar fractions in their lowest terms—

1. ·5. **2.** ·02. **3.** ·125. **4.** ·0125.
5. 6·4375. **6.** 10·0625. **7.** ·4163675. **8.** ·40125.
9. 367·375. **10.** 70·00004. **11.** 81·0005. **12.** 4·9125.
13. $\frac{\cdot 40125}{\cdot 56125}$. **14.** $\frac{54375}{47375}$. **15.** $\frac{\cdot 45703125}{2\cdot 77734375}$. **16.** $\frac{4\cdot 1875}{\cdot 5625}$.

97. A fraction, as $\frac{3}{5}$, applied to a unit, gives the quotient when 3 units are divided into 5 equal parts.

Example. Express $\frac{3}{5}$ of a unit *as a decimal.*

$$\left.\begin{array}{r} 5)\underline{3\cdot 0} \\ \cdot 6 \end{array}\right\} \text{Thus } \tfrac{3}{5} \text{ of a unit} = \cdot 6 \text{ units.}$$

Hence, to express a vulgar fraction as a decimal, **divide the numerator by the denominator.**

EXAMPLES. L.

Express in the decimal notation—

1. $\frac{1}{2}$. **2.** $\frac{1}{4}$. **3.** $\frac{3}{8}$. **4.** $\frac{5}{16}$. **5.** $\frac{9}{20}$.
6. $5\frac{21}{32}$. **7.** $6\frac{41}{40}$. **8.** $\frac{91}{800}$. **9.** $9\frac{21}{24}$. **10.** $\frac{35}{320}$.

Express as decimal fractions as far as the sixth decimal place—

11. $\frac{4}{9}$. 12. $\frac{5}{13}$. 13. $\frac{8}{21}$. 14. $\frac{5}{7}$. 15. $\frac{20}{91}$.

16. $3\frac{8}{11}$. 17. $4\frac{9}{14}$. 18. $11\frac{7}{9}$. 19. $20\frac{3}{7}$. 20. $3\frac{91}{92}$.

Find the greatest and least of the following, by expressing each fraction as a decimal as far as the 4th negative power of ten.

21. $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}$. 22. $\frac{5}{6}, \frac{6}{7}, \frac{17}{21}$. 23. $\frac{3}{5}, \frac{2}{3}, \frac{7}{12}$.

24. $\frac{4}{7}, \frac{5}{9}, \frac{11}{21}, \frac{23}{42}$. 25. $\frac{9}{11}, \frac{7}{9}, \frac{11}{13}$. 26. $\frac{4}{9}, \frac{7}{12}, \frac{11}{20}, \frac{13}{24}$.

Express as decimals—

27. $\$\frac{3}{4}$. 28. $\frac{5}{8}$ cwt. 29. $\frac{3}{16}$ lb. 30. $4\frac{3}{32}$ tons.

31. $\$27\frac{3}{20}$. 32. $4\frac{7}{16}$ cwts. 33. $3\frac{3}{40}$ yds. 34. $21\frac{7}{20}$ hours.

*MISCELLANEOUS EXAMPLES. LI.

1. Which is the greatest, $\frac{1}{13}$, $\frac{7}{90}$, or $\frac{3}{40}$?

2. Find the Least Common Denominator of $1\frac{1}{8}$, $\frac{6}{7}$, $\frac{7}{10}$, $\frac{2}{3}$ of $\frac{4}{5}$, $\frac{4}{5}$ of $\frac{1}{4}$.

3. The sum of $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2}}$ and $\dfrac{\frac{2}{5} \text{ of } \frac{5}{6}}{\frac{2}{9} \text{ of } 4\frac{1}{2}}$ is equal to how many times their difference?

4. The smaller of two numbers is $\dfrac{54\frac{3}{5}}{\frac{1}{5} \text{ of } 8\frac{2}{3}}$ and their difference is $\dfrac{1\frac{5}{9}}{\frac{9}{16}}$; what is the larger of the two numbers?

5. What number added to $17\frac{1}{35}$, $\frac{1}{7}$, $\frac{4}{21}$, $1\frac{7}{15}$ will make the sum 60?

6. What number divided by $4\frac{3}{5} - \frac{1}{4}$ of $3\frac{1}{10}$ will make the quotient 1?

7. What number multiplied by $9 - 3\frac{4}{25}$ will give 1 as product?

8. 48 is $\frac{5}{7}$ of some number; what is $\frac{3}{4}$ of the same number?

9. Add together the sum, difference, product and two quotients of $1\frac{1}{3}$ and $3\frac{1}{2}$.

10. How many dollars should be given in exchange for $\dfrac{\frac{4}{9} - \frac{1}{13}}{\frac{4}{9} + \frac{1}{13}}$ of \$20?

11. Express as a simple decimal $\dfrac{1{\cdot}875}{2{\cdot}1} \times \dfrac{3{\cdot}5}{3{\cdot}75}$.

12. Simplify $\dfrac{1{\cdot}5}{{\cdot}075} \times \dfrac{3{\cdot}25}{1\frac{1}{2}}$.

13. Simplify $\dfrac{{\cdot}321 \times {\cdot}321 - {\cdot}179 \times {\cdot}179}{{\cdot}321 - {\cdot}179}$ of \$5.

14. Add together $\dfrac{5{\cdot}7}{1{\cdot}52}$ and $\dfrac{{\cdot}0111}{{\cdot}74}$.

15. Subtract $\frac{\cdot 159}{\cdot 0212}$ from $\frac{1851\cdot 85}{25\cdot 9}$.

16. Find a decimal which shall differ from $\frac{19}{23}$ by less than $\frac{1}{1000000}$.

17. Show by expressing each fraction as a decimal that $\frac{6}{11}$ is greater and $\frac{8}{15}$ is less than $\frac{1}{2}+\frac{1}{2\times 3\times 4}+\frac{1}{2\times 3\times 4\times 5\times 6}$.

18. What (decimal) number multiplied by 125 is equal to the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, $\cdot 09375$ and $2\cdot 46$?

19. If the dividend is $\frac{7}{9}$ and the quotient $\frac{4}{35}$, what is the divisor?

20. Simplify $\frac{\frac{5}{9}\text{ of }4\frac{2}{13}\sim(4\frac{3}{7}+6\div 4\frac{2}{3})}{\frac{2}{3}\text{ of }1\frac{6}{7}\sim(18\frac{90}{91}\div 8)}$.

NOTE.—The **symbol** ~ is to be read '*the difference between.*'

21. Take from 1 the sum of its third, sixth, and ninth parts, divide the remainder by $5\frac{1}{3}-4\frac{1}{2}$, and find the difference between 1 and the result.

22. A person spending $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{8}$ of his money had $119 left; how much had he at first?

23. What is the least number which, when divided by each of the fractions $\frac{3}{5}$, $\frac{7}{10}$, $\frac{14}{15}$, and $\frac{8}{25}$, gives a whole number as quotient in each case?

24. One man's pace is $2\frac{5}{11}$ feet and another's is $2\frac{3}{13}$ feet; if they start walking together in step, how far will they walk before they are in step again?

25. If $\frac{4}{11}$ of a house be worth $1969.92, what is the value of $\frac{5}{16}$ of the house?

26. Three men own a house worth $6250; one has $\frac{3}{10}$, the second $\frac{3}{15}$ of it; what is the value of the third's share?

27. Three men own a house; one has $\frac{3}{16}$, the second $\frac{1}{4}$, and the third sells his share for $2800; what is the value of the house?

28. A father leaves some land to his three sons; to the first $\frac{3}{5}$, to the second $\frac{2}{15}$ of the whole; the third finds that his share is worth $1000 dollars more than the share of the second; what is the value of the land?

29. A man devised $\frac{5}{12}$ of his estate to one of his sons, and $\frac{1}{2}$ of the remainder to another; the surplus to his widow; the difference of the sons' legacies was $2352; how much did the widow receive?

30. After walking $4\frac{1}{2}$ miles a man has accomplished

$$\frac{2\frac{1}{2}-1\frac{1}{3}\text{ of }2\frac{1}{4}+1\frac{7}{9}}{(2\frac{1}{2}-1\frac{1}{3})\text{ of }(2\frac{1}{4}+1\frac{7}{9})}\text{ of }\frac{\frac{2}{11}+\frac{1}{2}}{\frac{4}{7}-\frac{3}{11}}$$

of his journey; how far has he still to walk?

31. A man having $271\frac{1}{2}$ acres of land, sold $\frac{1}{3}$ of it to one man and $\frac{3}{5}$ of it to another; what was the value of the remainder at $323.68 an acre?

32. For two parts of an examination there are 572 candidates, $\frac{8}{11}$ of the number entered for part 1, and $\frac{9}{13}$ for part 2; how many entered for both parts?

33. The driving-wheel of a locomotive is $15\frac{5}{16}$ feet, and a carriage wheel $9\frac{3}{8}$ feet in circumference; how far must a train move from rest before the same points on each of these wheels are again simultaneously in contact with the rail?

34. Find the greatest common divisor of 4, $2\frac{2}{9}$, $2\frac{2}{5}$, and $\frac{4}{90}$. (That is, find the greatest fraction which divides each, with quotient a whole number.)

35. A man gives away to each of five people $\frac{1}{10}$, $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{20}$, $\frac{3}{40}$ of a basket of apples; how many has he left, supposing he has only just enough apples to do the above operation without dividing an apple?

36. A man gives away to each of four people $\frac{1}{12}$, $\frac{3}{35}$, $\frac{3}{28}$, $\frac{2}{21}$ of a basket of apples, and has only just enough apples to be able to do this without dividing an apple; how many apples had he?

37. What is the least number by which, if $\frac{7}{9}$, $\frac{14}{27}$, and $\frac{28}{45}$ are multiplied respectively, the product in each case will be a whole number?

38. What is the greatest length which is contained a whole number of times exactly in both $18\frac{2}{5}$ feet and $57\frac{1}{2}$ feet?

CHAPTER V

COMPOUND QUANTITIES

98. In order that Arithmetic may be applied to quantities of different kinds it is necessary that for each kind of quantity a suitable **unit** should be selected.

The unit of **money** is a **dollar** (**$1**).

The unit of **length** is a **yard.**

The unit of **time** is a **day.**

The unit of **weight** is a **pound weight.**

The unit of **surface** is a square surface measuring a linear foot each way, called a **square foot.**

The unit of **solidity** is a rectangular solid measuring a linear foot each way, called a **cubic foot.**

The unit of **fluid capacity** is a **gallon.**

The unit of **angle** is a **right angle.**

99. When we are treating of large quantities it is convenient to use a large unit, and when of small quantities it is convenient to use a small unit. Hence has arisen the custom of using more than one unit for things of the same kind.

Example. The distance from Cambridge to London is said to be 52 **miles.** The height of St. Paul's dome is said to be 404 **feet.** The length of this page is about six and a half **inches.** Hence

From each principal unit, various **auxiliary** units are derived, either by dividing the principal or 'standard' unit into a number of equal parts, or by multiplying it a number of times.

100. A list of the relative magnitudes of the various auxiliary units which are in ordinary use, is called a **Table.**

A set of tables of auxiliary units of weights or measures, each of which is ten times the preceding, is called a decimal system of weights or measures.

101. The United States unit of money is the dollar; the tenth part of a dollar is a dime; the tenth part of a dime is a cent; thus a cent is the hundredth part of a dollar. Accounts are kept in dollars and cents only. Measures in a decimal system are expressed by decimals; thus 305 dollars 42 cents is written $305.42.

The French unit of money is a *Franc* (worth about 19.5 cents); the tenth part of a franc is a decime; the tenth part of a decime is a centime; thus a centime is the hundredth part of a franc. Accounts are kept in francs and centimes only. 1765.42 f. is read thus: 1765 francs 42 centimes.

EXAMPLES. LII.

Express in centimes—
1. 4.32 f. **2.** 300 f. **3.** 1.4 f.
Express in francs—
4. 4000 c. **5.** 263 c. **6.** 4.5 c.
Express in dollars—
7. 3641 cents. **8.** 103.5 cents.
Express in cents—
9. 2.36 dollars. **10.** 10.1 dollars.
Find how many dollars and cents there are in
11. 25 fr. **12.** 195 fr. **13.** 65 centimes.
Find how many francs and centimes there are in
14. $97.50. **15.** $5.85. **16.** $136.50.

102. The ordinary tables of weights and measures are not arranged on the decimal system. The principal are the following:—

LINEAR MEASURE.

12	Inches (12 in.)	make	1 Foot (1 ft.)
3	Feet	,,	1 Yard (1 yd.)
5½	Yards	,,	1 Pole (1 po.)
40	Poles (220 yds.)	,,	1 Furlong (1 fur.)
8	Furlongs (1760 yds.)	,,	1 Mile (1 mi.)
3	Miles	,,	1 League (1 lea.)

NOTE.—100 Links (=22 yards) make 1 Chain; 10 chains=1 fur.; 6080 feet=1 Nautical mile.

SQUARE MEASURE.

144	Square Inches (sq. in.)	make	1 Square Foot (1 sq. ft.)
9	Square Feet . .	,,	1 Square Yard (1 sq. yd.)
30¼	Square Yards . .	,,	1 Square Pole (1 sq. po.)
160	Square Poles . . .	,,	1 Acre (1 a.)
640	Acres . . .	,,	1 Square Mile.

NOTE.—10 square chains, or 4840 sq. yds., make 1 acre.
A square pole is sometimes called a (square) rod, or (square) perch.
A Rood (40 sq. poles) is not in ordinary use now.

CUBIC MEASURE.

1728	Cubic Inches	make	1 Cubic Foot (1 cub. ft.)
27	Cubic Feet	,,	1 Cubic Yard (1 cub. yd.)

MEASURES OF CAPACITY.

DRY MEASURE.

2	Pints (pt.)	make	1 Quart (qt.)
8	Quarts	,,	1 Peck (pk.)
4	Pecks	,,	1 Bushel (bus.)

NOTE.—The bushel contains 2150·42 cub. in.; therefore the quart contains 67⅕ cub. inches.

LIQUID MEASURE.

4	Gills (gi.)	make	1 Pint.
2	Pints	,,	1 Quart (qt.)
4	Quarts	,,	1 Gallon (gal.)

NOTE.—The gallon contains 231 cub. inches; therefore the quart (liquid measure) contains 57¾ cub. inches.

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT

(for weighing common goods).

16	Drams (16 drs.) . .	make	1 Ounce (1 oz.)
16	Ounces	,,	1 Pound (1 lb.)
100	Pounds	,,	1 Hundredweight (cwt.)
20	Hundredweight . .	,,	1 Ton.

---

28 lbs. = 1 Quarter (qr.)
112 lbs. = 1 long Hundredweight
2240 lbs. = 1 long Ton.

NOTE.—The quarter is used in Great Britain.

The long hundredweight and ton are the ordinary hundredweight and ton of Great Britain. In the United States they are used at the Custom House and in wholesale transactions in coal and iron.

TROY WEIGHT.

(The Ounce Troy is used for weighing Gold, Silver, etc.)

24 Grains (24 grs.) . make 1 Pennyweight (1 dwt.)
20 Pennyweights . ,, 1 Ounce (1 oz. *Troy*).
12 Ounces (5760 grs.) ,, 1 Pound (1 lb. *Troy*).

NOTE.—The grain Troy is the same as the grain Apothecaries; 5760 grains make 1 lb. Troy, 7000 grains make 1 lb. Avoirdupois.

APOTHECARIES' WEIGHT

(for weighing Medicines).

7000 Grains (gr.) make 16 oz. (*i.e.* 1 lb. Avoirdupois), so that 437½ grs. make 1 oz. Avoirdupois.

NOTE.—A cubic foot of water weighs about 997 oz. Avoirdupois.

ENGLISH MONEY.

4 Farthings . . . make 1 Penny (1*d.*)
12 Pence . . . . ,, 1 Shilling (1*s.* or 1/-).
20 Shillings (240*d.*) . . ,, 1 Pound (£1).

1, 2, and 3 farthings are indicated by ¼*d.*, ½*d.*, ¾*d.* respectively.

ANGULAR MEASURE.

60 Seconds (60″) . . . make 1 Minute (1′).
60 Minutes . . . . ,, 1 Degree (1°).
90 Degrees . . . . ,, 1 Right Angle (1 rt. gle.)

TIME.

60 Seconds (sec.) . . . make 1 Minute (min.)
60 Minutes . . . . ,, 1 Hour (1 hr.)
24 Hours . . . . ,, 1 Day.

NOTE.—Each day is considered to commence at midnight.

THE CALENDAR.

103. The solar year contains 365 days, 5 hours, 48 m. 48 secs. nearly. In four solar years there are 4 × 365 days + 1 day, nearly. Hence it is arranged that when the number expressing the year is divisible by 4, February shall contain 29 days instead of 28. Thus, every fourth year contains 366 days, and it is said to be a **Leap year.**

Thus 1888 was leap year; in 1888 February had 29 days.

The above assumes 365 days, 6 hours, as the length of the solar year; but this is a little too much; accordingly it is arranged that three out of four of those years which are represented in our calendar by complete hundreds (*e.g.* 2100) shall not be leap year. Every fourth complete century is, however, a leap year (*e.g.* 1600, 2000, etc.). [See Art. 154.]

EXAMPLES. LIII.

Which of the following years were leap years?—

1. 1760.
2. 1770.
3. 1800.
4. 1822.
5. 1815.
6. 1830.
7. 1842.
8. 1700.
9. 1856.
10. 1886.

104. **Thirty days hath September,**
April, June, and November.

The other months, except February, have 31 days each.

Reduction.

105. *DEF.* A **simple quantity** is a quantity expressed in terms of a single unit.

A **compound quantity** is a quantity expressed by means of more than one unit.

Example. 84 days 10 hrs. 6 min. is a compound quantity.

106. **Reduction** is the process by which a compound quantity is expressed as a simple quantity, or *vice versa.* The process will be best learned by examples.

Example i. *Reduce* 84 days 10 hrs. 6 min. *to minutes.*

From the Table

24 hrs. = 1 day.

Hence

84 days = 84 × 24 hrs. = 2016 hrs.

Thus

84 days 10 hrs. = 2016 + 10 hrs. = 2026 hrs.

From the Table

60 min. = 1 hr.

Hence

2026 hrs. = 2026 × 60 min. = 121560 min.

Thus

84 days 10 hrs. 6 min. = 121566 min.

Process,

84 days 10 hrs. 6 min.
24
——
2026 hrs.
60
——
121566 min.

Example ii. *Reduce* 1 ton 15 cwt. 3 qrs. 17 lbs. 13 oz. *to drams* (English measure).

20 cwt. = 1 ton.

4 qrs. = 1 cwt.

$\left.\begin{array}{l}28\text{ lbs.}\\=1\text{ qr.}\end{array}\right\}4\times7=28$

$\left.\begin{array}{l}16\text{ oz.}\\=1\text{ lb.}\end{array}\right\}4\times4=16$

$\left.\begin{array}{l}16\text{ drs.}\\=1\text{ oz.}\end{array}\right\}4\times4=16$

$$\begin{array}{rrl}
 & 1 & \text{ton}\\
 & 20 & \\ \hline
 & 35 & \text{cwt.}\\
 & 4 & \\ \hline
 & \left\{\begin{array}{r}143\ \text{qrs.}\\ 4\\ \hline 572\\ 7\\ \hline 4004\ \text{lbs.}\end{array}\right. & \\
\text{add} & 17 & \text{lbs.}\\ \hline
 & \left\{\begin{array}{r}4021\\ 4\\ \hline 16084\\ 4\\ \hline 64336\ \text{oz.}\end{array}\right. & \\
\text{add} & 13 & \text{oz.}\\ \hline
 & \left\{\begin{array}{r}64349\\ 4\\ \hline 257396\\ 4\\ \hline 1029584\ \text{drs.}\end{array}\right. &
\end{array}$$

or, shorter, thus:

$$\begin{array}{rl}
1 & \text{ton}\\
20 & \\ \hline
35 & \text{cwt.}\\
4 & \\ \hline
143 & \text{qrs.}\\
4 & \\ \hline
572 & \\
7 & \\ \hline
4021 & \\
16 & \\ \hline
64349 & \text{oz.}\\
16 & \\ \hline
1029584 & \text{drs.}
\end{array}$$

[See Art. 30.]

N.B.—The 17 lbs. is added to the result of multiplying by the 7; if it were added to the result of multiplying 143 qrs. by 4, the result would be the adding of 7 *times* 13 *lbs.*

EXAMPLES. LIV.

Reduce the following to inches:

1. 107 yds. 2 ft. 10 in. **2.** 1 mile 72 yds. 6 in. **3.** Half a mile.
4. A mile and a half. **5.** 20 yds. 1 ft. 6 in. **6.** 7 yds. 9 in.

How many hours are there in

7. One week? **8.** The month of January? **9.** 6 weeks 3 days?

Reduce to lbs.:

10. 83 tons 10 cwt. **11.** 46 tons 10 cwt. **12.** 107 tons 5 cwt. 10 lbs.

Reduce to ounces: **13.** 1 ton 3 cwt. 21 lbs. **14.** 15 cwt. 21 lbs.
15. 3 tons 1 cwt. 21 lbs. **16.** 4 tons 3 cwt. 34 lbs.

Reduce to drams: **17.** 6 cwt. 1 lb. 10 oz. **18.** 300 tons 2 cwt. 12 lbs.

Reduce to inches: **19.** 4 yds. 2 ft. 11 in. **20.** 1750 yds. 1 ft. 6 in.
21. 10 miles. **22.** 3 miles 220 yds. 2 ft. **23.** 8 miles 111 yds. 8 in.
Reduce to sq. ft.: **24.** 1 ac. 100 sq. yds. **25.** 120 ac. 50 sq. yds.
26. 131 ac. 2 roods. **27.** 450 ac. 1 r. 20 sq. yds. **28.** 50 ac. 100 sq. yds.
Reduce to pints: **29.** 5 bushels 3 pks. **30.** 7 bus. 2 pks. 6 qts.
31. 10 bus. 3 pks. 7 qts. **32.** 3 gal. 2 qts.
Reduce to cub. in.: **33.** 500 cub. yds. **34.** 174 c. yds. 22 c. ft.
35. 1000 c. yds. 20 c. ft. 1101 c. in. **36.** 11 c. yds. 21 c. ft.
Reduce to seconds: **37.** 12 d. 6 hrs. 25 min. **38.** 365 d. 6 hrs.
39. 2 wks. 11 hrs. 26 min. 24 sec. **40.** 4 wks. 3 d. 17 hrs. 27 min. 27 sec.
Reduce to seconds: **41.** 4 right angles. **42.** 120° 20′ 43″.
43. 60° 17′ 18″. **44.** 41° 50′. **45.** 470° 27′ 27″.
Reduce to grains: **46.** 5 lbs. 4 oz. (Troy). **47.** 10 lbs. 8 oz. (Troy).
48. 21 lb. 8 oz. 17 dwts. **49.** 9 cwt. 6 lbs. (Av.) **50.** 5 lbs. 4 oz. (Av).

N.B.—In reducing miles to yards, or acres to square yards, it is best to reduce to yards at once, unless we are prevented by the form of the question. In which case, we must proceed as follows:—

**Example* iii. *Reduce* 2 m. 3 f. 21 p. 3 yds. 2 ft. 11 in. *to inches.*

Proceeding as before, as far as the reduction to poles, we find that

2 m. 3 f. 21 p. 3 yds. 2 ft. 11 in. = 781 poles 3 yds. 2 ft. 11 in.

From the Table we find that $5\frac{1}{2}$ yds. = 1 pole, *i.e.* eleven (half yards) = 1 pole. We therefore multiply the number of poles by 11, thus,

781 poles
11
8591 (half yards).

To express half yards in yards, we must divide by 2, thus

2)8591 half yds.
4295 + 1 half yd. rem.
or 4295 + 1 ft. 6 in.

besides which there are in the question 3 yds. 2 ft. 11 in.

Hence the total length given above is 4299 yds. 1 ft. 5 in. which is 154781 inches.

The whole work may be arranged as below:—

2 m. 3 f. 21 p. 3 yds. 2 ft. 11 in.
8
19 furlongs
40
781 poles
11
2)8591 half yds.
4295 yds. + 1 ft. 6 in. rem.
3 yds. 2 ft. 11 in. added
4299 yds. 1 ft. 5 in.
3
12898 feet
12
154781 (*required no. of inches*).

*EXAMPLES. LV.

Reduce to feet: **1.** 2 mi. 3 fur. 4 p. 2 yds. **2.** 8 mi. 2 fur. 26 p. **3.** 21 mi. 1 fur. 8 po. 5 yds. **4.** 100 mi. 39 p. 1 yd. 1 ft. 6 in.

Reduce to inches: **5.** 10 mi. 3 fur. 10 po. **6.** 1 mi. 2 fur. 21 po. 4 yds. **7.** 18 mi. 7 fur. 11 po. 1 yd. **8.** 6 mi. 3 fur. 6 po. 4 yds. 6 in. **9.** 32 mi. 1 fur. 49 po. 4 yds. **10.** 7 fur. 49 po. 4 yds. 2 ft. 11 in.

Reduce to yards: **11.** 58 mi. 3 fur. 39 po. 1 ft. 6 in. **12.** 82 mi. 1 fur. 21 po. 1 yd. 1 ft. 6 in. **13.** 54 mi. 17 po. 4 yd. 1 ft. 6 in.

Reduce to sq. inches: **14.** 81 a. 3 r. 11 po. **15.** 12 a. 35 po. **16.** 100 a. 1 r. 1 po. 29 sq. yds. **17.** 3 r. 39 po. 25 sq. yds. 7 sq. ft. **18.** 121 a. 1 r. 29 po. **19.** 16 a. 3 r. 15 po. 3 sq. yds.

Reduce to sq. yds.: **20.** 1 a. 2 r. 3 po. 4 sq. yds. 2 sq. ft. 36 sq. in. **21.** 5 a. 3 r. 7 p. 10 y. 2 ft. 36 sq. in. **22.** 9 a. 29 po. 26 sq. y. 6 ft. 108 in. **23.** 224 a. 2 r. 26 p. 25 sq. y. $4\frac{1}{2}$ sq. ft. **24.** 70 a. 27 p. 7 sq. y. $2\frac{1}{4}$ sq. ft.

107. The following examples will explain the converse operation.

Example i. *Reduce* 12345 *inches to yds. ft. inches.*

Process,
12)12345 inches
3)1028 ft. + 9 inches
342 yds. + 2 feet
Hence 12345 inches = 342 yds. 2 ft. 9 in.

Example ii. *Reduce* 654321 *drams to long tons, etc.*

$$\begin{array}{ll}
16=4\times 4\left\{\begin{array}{l} 4)\underline{654321} \text{ drams} \\ 4)\underline{163580} \text{ sets of 4 drams each } +1 \text{ dram} \end{array}\right. & \\
16=4\times 4\left\{\begin{array}{l} 4)\underline{40895} \text{ oz.} \\ 4)\underline{10223} \text{ sets of 4 oz. each } +3 \text{ oz.} \end{array}\right. & \\
28=4\times 7\left\{\begin{array}{l} 4)\underline{2555} \text{ lbs. } +3 \text{ sets of 4 oz. each} \\ 7)\underline{638} \text{ sets of 4 lbs. each } +3 \text{ lbs.} \\ 4)\underline{91} \text{ qrs. } +1 \text{ set of 4 lbs.} \end{array}\right. & \\
\qquad 20)\underline{22} \text{ cwt. } +3 \text{ qrs.} &
\end{array}$$

(+3 oz. and +3 sets of 4 oz. each = 15 oz.; +3 lbs. and +1 set of 4 lbs. = 7 lbs.)

Hence 654321 drs. = 1 ton 2 cwt. 3 qrs. 15 oz. 1 dr.

NOTE.—If the question is to reduce inches to *miles*, or to miles, yds., etc.; then we may divide the yards by 1760 and so reduce at once to miles. Also, in reducing to acres, reduce at once from sq. yds. to acres (by dividing by 4840), when the question permits.

EXAMPLES. LVI.

Reduce to £ *s.* *d.*: **1.** 1000 pence. **2.** 12340 pence.
3. 6543 half pence. **4.** 12345 farthings. **5.** 10000000 farthings.

Reduce to dollars: **6.** 744 cents. **7.** A million cents. **8.** 7653 cents.

Reduce to tons, cwt. lbs. etc.: **9.** 10000 lbs. **10.** 4564 oz.
11. 83201 oz. **12.** 68540 drams. **13.** 42256 oz. **14.** 149328 oz.
15. 741,000,000 grs. **16.** 456789 oz. **17.** 3700102 drams.

Reduce to miles, yds. etc.: **18.** 40000 in. **19.** 8750 in.
20. 4567 ft. **21.** 4000001 in. **22.** 26826 in. **23.** 3706529 in.
24. 2000000 in. **25.** 700000 ft. **26.** 797979 in. **27.** 870870 in.

Reduce to acres, sq. yds. etc.: **28.** 888888 sq. in. **29.** 9123400 sq. in.
30. 432170 sq. in. **31.** 120924 sq. ft. **32.** 34512876 sq. in.

Reduce to cub. yds. etc.: **33.** 1000000 cub. in. **34.** 456000 cub. ft.
35. 8,156,740 cub. in. **36.** 4,790,123 cub. in. **37.** 33445566 cub. in.

Reduce to gals. etc.: **38.** Ten thousand pints. **39.** 100000 hlf. pts.

Reduce to bus. etc.: **40.** 5000 pints. **41.** 44567 pints.

Reduce to weeks, days, etc.: **42.** 40000 min. **43.** 1000000 secs.
44. Ten million secs. **45.** 749600 min. **46.** 8970000 secs.

Reduce to lbs. etc. (Troy): **47.** 370451 grs. **48.** 793684 grs.
49. 373700 grs. **50.** 419099 grs. **51.** 3674510 grs.

Reduce to cwt. lbs. etc., and also to lbs. Troy: **52.** 47910000 grs.
53. 89791000 grs. **54.** 3697971 grs. **55.** 89898900 grs.

Reduce to right angles, deg. etc.: **56.** 41234″. **57.** 56760″.
58. A million secs. **59.** 497643″. **60.** 12345″.

**Example* iii. *Reduce* 654392 *inches to miles, furlongs, poles, etc.*

Process, 12)654392 inches.
3)54532 ft. + 8 in.
18177 yds. 1 ft. 8 in.

Eleven (half yds.) make 1 pole; we must therefore reduce these yards to half yds., and we must notice that the remainder is 1 ft. 8 in *i.e.* half a yard and 2 in.

18177 yds. + half a yard + 2 in.
2

11)36355 half yds. + 2 in.
40)3305 poles + 2 in.
8)82 furlongs + 25 poles + 2 in.
10 miles 2 furlongs 25 p. 2 in.

*EXAMPLES. LVII.

Reduce to mi. fur. po.: **1.** 3825 yds. **2.** 733333 ft. **3.** 5144 yds.
4. 3132000 in. **5.** Ten million in. **6.** Fifty thousand yds.
7. 26826 in. **8.** 3706529 in. **9.** 5207653 in. **10.** 12595½ ft.
11. 4567890 in. **12.** 6789012 in. **13.** 887565 in. **14.** 379123 in.
Reduce to ac. r. po. etc.: **15.** 33080000 sq. ft. **16.** 8223700 sq. in.
17. Four million sq. in. **18.** 370000 sq. ft. **19.** 120924 sq. ft.
20. 34512876 sq. in. **21.** 9786531 sq. ft. **22.** 57625318 sq. in.
23. 9531756 sq. in. **24.** 7171717 sq. in. **25.** 300303892 sq. in.

108. The following examples may be noticed.

Example i. *Reduce* 7 *nautical miles to miles, yds. ft.*

A *nautical mile* contains 6080 ft., and a mile contains 1760 yards. Hence

6080
7 — 7 nautical miles contain

3)42560 feet
1760)14186 yds. + 2 ft.
8 miles + 106 yds.

1760)14186(8
14080

106

7 nautical miles = 8 miles 106 yards 2 feet.

Example ii. *Reduce* 1000 *weeks to years of* 365 *days.*

A week contains 7 days, and a year contains 365 days. Hence

1000 weeks contain
7

365)7000 days
19 yrs. 65 days.

365)7000(19
365

3350
3285

65

1000 weeks = 19 yrs. 65 days.

EXAMPLES. LVIII.

Reduce—
1. 3000 oz. of plate to cwt. lbs. **2.** 10000 oz. Troy to cwt., etc.
3. 9 tons to lbs. Troy. **4.** 1 cwt. to lbs. oz. dwt. grs.
5. 1000000 grains to cwt. **6.** 1 ton of water to gallons.
7. 1 mi. 2 f. 3 po. to chains. **8.** 7000 galls. of water to cwt.
9. 1000 chains to poles. **10.** 10000 chains to miles.
11. 400 oz. Avoir. to oz. Troy. **12.** 1300 weeks to years.
13. Express (1 lb. Avoir. – 1 lb. Troy) in grains.
14. Reduce 152 miles to nautical miles.

COMPOUND ADDITION.

109 Compound quantities are added as follows :—

Example. Add together 11 yds. 2ft. 9 in., 8 yds. 1 ft. 6 in., 21 yds. 10 in., 62 yds. 1 ft. 10 in.

The quantities should be arranged thus :—

Then adding the number of inches we find there are 35 in. ; we divide this number by 12, and obtain 2 ft. + 11 in. Adding the number of feet (including the 2 ft. from the inches) we find there are 6 ft. ; we divide 6 by 3, and obtain 2 yards. We then add the number of yards, and obtain 104 yds. We thus obtain the required sum : 104 yds. 11 in.

yds.	ft.	in.
11	2	9
8	1	6
21	0	10
62	1	10
104	0	11

EXAMPLES. LIX.

Add together the following :—

1.

cents.
71
62
35
89

2.

pence.
7
6½
5¼
11

3.

inches.
9
11
8
4
7

4.

ft.	in.
2	11
1	7
	9
1	6

5.

(Av.) lbs.	oz.
2	11
3	5
16	4
5	7
11	15

6.

ft.	in.
5	9
8	4
	11
10	6

7.

ft.	in.
8	8
10	6
7	9
4	11

8.

ft.	in.
3	9
17	6
18	4
16	8

9.

(Av.) lbs.	oz.
11	10
6	8
4	11
5	9

10.

s.	*d.*
4	7¼
8	9
11	6¾
4	9½
7	3¾

11.

days	hrs.
8	6½
9	10¼
4	5½
7	0¾
11	11¾

12.

ft.	in.
17	11¾
6	4½
3	7¼
19	11½
6	7¾

13.

(Troy) lbs.	oz.
7	0¾
4	6
5	9
7	10½
3	4½

14.

lb.	oz.	dwt.
3	4	9
5	8	4
20	3	7
1	7	9
32	1	6
4	8	4

15.

tons	cwt.	lbs.
7	4	11
27	3	4
5	0	11
38	15	6
100	13	4
7	11	3

16.

£	*s.*	*d.*
16	5	4
1	19	6
4	7	11
	8	5
42	11	9
30	1	1

	£	s.	d.
17.	3	4	9
		6	10½
			11¾
	20	14	7
		3	7¾

	£	s.	d.
18.	8	4	7½
	92	16	3
		17	7¾
	46	15	10
	3	8	7½

	£	s.	d.
19.	21	10	6
	4	13	6¾
	1	17	9½
	8	19	10
		4	11¼

	hrs.	min.	sec.
20.	41	13	6½
	172	19	7¾
	4732	17	11
	49	3	10½
	3	6	8
	74	13	4

	tons	cwt.	lbs.
21.	71	16	7¼
	432	19	7½
	84	12	1½
	3	6	8
	6	13	4
	11	3	9¼

	days	hrs.	min.
22.	49	1	0½
	7	3	9¼
	300	3	4
	47	16	8½
	536	10	6
	2	19	11¾

	days	hrs.	min.
23.	3	16	8½
	54	3	4
	267	11	9
	42	17	1½
	537	11	7
	49	18	4¼
	3	10	2½

	days	hrs.	min.
24.	5	3	8¼
	10	14	1½
	171	12	6
	4	19	11½
	561	12	6
	72	13	4
	417	2	3

	miles	yds.	in.
25.	7	17	7½
	84	13	4
	927	12	8
	29	3	7¾
	6	11	11½
	42	3	0½
	125	10	0

	£	s.	d.
26.	7	19	3½
	46	12	4
	276	4	7½
	77	7	9¾
	8760	10	6
	795	15	3¼
	20	4	4
	813	11	7¾
	99	19	11¾

	deg.	min.	sec.
27.	3271	15	2
	6132	6	5
	341	1	8
	2748	18	3
	12	5	10
	234	10	9
	4167	8	6
	329	18	5
	271	13	8

	cwt.	lbs.	oz.
28.	2769	10	8
	36	11	2
	472	13	10
	4792	18	4
	3279	15	8
	24	8	11
	429	17	5
	4198	15	4
	263	10	6

	tons	cwts.	lbs.
29.	3672	6	9
	4278	13	6
	236	4	1
	5982	14	6
	3716	8	4
	410	7	10
	6759	0	5
	4917	18	0
	427	12	6
	218	8	5
	29	15	8
	6374	8	11
	7109	15	7
	492	17	5

	days	hrs.	min.
30.	2976	7	5
	427	18	3
	8729	15	6
	6298	5	8
	417	18	6
	309	15	0
	8792	6	11
	4928	11	3
	29	8	2
	58	16	3
	8684	9	9
	6	8	10
	928	4	5
	5432	7	6

	cwt.	lbs.	oz.
31.	527	9	8
	8436	15	2
	4167	9	8
	429	8	3
	927	15	7
	8	1	2
	78	17	9
	429	0	5
	7283	18	6
	5432	12	3
	710	10	6
	636	8	2
	42	3	7
	9245	8	6

	tons	lbs.	oz.
32.	274	8	6
	329	14	5
	7346	15	8
	2478	3	5
	724	13	6
	29	15	2
	3456	8	11
	7263	5	2
	827	6	9
	9	11	5
	2345	7	2
	621	5	9
	1070	2	8
	294	15	6

	days	hrs.	min.
33.	8274	5	7
	329	8	6
	415	2	9
	42	5	10
	2736	17	4
	9	15	7
	8138	14	4
	725	4	6
	87	9	11
	234	15	4
	9027	5	9
	4378	9	3
	274	2	5
	42	9	7

	days	hrs.	min.
34.	472	15	7
	7924	8	10
	291	4	5
	468	0	8
	9	15	2
	6824	14	5
	8021	3	8
	270	17	4
	986	15	6
	46	19	8
	12	0	1
	9207	5	8
	431	7	6
	79	18	11

	lbs.	oz.	dr.
35.	3	7	10
	21	8	6
	4	11	7
	9	5	0
	12	15	17

	cwt.	lb.	oz.
36.	2	21	14
	3	7	8
	1	4	5
	3	27	6
	1	9	9

	(long) tons	cwt.	qr.	lbs.
37.	21	11	2	14
	471	3	3	7
	2	14	1	23
	75	6	0	21
	666	17	3	9

	(long) cwt.	qr.	lbs.
38.	10	2	20
	17	1	9
	19	3	27
	4	0	16
	5	3	8
	12	2	15

	(long) cwt.	qr.	lbs.
39.	77	2	13
	85	1	7
	90	3	27
	4	1	14
	31	3	4
	7	3	26

	(long) tons	cwt.	qr.	lbs.
40.	5	17	2	25
	47	18	1	1
	371	10	2	0
	37	11	3	15
	42	3	2	19
	77	7	3	27

	oz.	dwt.	gr.
41.	7	7	7
	4	3	23
	5	17	15
	1	12	13

	oz.	dwt.	gr.
42.	9	4	0
	3	7	5
	10	14	13
	5	4	10

	lbs.	oz.	dwt.	gr.
43.	3	3	4	0
		2	16	13
	5	0	0	0
	1	9	7	14

	yds.	ft.	in.
44.	4	2	8
	21	1	10
	3	2	6
	51	1	11

	mi.	fur.	p.
45.	10	7	27
	3	6	31
	41	3	9
	61	1	39

	mi.	fur.	p.	yds.
46.	1	3	21	3
	2	5	19	4½
	6	7	4	2
	8	2	37	3½

	sq.yd.	sq.ft.	sq.in.
47.	20	7	37
	41	6	108
	7	8	96
	4	5	120

	ac.	r.	p.
48.	171	3	25
	40	2	35
	6	3	29
	77	2	37

	ac.	r.	p.	sq.yd.
49.	81	3	4	22
	9	2	35	27
	84	1	37	21
	9	0	25	16

	cub.yd.	cub.ft.	cub.in.
50.	61	13	550
	37	20	864
	3	5	72
	65	21	960

	cub.yd.	cub.ft.	cub.in.
51.	3	19	144
	25	7	360
	27	26	960
	8	3	864

	cub.yd.	cub.ft.	cub.in.
52.	7	8	12
	30	21	96
	49	25	972
	61	20	36

	gall.	qt.	pt.
53.	3	2	1
	4	3	1½
	10	2	0
	5	1	1½

	bus.	pk.	qts.
54.	4	2	1
	5	3	1½
	3	1	1
	6	2	1½

	bus.	pk.	qts.
55.	6724	3	1
	8137	2	1½
	965	3	0
	2506	1	1

	hrs.	min.	sec.
56.	21	31	25
	4	21	30
	20	29	45
	4	45	36

	da.	hr.	min.
57.	121	13	40
	25	21	45
	3	6	25
	4	23	49

	wk.	da.	hr.	min.
58.	1	4	21	30
	4	5	5	15
	8	6	23	45
	10	3	10	30

	deg.	min.	sec.
59.	3	12	15
	25	35	45
	18	9	30
	12	15	47

	deg.	min.	sec.
60.	4	9	30
	14	17	37
	29	39	45
	6	27	59

	deg.	min.	sec.
61.	11	14	45
	6	39	5
	4	1	45
	46	17	7

COMPOUND SUBTRACTION.

110. Compound quantities are subtracted as follows :—

Example. Subtract 6 cwt. 52 lbs. 8 oz. from 11 cwt. 50 lbs. 6 oz.

	cwt.	lbs.	oz.	
Process,	11	50	6	
	6	52	8	
	4	97	14	(Difference required).

The *mental* process is as follows :—

8 oz. and **14 oz.** = 1 lb. 6 oz. ; put down 14 oz. and carry 1 lb.

1 lb., 52 lb., and **97 lb.** = 1 cwt. 50 lbs. ; put down 97 lb. and carry 1 cwt.

1 cwt., 6 cwt., and **4 cwt.** = 11 cwt. ; put down **4 cwt.**

NOTE.—'**Carry 1 lb.**' indicates that having made the lower line up to 1 lb. 6 oz. we are '1 lb. to the good' towards making the lower line up to the top line ; '**carry 1 cwt.**' indicates that having made the lower line up to 1 cwt. 50 lbs. we are '1 cwt. to the good' towards making the lower line up to the top line.

EXAMPLES. LX.

1. From 26 yds. 2 ft. 8 in.
subtract 25 yds. 2 ft. 9 in.

2. From 36 lbs. 10 oz. 18 dwt.
subtract 24 lbs. 10 oz. 19 dwt.

3. From $365.64
subtract $216.72.

4. From $300
subtract $76.81.

Subtract—

5. 5 yds. 2 ft. 9 in. from 8 yds. 1 ft. 6 in.
6. 4 yrs. 60 days 22 hrs. 54 min. from 6 yrs. 64 days 20 hrs. 50 min.
7. £124. 1*s.* 10*d.* from £150. 0*s.* 0*d.*
8. 3 sq. yds. 6 sq. ft. 70 sq. in. from 10 sq. yds.

Find the difference between

9. £32. 14*s.* 7½*d.* and £120. 12*s.* 1¼*d.*
10. $64.72 and $72.64.
11. 1 cwt. 12 lbs. 10 oz. and 2 cwt.
12. 23 tons 14 cwt. 21 lbs. and 30 tons 10 cwt.
13. 1 ton 16 cwt. 1 lb. and 2 tons 10 cwt.
14. 4 yds. 1 ft. 9 in. and 5 yds.

By how much does the first of each of the following quantities exceed the second?—

15. 4 mi. and 3 mi. 6 fur. 20 p.
16. 4 mi. 25 yds. and 3 mi. 200 yds. 2 ft.
17. 100 sq. yds. and 21 sq. yds. 4 sq. ft. 83 sq. in.
18. 12 sq. yds. 4 sq. ft. 72 sq. in. and 10 sq. yds. 7 sq. ft. 99 sq. in.

What must be added to one of each of the following quantities to make the other?—

19. 4 ac. 120 sq. yds. 4 sq. ft. 72 sq. in. and 5 ac.
20. 120 ac. 2 r. 25 p. 4¼ sq. yds. and 121 ac. 3 r. 10 p. 2½ sq. yds.
21. 20 cub. yds. 10 cub. ft. 144 cub. in. and 25 cub. yds.
22. 3 galls. 2 qts. 1 pt. and 10 galls.
23. 5 bus. 1 pk. and 4 bus. 2 pks. 1 qt.
24. 21 hrs. 21 min. 30 secs. and 1 day.
25. 2 weeks and 1 wk. 4 da. 21 hrs. 30 min.
26. 9 wks. 4 da. 16 hrs. 35 min. and 73 days.
27. 3 deg. 20 min. 30 sec. and a right angle.
28. 29° 41′ 22″ and 31° 21′ 20″.

111. The following method of subtracting the sum of several given compound quantities from a given compound quantity may be noticed. [See Art. 23.]

Example. Subtract the sum of £4. 16*s.* 10*d.*, £21. 14*s.* 7*d.*, £39. 18*s.* 4*d.*, *and* £121. 13*s.* 11*d.* *from* £222. 10*s.* 6*d.*

	£	*s.*	*d.*
Process,	222	10	6
	4	16	10
	21	14	7
	39	18	4
	121	13	11
Result,	34	6	10

The mental process is as follows :—

11, 15, 22, 32*d.* ; 2*s.* 8*d.* and **10***d.* = **3***s.* 6*d.* ; put down 10, *carry* **3*s.*** ;
3, 6, 14, 18, 24*s.* and **6***s.* = 30*s.* ; put down **6**, *carry* **30** ;
30, 40, 50, 60, 70*s.* and **0** = £**3** : 10*s.* ; *carry* £**3** ;
3, 4, 13, 14, 18, and **4** = **2**2 ; put down 4, *carry* **2** ;
2, 4, 7, 9, and **3** = 12 ; put down 3, *carry* **1** ;
1, 2, and **0** = **2**.

EXAMPLES. LXI.

1. Subtract the sum of 4 days 10 hrs. 6 min., 21 days 14 hrs. 5 min., and 32 days 21 hrs. 7 min. from 60 days.

2. Subtract the sum of $21.19, $42.13, $171.15, $49.40 from $300.

3. Subtract the sum of 6 yds. 2 ft. 10 in., 71 yds. 1 ft. 9 in., 32 yds. 11 in. from 200 yds. 2 ft. 8 in.

4. In one year a man spends 35 days in Boston, 67 days 5 hrs. in Philadelphia, 12 hrs. 25 mins. on the journey from Boston to Philadelphia, 2 hrs. 55 min. on the journey from Philadelphia to Baltimore, and the rest of the year in Baltimore ; how long is he there ?

5. A man's balance at his banker's is $2569.72 ; what will it be after he has drawn out $72.16, $65.23, and $96.34 ?

6. Subtract the sum of £2. 15*s.* 9*d.*, £1. 14*s.* 8*d.*, 14*s.* 7*d.*, 15*s.* 9½*d.*, £8. 15*s.*, and 3*s.* 6*d.* from £15. 14*s.* 7½*d.*

7. Out of a 3-gallon vessel I fill 4 vessels whose capacities are 1 gal. 1 pt., 3 qts., 1 pt., and 1 qt. 1 pt. ; how much is left ?

COMPOUND MULTIPLICATION.

112. The method by which the sum of a given number of repetitions of a compound quantity is found is called **compound multiplication.**

Compound Multiplication is thus an abbreviation of Compound Addition.

Example. Multiply 16 yds. 2 ft. 11 in. *by* 6.

That is, find the sum of 6 repetitions of 16 yds. 2 ft. 11 in.

$$\left.\begin{array}{r} \text{yds. ft. in.} \\ 16\,.\,2\,.\,11 \\ 16\,.\,2\,.\,11 \\ 16\,.\,2\,.\,11 \\ 16\,.\,2\,.\,11 \\ 16\,.\,2\,.\,11 \\ 16\,.\,2\,.\,11 \\ \hline 101\,.\,2\,.\;\;6 \end{array}\right\}\text{is abbreviated}\left\{\begin{array}{r} \text{yds. ft. in.} \\ 16\,.\,2\,.\,11 \\ 6 \\ \hline 101\,.\,2\,.\;\;6 \end{array}\right.$$

The multiplication table being used as on page 19.

EXAMPLES. LXII.

Multiply—

1. 13 ft. 6 in. by 4.
2. 13 lb. 4 oz. by 5.
3. £4. 8*s*. 4*d*. by 9.
4. £8. 9*s*. 6*d*. by 8.
5. 6 days 3 hrs. 21 min. by 11.
6. 2 miles 85 yds. 7 in. by 7.
7. 3 cwt. 21 lbs. 7 oz. by 8.
8. $72.83 by 5.
9. 12 yds. 1 ft. 5 in. by 3.
10. 10 cwt. 2 qrs. 20 lbs. by 12.
11. 5 wks. 5 days 5 hrs. by 10.
12. 36 deg. 54 min. 21 sec. by 11.

113. When the multiplier exceeds 12 and can be expressed as a product of factors each less than 13, the following method should be used.

Example. Multiply 7 yds. 2 ft. 3 in. *by* 42.

$$\begin{array}{ll} & \begin{array}{r} \text{yds. ft. in.} \end{array} \\ \text{Process,} & \begin{array}{r} 7\,.\,2\,.\,3 \end{array} \\ \quad(42=6\times 7) & \begin{array}{r} 6 \end{array} \\ \hline & 46\,.\,1\,.\,6 = 6 \text{ times 7 yds. 2 ft. 3 in.} \\ & \qquad\quad\;\; 7 \\ \hline & 325\,.\,1\,.\,6 = 7\times 6 \text{ times 7 yds. 2 ft. 3 in.} \end{array}$$

EXAMPLES. LXIII.

Multiply—

1. 5 wks. 4 days 8 hrs. by 24.
2. 7 yds. 2 ft. 11 in. by 32.
3. 12 lbs. 4 oz. 6 dwt. by 42.
4. 11 yds. 1 ft. 7 in. by 48.
5. 1 wk. 3 days 2 hrs. by 81.
6. 3 qrs. 17 lbs. 6 oz. by 63.
7. $21.14 by 56.
8. 2 days 5 hrs. 12 min. by 100.
9. 1 sq. yd. 7 sq. ft. 8 sq. in. by 56.
10. 17 cwt. 64 lbs. by 1000.
11. £8. 16*s.* 8*d.* by 10000.
12. 2 mi. 6 fur. 11 p. by 10000.
13. 14 sq. yd. 20 sq. in. by 4000.
14. 1 sq. mi. 25 acres by 5000.
15. £2. 11*s.* 7*d.* by 7000.
16. 7 bus. 3 pks. 6 qts. by 2400.
17. 1 ton 3 cwt. 4 lbs. by 36.
18. 4 tons 12 cwt. 70 lbs. by 42.
19. 17 tons 1 cwt. 8 lbs. by 40.
20. 3 tons 6 cwt. 54 lbs. by 70.
21. 4 lbs. 10 oz. 4 drs. by 25.
22. 3 cwt. 6 lbs. 5 oz. by 35.
23. 4 oz. 5 dwts. 6 grs. by 80.
24. 2 lbs. 9 oz. 9 dwts. by 90.
25. 5 fur. 78 yds. 2 ft. by 45.
26. 1 m. 100 yds. 2 ft. by 100.
27. 5 a. 3 r. 10 p. by 72.
28. 10 a. 2 r. 20 p. by 75.
29. 3 sq. yds. 7 sq. ft. 120 sq. in. by 80.
30. 30 sq. yds. 4 sq. ft. 100 sq. in. by 100.
31. 30 gals. 3 qts. 1 pt. by 40.
32. 10 gals. 2 qts. 1 pt. by 108.
33. 6 bus. 1 pk. 1 pt. by 120.
34. 2 bus. 5 qts. 1 pt. by 120.
35. 16 h. 30 m. 20 s. by 55.
36. 3 yrs. 110 d. 17 h. by 2400.

114. When the multiplier exceeds 12 and cannot be expressed as the product of simple factors, the following method should be used.

Example. ***Multiply*** 11 sq. yds. 3 sq. ft. 8 sq. in. ***by*** 365.

$$\begin{array}{lrl}
 & \text{sq.yds. sq.ft. sq.in.} & \\
\text{Process,} & 11\ .\ 3\ .\ \ 8 & \\
 & 10 & \\
\hline
 & 113\ .\ 3\ .\ 80 & = 10 \text{ times 11 sq. yds. 3 sq. ft. 8 sq. in.} \\
 & 10 & \\
\hline
 & 1133\ .\ 8\ .\ 80 & = 100 \quad ,, \quad ,, \quad ,, \\
 & 3 & \\
\hline
 & 3401\ .\ 7\ .\ 96 & = 300 \quad ,, \quad ,, \quad ,, \\
\text{Multip. 2d line by 6} & 680\ .\ 3\ .\ 48 & = 60 \quad ,, \quad ,, \quad ,, \\
\text{,, top line by 5} & 56\ .\ 6\ .\ 40 & = 5 \quad ,, \quad ,, \quad ,, \\
\hline
 & \underline{\underline{4138\ .\ 8\ .\ 40}} & = 365 \quad ,, \quad ,, \quad ,,
\end{array}$$

EXAMPLES. LXIV.

Multiply—

1. $6.25 by 365.
2. $75.21 by 443.
3. $91.23 by 111.
4. $53.62 by 211.
5. $72.72 by 407.
6. $311.81 by 1031.
7. $112.34 by 2111.
8. $75.61 by 10101.
9. 3 yds. 1 ft. 11 in. by 2201.
10. $98.98 by 30403.
11. 3 tons 1 cwt. 14 lbs. by 411.
12. 2 tons 3 cwt. 27 lbs. by 3213.
13. 4 m. 3 f. 120 yds. by 1530.
14. 1 m. 150 yds. by 503.
15. 2 cub. yds. 3 cub. ft. 100 cub. in. by 301.
16. 2 acres 100 sq. yds. by 601.

EXAMPLES. LXV.

Find the value of

1. 40 things at 10c. each.
2. 72 things at 9c. each.
3. 120 ....at 7c. ...
4. 84 ....at 11c. ...
5. 54 ....at $1.91 ...
6. 63 ....at $3.11 ...
7. 121 ....at $5.11 ...
8. 100 ....at $1.75 ...
9. 300 ....at $2.31 ...
10. 144 ....at $2.85 ...
11. 3 ....at £2. 6*s.* 8*d.*
12. 7 ....at $76.91 ...
13. 24 ....at £7. 5*s.* 9*d.*
14. 42 ....at $15.62 ...
15. 73 ....at £1. 2*s.* 6*d.*
16. 83 ....at $71.17 ...
17. 131 ....at £2. 3*s.* 4*d.*
18. 141 ....at $39.18 ...
19. 1101 ....at $7.17 ...
20. 401 ....at £21. 3*s.* 4*d.*
21. 1000 ....at 8c. ...
22. A gross at 6c. ...

23. How many tons of coal will supply 12 fires for 28 weeks, each fire consuming 1 ton 42 lbs. per week?

24. Find the cost of 365 tons at $6.36 per ton.

25. Find the value of 31 tons 5 cwt. at 45c. per cwt.

26. What will $3.25 per day amount to at the end of a year?

27. A farmer sold 320 sheep at a fair for $18.36 each and bought 54 heifers at $77.25 each; how much money did he carry home?

COMPOUND DIVISION.

115. **Compound Division** is the converse of compound multiplication, and may be explained thus.

A given compound quantity is to be divided into a given number of equal parts; to find one of the parts.

Example. *Divide* 17 yds. 2 ft. 4 in. by 5.

yds. ft. in.
Process, 5)17 . 2 . 4
3 . 1 . 8 *Quotient.*

The process is based upon the *repetition of subtraction;* it states that if 5 times 3 yds. 1 ft. 8 in. is subtracted from 17 yds. 2 ft. 4 in. there will be no remainder. The greatest multiple of 5 that can be subtracted from 7 is 15, that is 3 × 5. And if 5 times 3 yds. be subtracted from 17 yds., the remainder is 2 yds., *i.e.* 6 ft.

There are now 6 ft. + 2 ft., *i.e.* 8 ft., from which 5 times 1 ft. can be subtracted, leaving 3 ft. remainder, *i.e.* 36 inches.

There are now 36 in. + 4 in., *i.e.* 40 in., from which 5 times 8 in. can be subtracted; there is no remainder. Hence 5 times 3 yds. 1 ft. 8 in. subtracted from 17 yds. 2 ft. 4 in. leaves no remainder.

This is what is meant when we say that 17 yds. 2 ft. 4 in. divided by 5 gives *Quotient* 3 yds. 1 ft. 8 in.

EXAMPLES. LXVI.

Divide—

1. $7.35 by 3.
2. $71.34 by 6.
3. $61.74 by 9.
4. $36.20 by 10.
5. $76.35 by 5.
6. $76.56 by 8.
7. $62.64 by 12.
8. $100.44 by 12.
9. $48.33 by 9.
10. $762.41 by 11.
11. $36.96 by 11.
12. $90.84 by 12.
13. $762.75 by 9.
14. $231.55 by 11.

Find to the nearest inch the result of dividing

yds. ft. in.
15. 23 . 2 . 10 by 3.
16. 7 . 1 . 2 by 12.
17. 29 . 0 . 11 by 9.
18. 98 . 2 . 5 by 11.
19. 71 . 1 . 3 by 8.
20. 67 . 0 . 5 by 7.

NOTE.—When there is a **remainder** after the division, we learn that the quotient, when multiplied by the divisor, gives a product a little *smaller* than the dividend; also that if the quotient were increased by 1 inch, its product by the divisor would be a little *larger* than the dividend. A quotient can thus always be found, which is correct **to the nearest inch.**

Example. Find to the nearest second the result of dividing 47632 days 11 hrs. 56 sec. by 100.

```
            days  hrs. min. sec.
Process, 100)476,32  11   0   56(days 476.7 hrs. 47 min. 25 sec., Quotient.
               24
             ----
             7,79
               60
             -----
             47,40
                60
             -------
               24,56
```

To find the quotient when dividing by 100 we omit the last two digits in the dividend.

The remainder is 56 sec.; that is, 100 sec. all but 44 sec. Hence if we multiply 476 days 7 hrs. 47 min. 24 sec. by 100 the product will be 56 sec. too little; and if we multiply 476 days 7 hrs. 47 min. 25 sec. by 100 the product will be 44 sec. too much. The last is therefore the *nearest*.

EXAMPLES. LXVII.

Divide—

1. $492 by 100.

2. $7967.4 by 10.

3. £56. 13*s*. 4*d*. by 100.

4. £523. 19*s*. 2*d*. by 100.

5. 813 yds. 32 in. by 100.

6. 363 yds. 1 ft. 7 in. by 100.

Find to the nearest cent the result of dividing

7. $765.42 by 10.

8. $962.38 by 10.

9. $9321.76 by 10.

10. $7.01 by 10.

11. $864.32 by 100.

12. $1929.10 by 100.

13. $9269.98 by 100.

14. $9264.82 by 100.

15. 98c. by 100.

16. 32c. by 100.

Divide each of the following as nearly as possible into the given number of equal parts:—

17. 1 ton 11 cwt. 63 lbs. into 10 equal parts.

18. 3 mi. 127 yds. 2 ft. into 100 equal parts.

19. 3 a. 1230 sq. yds. into 100 equal parts.

20. 100 a. 1179 sq. yds. 6 sq. ft. 108 sq. in. into 10 equal parts.

Example. *Divide* 2318 oz. 13 dwt. 21 grs. *by* 139.

```
     oz.   dwt. grs. oz.
139)2318   13   21(16
     928
     ---
      94
      20
    ----
139)1893(13 dwt.
     503
     ---
      86
      24
    ----
139)2085(15 grs.
     695
     ---
```

The process states that 139 times 16 oz. 13 dwt. 15 grs. subtracted from 2318 oz. 13 dwt. 21 grs. leaves no remainder.

139 times 16 oz. subtracted from 2318 oz. leaves 94 oz. remainder.

This 94 oz. with the 13 dwt. gives 1893 dwt. 139 times 13 dwt. subtracted from 1893 dwt. leaves 86 dwt. remainder.

This 86 dwt. with the 21 grs. gives 2085 grs. 139 times 15 grs. subtracted from 2085 grs. leaves no remainder.

Hence 16 oz. 13 dwt. 15 grs. is the required *Quotient*.

EXAMPLES. LXVIII.

Divide—

1. $144.96 by 96. **2.** $724.80 by 151.
3. $5939.28 by 678. **4.** $989.88 by 876.
5. 69 tons 7 cwt. 14 lbs. by 411. **6.** 480 tons 10 cwt. 57 lbs. by 3213.
7. 6798 mi. 120 yds. by 1530. **8.** 545 mi. 6 fur. 210 yds. by 503.
9. 636 cu. yds. 2 cu. ft. 724 cu. in. by 301. **10.** £63. 17*s*. 6*d*. by 365.

Find to the nearest cent the result of dividing

11. $37651.23 by 117. **12.** $2345.67 by 337.
13. $4300.25 by 213. **14.** $7654.32 by 179.
15. $6245.31 by 791. **16.** $10000 by 561.

Divide each of the following as nearly as possible into the given number of equal parts :—

17. $33333.33 into 313 parts. **18.** $65040.31 into 703 parts.
19. $91919.19 into 901 parts. **20.** $999.99 into 652 parts.
21. $65.21 into 71 parts. **22.** $762.31 into 75 parts.
23. $988 into 63 parts. **24.** £21. 13*s*. 4*d*. into 571 parts.

Find

25. 1 ton 5 cwt. ÷ 7. **26.** 3 tons 1 cwt. 64 lbs. ÷ 8.
27. 33 tons 2 cwt. 95 lbs. 15 oz. ÷ 11.
28. 228 tons 18 cwt. 73 lbs. 12 oz. ÷ 76.
29. 1 m. 297 yds. 1 ft. 2 in. ÷ 29. **30.** 1111 a. 4598 sq. yds. ÷ 72.
31. 37 m. 4 fur. 16 yds. 8 in. ÷ 56. **32.** 834 a. 1 r. 10 p. ÷ 70.
33. 479 sq. yds. 1 sq. ft. 112 sq. in. ÷ 80. **34.** 3708 gals. ÷ 96.
35. 328 d. 6 h. 40 m. ÷ 120. **36.** 35° ÷ 144.

*EXAMPLES. LXIX.

Multiply—

1. 4 m. 11 p. 4 yds. (i) by 10, (ii) by 48.
2. 21 m. 3 fur. 3 yds. 1 ft. 6 in. (i) by 32, (ii) by 23.
3. 43 m. 3 f. 4 p. 1 yd. 6 in. (i) by 125, (ii) by 521.
4. 3 fur. 27 p. 4 yds. 2 ft. 9 in. (i) by 37, (ii) by 1421.
5. 4 a. 3 r. 20 p. 5 sq. yds. (i) by 10, (ii) by 72.
6. 27 a. 21 p. 27 sq. yds. 100 sq. in. (i) by 56, (ii) by 37.
7. 49 a. 2 r. 37 p. 30 sq. yds. (i) by 132, (ii) by 437.
8. 171 a. 39 p. 30 sq. yds. 1 sq. ft. (i) by 47, (ii) by 379.

Divide—

9. 10 m. 1 f. 6 p. (i) by 10, (ii) by 22.
10. 4 m. 11 p. 4 yds. 1 ft. 6 in. (i) by 32, (ii) by 29.
11. 109 m. 2 f. 17 p. 2 yds. 2 ft. (i) by 56, (ii) by 59.
12. 47 m. 1 f. 20 p. 1 in. (i) by 67, (ii) by 167.
13. 26814 a. 1 r. 32 p. 1 sq. ft. 3 sq. in. (i) by 10, (ii) by 72.
14. 18917 a. 2 r. 7 p. 7 sq. yds. 3 sq. ft. 108 sq. in. (i) by 25, (ii) by 54.
15. 33 a. 2 r. 20 p. (i) by 37, (ii) by 127.
16. 109 a. 2 r. 15 p. 12 yds. 2 ft. 6 in. (i) by 57, (ii) by 159.

116. In Art. 32, it is pointed out that **Division** may be regarded in either of **two ways**.

A quantity is to be divided into a number of equal parts, then:

I. If we are told the number of the parts, we may, by division, find the value of one of the parts (Art. 115).

II. If we are told the value of a part, we may, by division, find the number of the parts, as follows:

Example. Divide 2318 oz. 13 dwt. 21 grs. *into equal parts, each equal to* 16 oz. 13 dwt. 15 grs.

$$\begin{array}{rrrl}
\text{oz.} & \text{dwt.} & \text{grs.} & \\
16\,. & 13\,. & 15 & (1) \\
 & & 10 & \\
\hline
166\,. & 16\,. & 6 & (10) \\
 & & 10 & \\
\hline
1668\,. & 2\,. & 12 & (100)
\end{array}
\qquad
\begin{array}{rrrl}
\text{oz.} & \text{dwt.} & \text{grs.} & \qquad\qquad\text{oz. dwt. grs.} \\
2318\,. & 13\,. & 21 & \\
1668\,. & 2\,. & 12 & = 100 \times 16\,.\,13\,.\,15 \\
\hline
650\,. & 11\,. & 9 & \\
500\,. & 8\,. & 18 & = 30 \times 16\,.\,13\,.\,15 \\
\hline
150\,. & 2\,. & 15 & \\
150\,. & 2\,. & 15 & = 9 \times 16\,.\,13\,.\,15 \\
\hline
 & & & \qquad\underline{\overline{139}}
\end{array}$$

Thus 2318 oz. 13 dwt. 21 grs. contains 139 such parts.

In the above process, we multiply the divisor by 10 and the product by 10 as often as is necessary; whence each digit in the quotient is found by multiplication; and then the number of parts so obtained is **subtracted** from the dividend.

NOTE.—Comparing the above with the Example given below, we see that the divisor and quotient are interchangeable [Art. 34]. The process of Art. 115 is called **partition**, the above process is called **quotition**.

EXAMPLES. LXX.

Divide—

1. \$75.03 by \$1.23.

2. \$5678.70 by \$82.30.

3. \$4792.69 by \$97.81.

4. £30. 6*s.* 8*d.* by £2. 6*s.* 8*d.*

5. 84 yds. 1 in. by 7 yds. 1 ft. 11 in.

6. 329 yds. 2 ft. 4 in. by 3 yds. 4 in.

Find the quotient and the remainder in the division of

7. 763 yds. 2 ft. 6 in. by 71 yds. 1 ft. 11 in.

8. 10000 yds. by 24 yds. 1 ft. 6 in.

9. 13456 days 6 hrs. by 365 days 6 hrs.

10. 6732 days 6 hrs. 20 min. by 1 day 6 hrs. 20 min.

11. 16 tons 11 cwt. 16 lbs. by 1 ton 1 cwt. 1 lb.

12. 110 m. 6 fur. 70 yds. by 2 m. 1 fur. 16 yds.

Divide—

13. 7 tons 7 cwt. 25 lbs. 2 oz. by 16 cwt. 36 lbs. 2 oz.

14. 30 tons 18 cwt. 76 lbs. 8 oz. by 3 cwt. 68 lbs. 5 oz.

15. 52 tons 17 cwt. 84 lbs. by 37 cwt. 78 lbs.

16. How many bales of cloth at £1. 2*s.* 11½*d.* each, can be bought for £126. 5*s.* 5*d.*?

17. How many parcels each weighing 11 lbs. 4 oz. can be made up out of goods weighing 1 cwt. 1 lb. 4 oz.?

18. How many parcels, each weighing 41 lbs. 8 oz. can be made up out of goods weighing 1 ton, and what weight will be remaining?

The division of one quantity by another of the same kind may also be performed by ordinary *long division*, each quantity being first reduced to the same unit.

Example. Find how many times 16 oz. 13 dwt. 15 grs. *is contained in* 2318 oz. 13 dwt. 21 grs.

Reduce each of these quantities to a simple quantity in terms of the same unit; thus

16 oz. 13 dwt. 15 grs. = 8007 grs.
2318 oz. 13 dwt. 21 grs. = 1112973 grs.

By dividing, we find that 1112973÷8007=139; that is [Art. 33] 139 groups of 8007 grs. make up 1112973 grs.; or, 16 oz. 13 dwt. 15 grs. is contained 139 times in 2318 oz. 13 dwt. 21 grs.

EXAMPLES. LXXI.

How many times is

1. 1 cwt. 12 lbs. 3 oz. contained in 1 ton 14 cwt. 77 lbs. 13 oz.?
2. 1 mi. 100 yds. 2 ft. contained in 105 mi. 1266 yds. 2 ft.?
3. 16 hrs. 30 min. 20 sec. contained in 37 days 19 hrs. 48 min. 20 sec.?
4. £39. 18*s*. 1½*d*. contained in £1356. 16*s*. 3*d*.?
5. 7 yds. 2 ft. 9 in. contained in 174 yds. 6 in.?
6. 2 ac. 3 r. 32 p. contained in 44 ac. 1 r.?
7. 21 cub. yds. 22 cub. ft. 119 cub. in. contained in 1156 cub. yds. 8 cub. ft. 1123 cub. in.?
8. 3 cwt. 26 lbs. 8 oz. contained in 4 tons 8 cwt. 15 lbs. 8 oz.?

MISCELLANEOUS EXAMPLES. LXXII.

1. A gold coin weighs 123 grains (Troy), how many can be made of 1 lb. Avoir. of gold?

2. Divide $525 equally to the nearest cent among 17 persons.

3. If a person spend $2.09 per day, and during the year 1888 has saved $205.72, what was his yearly income?

4. A man has to distribute 100 lbs. of sugar to the nearest oz. among 27 people, taking the remainder for his trouble. How much does he get?

5. A man buys 6 chests of tea weighing 400 lbs. each for $1400, and sells the tea at 72 cents per lb.; how much does he gain?

6. Four tourists in England spend £21. 12*s*. in 9 days; what does each tourist spend per day on the average?

7. How much would 6 tourists spend in 16 days if they spent as much per day as in question 6?

8. A silversmith melts a silver dish weighing 96 oz., and forms of the metal 3 doz. dessert spoons, and 1 doz. table spoons; the doz. table spoons weigh 30 oz.; find the value of a dessert spoon at 5s. per oz.

9. A stationer bought 50 reams of paper at $3 a ream, and 70 reams at $4.50 a ream; find the whole cost, and how much he would lose if he sold the whole at $3.75 a ream.

10. A grocer buys 30 lbs. of tea at 48 cents a pound, and 50 lbs. of tea at 64 cents a pound, and having mixed them sells 40 lbs. of the mixture at 56 cents; at what price per lb. must he sell the remainder that he may neither gain nor lose?

11. A wine merchant pays $450 for a pipe (*i.e.* 126 galls.) of wine, and bottles it into an equal number of quart, pint, and half-pint bottles; how many bottles of each size has he?

12. £1389. 14*s.* 8*d.* is composed of an equal number of groats (4d. pieces) and of florins (2s.); how many of each coin are there?

13. Goods are bought at 21 cents per lb., carriage and other expenses amount to 10 cents per lb., and they are sold at $21.60 per cwt.; what is the gain or loss per cwt.?

14. A man bought 1765 yards of linen at 56 cents per yard; 20 yds. were soiled, and were sold at 48 cents per yard; he sold the rest at 60 cents per yard; what did he gain?

15. Distribute $169.33 among 23 men, giving 40 cents more to each of three of them than to each of the others.

16. Distribute $286.73 among 50 men, giving twice as much to each of three of them as to each of the others.

17. Distribute $4810.33 among 73 persons, giving $25 more to each of two of them than to each of the others.

18. Divide 76 yds. 2 ft. 9 inches of cloth between two persons, giving one twice as much as the other.

19. Divide 179 yds. 2 ft. 6 inches of cloth among A, B, and C so that for every inch given to A, B may have 3 inches, and C 2 inches.

20. Divide $18765.45 among 5 persons, so that one of them may have only half as much as each of the others.

21. The total expense of 12 tourists at a hotel in England is £485. 17*s.* 6*d.*; 4 of them stayed 10 weeks, 4 of them stayed 11 weeks, and 4 of them stayed 9 weeks; how much did it cost per week for one person?

22. Find the expense of covering the floor of a room containing 59 sq. yds. 2 sq. ft. at 18 cents per sq. ft.

23. The walls of a room contain 121 sq. yds. 8 sq. ft. 72 sq. in.; find the cost of papering at 20 cents per sq. ft.

24. A man walks 100 yards taking 131 paces; how many paces would he take in walking round the world, supposing the circumference to be 25,000 miles?

25. A person buys 325 gallons of spirit at $2 a gallon, how much water must he add to it that he may gain $40 and retail the mixture at $1.50 per gallon?

26. A man gives away $76.23, which is 9 cents for every $5 in his income; what is his income?

27. A, B, and C have $197 among them; B and C have together $131.59, and A and C have $164.24 together. How much has C?

28. What was the day of the month 100 days after January 1, 1885?

29. The 15th of August 1885 was a Sunday; what day of the week was January 1, 1886?

30. Find the number of days between March 1, 1884 and August 1, 1886, including one of the days named.

31. Find the number of days from December 1, 1861 to June 1, 1862 (including one of them).

32. The London 'Times' on Thursday the 25th June 1885 was numbered 31,482; what was it numbered on the 1st of January 1886?

33. The 1st of September 1885 was a Tuesday; what day of the week was January 1, 1885?

34. The London 'Standard' on Monday, August 31, 1885, was numbered 19,073; supposing the paper to have been published every week day, on what date was it numbered 18,000?

35. What was the day of the week on which the first number of the London 'Standard' (Question 34) was published?

36. Find the number of hours from noon on June 1, 1881 to noon on April 1, 1882.

37. A tank which holds 1000 gallons weighs when empty 4 cwt. 20 lbs.; what is the weight of the tank when full of water?

[N.B.—*A pint of water weighs a pound and a quarter, English measure.*]

38. What angle does the large hand of a clock describe in 10 minutes? what angle does the small hand describe in 10 minutes?

39. A, B, and C have $764; A has twice as much as B, and B has three times as much as C. How much has each?

40. Prove that if a man puts by a certain number of pence for each day in the year, he will at the end of a year have saved that number of sovereigns + the same number of half-sovereigns + five times same number of pence.

41. If the earth made a complete revolution every 24 hours, through what angle would it turn in 5 minutes?

42. If the moon made a complete revolution in her orbit in 28 days 12 hours, how long would she take to describe a right angle in her orbit; and how long to describe 2 minutes?

43. The circumferences of the large and small wheels of a bicycle are respectively 165 in. and 40 in.; how many more turns will the latter have made than the former in going a distance of 15 miles?

44. Taking the length of the year as exactly 365 days 5 hrs. 48 min. 48 secs., what will be the error in the calendar at the end of 400 years, in which there are 97 leap years?

117. The following are examples of the multiplication and division by fractions of compound quantities :—

Example i. *Multiply* 6 yds. 2 ft. 10 in. *by* $3\frac{2}{15}$.

That is, find the value of $(3+\frac{2}{15})$ repetitions of 6 yds. 2 ft. 10 in.

$$\begin{array}{llcccccl}
 & & \text{yds.} & & \text{ft.} & & \text{in.} & \\
\text{Process,} & & 6 & . & 2 & . & 10 & \\
 & & & & & & 2 & \\
\hline
 & 3) & 13 & . & 2 & . & 8 & = 6 \text{ yds. } 2 \text{ ft. } 10 \text{ in.} \times 2. \\
\hline
 & 5) & 4 & . & 1 & . & 10\frac{2}{3} & = \quad ,, \quad ,, \quad \times \frac{2}{3}. \\
\hline
 & & & & 2 & . & 9\frac{1}{3} & = \quad ,, \quad ,, \quad \times \frac{2}{15}. \\
 & & 20 & . & 2 & . & 6 & = \quad ,, \quad ,, \quad \times 3. \\
\hline
\text{Adding} & & 21 & . & 2 & . & 3\frac{1}{3} & = \quad ,, \quad ,, \quad \times (3+\frac{2}{15}). \\
\hline
\end{array}$$

NOTE.—To multiply by $3\frac{2}{11}$, that is, $\frac{35}{11}$, or $\frac{5\times 7}{11}$, we may conveniently first multiply by 5 and by 7, and then divide the result by 11.

Example ii. *Divide* 8 yds. 1 ft. $10\frac{1}{2}$ in. *by* $4\frac{1}{12}$ *of* $1\frac{3}{7}$.

Since $$4\frac{1}{12}\times 1\frac{3}{7}=\frac{49}{12}\times\frac{10}{7}=\frac{7\times 5}{6}=5\frac{5}{6},$$

therefore, to divide by $5\frac{5}{6}$, we must multiply by $\frac{6}{35}$, that is, we must multiply by 6 and divide by 35.

$$\begin{array}{llcccccl}
 & & \text{yds.} & & \text{ft.} & & \text{in.} & \\
\text{Process,} & & 8 & . & 1 & . & 10\frac{1}{2} & \\
 & & & & & & 6 & \\
\hline
 & 5) & 51 & . & 2 & . & 3 & = 8 \text{ yds. } 1 \text{ ft. } 10\frac{1}{2} \text{ in.} \times 6. \\
\hline
 & 7) & 10 & . & 1 & . & 0\frac{3}{5} & = \quad ,, \quad ,, \quad \times \frac{6}{5}. \\
\hline
 & & 1 & . & 1 & . & 5\frac{8}{35} & = \quad ,, \quad ,, \quad \times \frac{6}{35}. \\
\hline
\end{array}$$

NOTE.—Having divided the inches by 7, remainder is $1\frac{3}{5}$ in. $=\frac{8}{5}$ in. which, divided by 7, gives $\frac{8}{35}$ in.

EXAMPLES. LXXIII.

Find the value of

1. $\frac{4}{9}$ of 1 yd. 1 ft. 7 in. **2.** \$6.85 × $\frac{3}{8}$. **3.** $3\frac{1}{8}$ of 3 cwt. 21 lbs.

4. $4\frac{3}{5}$ of 7 days 6 hrs. **5.** $\frac{4}{21}$ of \$78.63. **6.** $5\frac{3}{5}$ of 11 yds. 9 in.

7. $4\frac{3}{8}$ of 1 ton 2 cwt. 3 lbs. **8.** $8\frac{4}{7}$ of 7 tons 1 cwt. 22 lbs.

9. \$564.32 × $2\frac{1}{7}$. **10.** \$16.42 × $4\frac{4}{11}$.

11. $(3\frac{1}{4}+5\frac{1}{8})$ of \$50. **12.** \$$5\frac{3}{4}$ + \$$8\frac{3}{10}$.

13. 3 mi. 200 yds. ÷ $\frac{3}{4}$. **14.** 671 yds. 1 ft. 10 in. ÷ $3\frac{7}{8}$.

15. \$965.13 ÷ $8\frac{1}{3}$. **16.** \$721.10 ÷ $7\frac{1}{9}$.

17. 10 yds. 1 ft. $6\frac{4}{5}$ in. $\div 5\frac{3}{5}$.

18. 49 yds. 1 ft. $6\frac{2}{5}$ in. $\div 2\frac{4}{5}$.

19. \$166.54 $\div 4\frac{4}{5}$.

20. 3 days 14 hrs. $\div 5\frac{3}{5}$.

21. $\frac{2}{5}$ of 4 tons 3 cwt.

22. $\frac{3}{5}$ of 3 cwt. 35 lbs.

23. $\frac{4}{5}$ of 5 a. 100 sq. yds.

24. $5\frac{1}{5}$ of $\dfrac{1\frac{1}{3}}{1\frac{1}{12}}$ of 12 yds. 1 ft. 3 in.

25. $48\frac{3}{4}$ of 12 c. yd. 100 c. in.

26. $24\,(1\frac{1}{3} - \frac{5}{6})$ of 3 bus. 2 pks.

27. 4 a. 130 p. $\div \frac{4}{15}$.

28. 5 a. 1243 yds. 3 ft. $\div \frac{5}{24}$.

29. 4 mi. 1320 yds. $\div 2\frac{1}{7}$.

30. 4 bus. 3 pks. 4 qts. $\div (\frac{1}{3} - \frac{1}{4})$.

31. \$365.42 $\div (3\frac{1}{4}$ of $5\frac{1}{13})$.

32. \$10,000 $\div (3\frac{3}{28}$ of $\frac{4}{5})$.

33. $(3\frac{7}{9} + 4\frac{6}{7})$ of $1\frac{1}{17}$ of £1. 1*s*. 1*d*.

34. $\frac{3}{5}$ of $\frac{5}{4}$ of $\frac{12}{25}$ of £10. 10*s*.

35. $3\frac{1}{2}$ of $3\frac{1}{3}$ of 89 yds. 1 ft. $7\frac{5}{8}$ in.

36. $5\frac{1}{8}$ of $\dfrac{1\frac{1}{2}}{3\frac{1}{3} + 2\frac{1}{6}} \times$ £4. 6*s*. 8*d*.

37. $27\frac{1}{3}$ of 4 d. 6 h. 20 min.

38. \$42.66 $\div \dfrac{2\frac{1}{2} - 1\frac{3}{4}}{4\frac{1}{5} - 3\frac{1}{4}}$.

39. $3\frac{1}{4}$ of $5\frac{1}{5}$ of \$7 $- 5\frac{1}{3}$ of $4\frac{1}{2}$ of \$3.50.

40. $4\frac{2}{7}$ of \$65.32 $+ 5\frac{1}{7}$ of \$71.21 $- \frac{11}{14}$ of \$560.72.

41. $\frac{4}{5}$ of \$1 $+ \frac{5}{12}$ of \$4.62 $- \frac{3}{20}$ of \$76.15.

42. $2\frac{1}{3}$ of $4\frac{1}{5}$ of \$61.35 $- 5\frac{1}{7}$ of $9\frac{5}{8}$ of \$8.82.

43. $\frac{3}{4}$ of 12 yds. 1 ft. 10 in. $- \frac{1}{15}$ of 2 yds. $- \frac{2}{11}$ of 20 yds. 6 in.

44. $7\frac{1}{4}$ of 1 mile $+ 2\frac{1}{10}$ of 1 furlong $- 3\frac{1}{3}$ of 1 mile 1 furlong.

118. One quantity is said to be expressed as **the fraction of** another quantity of the same kind when the first quantity is expressed *in terms of the* second quantity *as unit.*

The *number*, whole or fractional, which indicates a quantity in terms of a second quantity as unit, is the *number of times* which the first quantity is of the second; hence,

To express one quantity as the fraction of another we must find how many times the first contains the second.

Example i. *Express* 6 lbs. 10 oz. $12\frac{1}{2}$ dwt. *as the fraction of* 1 lb.
In other words, *reduce* 6 lbs. 10 oz. $12\frac{1}{2}$ dwt. to lbs.

METHOD I. $12\frac{1}{2}$ dwt. $= \frac{25}{2}$ dwt. $= \frac{25}{40}$ oz. $= \frac{5}{8}$ oz.; hence 10 oz. $12\frac{1}{2}$ dwt. $= 10\frac{5}{8}$ oz. $= \frac{85}{8}$ oz. $= \frac{85}{96}$ lb.; thus, 6 lbs. 10 oz. $12\frac{1}{2}$ dwt. $= 6\frac{85}{96}$ of 1 lb.

Or shortly thus: 6 lbs. 10 oz. $12\frac{1}{2}$ dwt. $= 6\dfrac{10\frac{25}{40}}{12}$ lbs. $= 6\dfrac{10\frac{5}{8}}{12}$ lbs. $= 6\frac{85}{96}$ lbs.

METHOD II. 10 oz. $12\frac{1}{2}$ dwt. $= \frac{425}{2}$ dwt.; 1 lb. $= 240$ dwt.; hence, 10 oz. $12\frac{1}{2}$ dwt. $= \dfrac{425}{2 \times 240}$ of 240 dwt. $= \frac{85}{96}$ of 1 lb.; therefore, 6 lbs. 10 oz. $12\frac{1}{2}$ dwt. $= 6\frac{85}{96}$ lbs. $= 6\frac{85}{96}$ of 1 lb.

Example ii. Express 3 days 10 hrs. 21 min. as the fraction of 14 hrs. We must express each compound quantity as a simple quantity in terms of the same unit.

METHOD I. Express each quantity in terms of the *highest* denomination, that is, in terms of 1 day.

Thus 3 days 10 hrs. 21 min.

$$=3\frac{10\frac{21}{60}}{24}\text{ days}=3\frac{10\frac{7}{20}}{24}\text{ days}=3\frac{207}{24\times 20}\text{ days}=3\frac{69}{160}\text{days}=\frac{549}{160}\text{ days,}$$

$$\text{also 14 hrs.}=\frac{14}{24}\text{ day}=\frac{7}{12}\text{ day.}$$

$$\text{Then 3 days 10 hrs. 21 min.}=\frac{549}{160}\text{ day}=\frac{\frac{549}{160}}{\frac{7}{12}}\text{ of }\frac{7}{12}\text{ day}$$

$$=\frac{549\times 12}{160\times 7}\text{, that is }\frac{549\times 3}{40\times 7}\text{, that is }\frac{1647}{280}\text{ of 14 hrs.}$$

METHOD II. Express each quantity in terms of the *lowest* denomination, that is, in terms of 1 minute.

$$\text{Thus 3 days 10 hrs. 21 min.}=4941\text{ min.,}$$
$$\text{and 14 hrs.}=840\text{ min.}$$

$$\text{Then 3 days 10 hrs. 21 min.}=\frac{4941}{840}\text{ of 840 min.}$$
$$=\frac{1647}{280}\text{ of 14 hrs.}$$

119. *DEF.* The **ratio** of one quantity to another of the same nature is that relation of the quantities to one another which is expressed by the **number** (whole or fractional) of times which the first quantity is of the second.

Thus, the **ratio** of one quantity to another is that which expresses their **relative size.**

Example. The ratio of \$21 *to* \$6 is $\frac{21}{6}$; for \$21 is $\frac{21}{6}$ times \$6.

The ratio of \$21 *to* \$6 is written thus, \$21 : \$6, or thus, $\frac{\$21}{\$6}$, so that

\$4.50 : \$3.25, or $\frac{\$4.50}{\$3.25}$ is a convenient abbreviation for

(i) the number of times which \$4.50 contains \$3.25,
or (ii) the quotient when \$4.50 is divided by \$3.25,
or (iii) the number expressing \$4.50 in terms of \$3.25,
or (iv) the measure of \$4.50 in terms of \$3.25,
or (v) the fraction which \$4.50 is of \$3.25,
or (vi) the ratio of \$4.50 to \$3.25.

EXAMPLES. LXXIV.

1. Reduce \$7.50 to the fraction of \$1.

2. Reduce \$18.60 to the fraction of \$1.

3. Express \$17.50 as the fraction of \$5.

4. What fraction of \$5.50 is \$13.20 ?

5. How many times does \$102.25 contain \$6.25 ?

6. What part of \$6.51 is \$1.56 ?

Express each of the following 12 quantities as the fraction of its *highest* denomination.

7. 1 yd. $5\frac{1}{4}$ in. **8.** 3 cwt. 7 lbs. $4\frac{1}{2}$ oz. **9.** 21 yds. $31\frac{1}{2}$ in.
10. 1 yd. 1 ft. 1 in. **11.** 5 cwt. 70 lbs. 10 oz.
12. 2 tons 11 cwt. 87 lbs. 8 oz. **13.** 3 mi. 168 yds. 2 ft.
14. 3 yds. 2 ft. 9 in. **15.** 1 cu. yd. 20 cu. ft. 144 cu. in.
16. 3 days 4 hrs. 10 min. **17.** 3 a. 5 p. 11 sq. yds.
18. 3° 21′ 45″.

19. Reduce 1 ft. 1 in. to the fraction of 13 yds. 1 ft. 1 in.

20. Reduce 3 cwt. 62 lbs. 8 oz. to the fraction of 1 cwt. 75 lbs.

21. Reduce 4 tons 5 cwt. 31 lbs. 4 oz. to the fraction of 5 tons.

22. What fraction of 3 mi. 4 fur. is 1 mi. 220 yds. 2 ft. ?

23. What fraction of 4 cu. yds. 9 cu. ft. is 8 cu. ft. 432 cu. in. ?

24. What fraction of 1 ac. 3 r. is 480 sq. yds. 4 sq. ft. 72 sq. in. ?

Simplify—

25. $\dfrac{\text{2 yds. 6 in.}}{\text{2 mi. 96 yds.}}$ **26.** $\dfrac{\text{96 cents.}}{\text{69 dollars.}}$ **27.** $\dfrac{\text{3 yds. 1 ft.}}{\text{4 yds. 6 in.}}$

28. $\dfrac{\text{3 cwt. 62 lbs. 8 oz.}}{\text{1 cwt. 75 lbs.}}$ **29.** $\dfrac{\text{4 tons 5 cwt. 31 lbs. 4 oz.}}{\text{5 tons 10 cwt.}}$

30. Reduce 1 ton 2 cwt. 10 lbs. to the fraction of 1 cwt. 5 oz.

31. Reduce 2 yds. 2 ft. 11 in. to the fraction of 14 yds. 2 ft. 7 in.

32. What part of 7 yds. 2 ft. 10 in. is 10 yds. 2 ft. 7 in. ?

33. When the unit is \$10, what does $\frac{3}{4}$ represent ?

34. 5 lbs. 5 oz. 6 dwt. is represented in terms of a certain unit by $\frac{1}{5}$; what is that unit ?

35. Express 2 cwt. 87 lb. 8 oz. in terms of 5 tons as unit.

36. Express 5 tons in terms of 2 cwt. 87 lbs. 8 oz. as unit.

37. What part of a nautical mile is 7 furlongs ?

38. When \$100 is the unit, how much is $37\frac{5}{20}$ (units) ?

39. What is the ratio of 4 tons 17 cwt. 62 lbs. 8 oz. to $\frac{1}{2}$ cwt. ?

40. What is the ratio of 3 yds. 2 ft. 9 in. to 1 mi. ?

41. What is the ratio of \$4.75 to \$5.86 ?

Example. Reduce $3\frac{3}{7}$ of 2 oz. 15 dwt. to the fraction of $1\frac{5}{7}$ of 3 oz. 17 dwt.

$3\frac{3}{7}$ of 2 oz. 15 dwt. is $\dfrac{3\frac{3}{7} \text{ of 2 oz. 15 dwt.}}{1\frac{5}{7} \text{ of 3 oz. 17 dwt.}}$ of $1\frac{5}{7}$ of 3 oz. 17 dwt.

[Art. 119.]

Hence, the required fraction is

$$\frac{3\frac{3}{7} \text{ of 2 oz. 15 dwt.}}{1\frac{5}{7} \text{ of 3 oz. 17 dwt.}} = \frac{3\frac{3}{7} \text{ of 55 dwt.}}{1\frac{5}{7} \text{ of 77 dwt.}} = \frac{24 \times 7 \times 55}{12 \times 7 \times 77} = \frac{10}{7}.$$

Thus $3\frac{3}{7}$ of 2 oz. 15 dwt. is $\frac{10}{7}$ of ($1\frac{5}{7}$ of 3 oz. 17 dwt.)

EXAMPLES. LXXV.

1. Express $4\frac{1}{12}$ of \$32.16 as the fraction of $5\frac{1}{16}$ of \$38.40.

2. Express $1\frac{1}{6}$ of \$2.22 as the fraction of $\frac{4}{15}$ of \$5.25.

3. Express $2\frac{1}{4}$ of 2 ft. $4\frac{1}{2}$ in. as the fraction of $5\frac{1}{2}$ of 3 yds. 1 ft.

4. Reduce $4\frac{1}{5}$ of $2\frac{3}{7}$ of \$427 to the fraction of $13\frac{1}{3}$ of $4\frac{1}{4}$ of \$45.61.

5. How many times is 2 lbs. 5 oz. contained in $3\frac{1}{5}$ of $9\frac{1}{4}$ cwt.?

6. What fraction of $4\frac{3}{15}$ of 5 cwt. 88 lbs. is $2\frac{1}{3}$ of 1 lb. 5 oz.?

7. What fraction is $3\frac{1}{9}$ of 1 ton $3\frac{1}{4}$ cwt. of $5\frac{3}{5} \times 4$ tons $5\frac{1}{2}$ cwt.?

Simplify—

8. $\dfrac{3\frac{1}{8} \text{ of 7 yds. 4 in.}}{\frac{1}{16} \text{ of 3 yds. 1 ft. 5 in.}}$

9. $\dfrac{\frac{3}{5} \text{ of } \frac{2}{7} \text{ of 1 ft.}}{1\frac{1}{2} \text{ yds.}}$

10. $\dfrac{2\frac{1}{4} \text{ of \$68.92}}{9\frac{1}{4} \times \$87.81}.$

11. $\dfrac{6\frac{1}{6} \text{ of 29 sq. ft.}}{\frac{3}{4} \text{ of 17 sq. yds.}}$

12. $\dfrac{4\frac{5}{9} \text{ of } 6\frac{3}{7} \text{ of 1 ft. 5 in.}}{3\frac{2}{5} \times 5\frac{6}{7} \text{ of } 6\frac{1}{4} \text{ in.}}$

13. $\dfrac{\frac{2}{5} \text{ of } 1\frac{2}{3} \text{ of 37 yds. } 3\frac{1}{4} \text{ in.}}{\frac{2}{7} \text{ of } 1\frac{1}{6} \text{ of 6 ft. 5 in.}}$

14. What fraction of $3\frac{5}{9}$ of 171 yds. 2 ft. $9\frac{3}{4}$ in. is equal to $2\frac{2}{3}$ of 286 yds. 1 ft. $8\frac{1}{4}$ in.?

15. What part of \$5.15$\frac{5}{8}$ is $\frac{5}{12}$ of \$24.75?

16. Express $1\frac{1}{3}$ of 7 yds. in terms of $1\frac{1}{6}$ of $5\frac{1}{2}$ poles.

17. The unit is 21 tons 7 cwt., and the number applied to the unit is $\frac{3}{35}$ of $7\frac{7}{9}$, what is the quantity?

18. Express the sum of $\frac{1}{62}$ of the number of days in January 1886, $\frac{9}{58}$ of the number of days in February 1888, and $\frac{3}{4}$ of $\frac{2}{3}$ of the number of days in the year 1900, as the fraction of the number of days in the year 1880.

19. What is the ratio of an oz. Avoir. to 1 oz. Troy?

20. Express 21 m. 3 fur. 23 p. as the fraction of a mile, and multiply 43 yds. 1 ft. 6 in. by the result.

21. Find the value of 7 m. 330 yds. of telegraph wire at \$28.32 per mile.

22. Find the value of 52 a. 130 p. at \$756.64 per acre.

120. A decimal system of weights and measures, called the **Metric System** (see p. 143) is used in scientific treatises, and is in general use on the continent of Europe.

In the **Metric** system the unit of length is called a **Metre** (it contains 39·37079 inches).

The Table is as follows :—

10 metres = 1 decametre.
10 decametres = 1 hectometre.
10 hectometres = 1 kilometre.
etc.

$\frac{1}{10}$ of a metre = 1 decimetre.
$\frac{1}{10}$ of a decimetre = 1 centimetre.
$\frac{1}{10}$ of a centimetre = 1 millimetre.
etc.

Example. A certain line *contains* 4700·125 metres; *express its length as a* compound quantity.

Result. 4 kilometres, 7 hectometres, 1 decimetre, 2 centimetres, 5 millimetres.

EXAMPLES. LXXVI.

Express in kilometres—

1. 3742 metres.
2. 40000 metres.
3. 3·42 metres.

Express in centimetres—

4. 12·3 metres.
5. 403·36 metres.
6. ·09 metre.
7. 1 kilometre.
8. ·031 kilometre.
9. ·0003 kilometre.

121. A decimal system of weights and measures is practically used whenever any quantity is expressed in terms of a *single unit in the decimal notation.*

It is quite unnecessary to assign a *name* to every decimal multiple or submultiple of the unit.

Example. A length may be recorded as containing 39·1393 inches.

122. We give some examples of the **reduction** of simple decimal quantities from one unit to another.

Example i.

Reduce £4·11875 *to* pence.

Process, 4·11875 pounds
20
$82\cdot375\cancel{0}$ shillings
12
Result, $\underline{988\cdot5\cancel{00}}$ pence.

Example ii.

Reduce 143564 oz. to tons.

Process, $\begin{cases} 4)143564 \text{ oz.} \\ 4)35891 \end{cases}$
100)8972·75 lbs.
20)89·7275 cwt.
Result, $\underline{4\cdot486375}$ tons.

EXAMPLES. LXXVII.

Reduce—

1. 45·6 lbs. to oz.
2. ·00625 of 1 sq. yd. to sq. in.
3. ·36875 days to minutes.
4. ·625 of a guinea to pence.
5. 4·3727 tons to lbs.
6. 3·59375 of 5 yds. to inches.
7. 7·6375 acres to poles.
8. 4·609375 tons to lbs.
9. ·46875 cwt. to lbs.
10. 3·68125 days to seconds.
11. 2·4123 miles to yds.
12. 3·412 acres to sq. yds.
13. 4563 minutes to days.
14. 365·241 pence to pounds.
15. 34567·4 lbs. to tons.
16. 7 lbs. to tons.
17. 123471 secs. to days.
18. 11000 yds. to miles.
19. 121 lbs. Avoir. to lbs. Troy.
20. 2345 pints to bushels.
21. 490 lbs. to tons.
22. 270 secs. to days.
23. 123458 grains to tons.
24. ·891 sec. to degrees.

123. Compound Quantities may be **reduced** to Simple Quantities, and *vice versa.*

Example i. *Reduce* £4·11875 *to* £ *s. d.*

Process,
$$\begin{array}{l} 4{\cdot}11875 \text{ pounds.} \\ \underline{20\phantom{{\cdot}0000}} \\ 2{\cdot}375\cancel{0}\cancel{0} \text{ shillings.} \\ \underline{12\phantom{{\cdot}000}} \\ 4{\cdot}5\cancel{0}\cancel{0} \text{ pence.} \end{array}$$

The £4 is not reduced to shillings; the 2*s.* is not reduced to pence.

Hence, £4·11875 = £4. 2*s.* 4¼*d.*

Example ii. *Reduce* 6 miles 4 fur. 38 yds. 18 in. *to* miles.

Process,
$$\begin{array}{l} \left\{\begin{array}{l} 6\overline{)18} \text{ in.} \\ 6\overline{)3} \end{array}\right. \\ \left\{\begin{array}{l} 11\overline{)38{\cdot}5} \text{ yds.} \\ 20\overline{)3{\cdot}5} \end{array}\right. \\ 8\overline{)4{\cdot}175} \text{ fur.} \end{array}$$

Result, $\underline{6{\cdot}521875}$ miles.

18 in. is reduced to yds. by dividing by 6 × 6; the 38 yds. is added to it; and so on.

EXAMPLES. LXXVIII.

Express as compound quantities—

1. £3·45.
2. 1·41625 yds.
3. 21·36875 miles.
4. 13·4375 acres.
5. 2·375 tons.
6. ·365625 of 5 yds.
7. 2·459375 of 2 tons.
8. 6·84375 of ½ ton.
9. 3·65625 of 36 lbs.
10. ·431246 of 100 days.
11. ·474609375 of 256 days.
12. 1·2371 of 2 years.
13. 2·8175 of 21 cwt.

Express as simple decimal quantities in terms of the unit given—

14. 3 yds. 1 ft. 6 in. as dec. of 1 yd. **15.** \$36·96 as dec. of \$2·31.
16. 5 yds. 2 ft. 9 in. ,, ⅝ yd. **17.** 21 lb. ,, 1 ton.
18. 1⅓ in. as dec. of 9 yds. 2 ft. **19.** 216 sec. ,, 1 day.

Express each of the following as a decimal of its highest denomination—

20. 1 ton 4 cwt. 37 lbs. 8 oz. **21.** 3 miles 3 fur. 121 yds.
22. 1 week 2 days 15 hours. **23.** 1 day 3 hrs. 9 min. 27 sec.
24. 1 bush. 1 pk. 3 qts. 1 pt. **25.** 10 miles 3 fur. 33 yds.
26. 11 tons 12 cwt. 2 lbs. **27.** 1 pole 2 yds. 2½ in.
28. 2 gals. 2 qts. 1 gill. **29.** 1 ac. 3 r. 15·45 p.

124. To express one compound quantity as the decimal of another we proceed as in Art. 118, Example ii, and convert the resulting vulgar fraction to a decimal.

Example i. *Reduce* 1 yd. 1 ft. 4 in. *to* the decimal of 2 yds. 1 ft. 9 in.

The required decimal is equal to the fraction $\frac{\text{1 yd. 1 ft. 4 in.}}{\text{2 yds. 1 ft. 9 in.}}$, that is to $\frac{52}{93}=\cdot 559139\dot{7}$ etc.

Hence 1 yd. 1 ft. 4 in. $=\underline{\cdot 559139}$ etc. of 2 yds. 1 ft. 9 in.

Example ii. *Find the value of* ·625 *of* 34 days 9 hrs. 20 min.

$$\cdot 625=\frac{625}{1000}=\frac{25}{40}=\frac{5}{8};$$

hence ·625 of 34 days 9 hrs. 20 min. $=\frac{5}{8}$ of 34 days 9 hrs. 20 min.
$=\underline{\text{21 days 11 hrs. 50 min.}}$
[Art. 117.

Example iii. *Find the value of* ·627 *of* 2 tons 11 cwt. 81 lbs.

$$\begin{array}{lr}
\text{Reducing 2 tons 11 cwt. 81 lbs.} & =5181\text{ lbs.}\\
\text{Multiplying by} & \cdot 627\\ \hline
 & 36267\\
 & 10362\\
 & 31086\\ \hline
\text{Reducing to tons, cwt. lbs.} & 100\overline{)3248\cdot 487}\text{ lbs.}\\
 & 20\overline{)32\cdot 48}\text{ lbs.}\\
 & 1\cdot 12
\end{array}$$

Hence ·627 of 2 tons 11 cwt. 81 lbs. $=\underline{\text{1 ton 12 cwt. 48·487 lbs.}}$
[See Arts. 130, 155.

Example iv. *Reduce* 5¾ *of* 5 miles 20 yds. *to the decimal of* 5¼ *of* 1 mile 130 yds.

Here 5¾ of 5 miles 20 yds. is $\frac{5\frac{3}{4} \text{ of } 5 \text{ miles } 20 \text{ yds.}}{5\frac{1}{4} \text{ of } 1 \text{ mile } 130 \text{ yds.}}$ of (5¼ of 1 mile 130 yds.) [Art. 119.

This fraction is $\frac{23 \times 8820}{21 \times 1890}$, or $\frac{23 \times 21 \times 21 \times 20}{21 \times 21 \times 90}$, or $\frac{46}{9}$, or 5·111 . . .

Hence 5¾ of 5 miles 20 yds. is $\underline{5{\cdot}111}$ . . . of (5¼ of 1 mile 130 yds.)

EXAMPLES. LXXIX.

Reduce—

1. 9 in. to the decimal of 2 ft.

2. 33 yds. 7 in. to the decimal of 53 yds. 4 in.

3. 5 yds. 1 ft. 1½ in. to the decimal of 17 yds. 2 ft. 9 in.

4. 6 ft. 6¾ in. to the decimal of 5 ft. 3 in.

5. 1 yd. 3¾ in. to the decimal of 35 yds. 1 ft. 8 in.

6. 1 day 7 hrs. to the decimal of 31 days.

7. 1 day 1 hr. 45 min. to the decimal of 3 days 11 hrs. 20 min.

8. 2 cwt. 8 lbs. to the decimal of 81 lbs. 4 oz.

9. 14 lbs. 15 oz. to the decimal of 3 lbs. 12 oz.

10. 6 cwt. 84 lbs. 6 oz. to the decimal of 1 ton 2 cwt. 22 lbs.

11. 2 tons 3 cwt. 75 lbs. 2 oz. to the decimal of 10 tons 7 cwt. 23 lbs. 10 oz.

12. 5 ft. 5 in. to the decimal of 8 ft. 8 in.

13. 1 mi. 550 yds. to the decimal of 1 mi. 240 yds.

14. 5 a. 3 r. 15 p. to the decimal of 1 a. 2 r. 32 p.

15. 1 day 19 hrs. 10 m. 30 sec. to the decimal of 1 day 3 hrs. 46 m. 40 sec.

16. 7 bus. 3 pks. 4 qts. to the decimal of 31 bus. 1 pk.

17. 3 cu. yds. 6 cu. ft. 111 cu. in. to the decimal of 5 cu. ft. 1360 cu. in.

18. 27 sq. yds. 8 sq. ft. 123 sq. in. to the decimal of 7 sq. yds. 4 sq. ft. 64 sq. in.

Find the value of—

19. ·375 of 2 yds. 8 in.

20. ·275 of 56 days 16 hrs.

21. ·01875 of 1 cwt. 40 lbs.

22. 37·375 of 3 tons 4 cwt.

23. 9·68125 of 5 hrs. 20 min.

24. 3·4375 of 3 sq. yds. 6 sq. ft. 104 sq. in.

25. ·12345 of 27 tons 1 cwt.

26. 3·742 of 1 cwt. 75 lbs.

27. 4·1275 of 1 yd. 2 ft. 9 in.

28. 6·7425 of 1 day 1 hr. 1 m.

29. 37·437 of 41 days 23 hrs.

Express as compound quantities—

30. ·93375 of 5 tons − ·628125 of 1 ton.

31. ·5 cwt. + ·7 of 5 cwt. + ·125 of 1 ton.

32. ·006 tons + ·03 cwt. + ·09 of 25 lbs.

33. ·00465 mi. + 2·745 fur. **34.** 3·64 mi. − 6·42 fur.

35. 2·325 cwt. − ·075 tons. **36.** 31·14 gal. − ·357 pts.

37. 3·075 fur. − $\frac{13}{80}$ mile.

38. $\frac{1}{17}$ of 20 lbs. 3 oz. + ·875 of 1 lb. 8 oz.

39. ·148 of 1 ton + ·314 of 1 cwt. + 10·25 of 1 lb.

40. $\frac{3}{5}$ of 76 days 6 hrs. + ·855 of 30 days.

41. ·2375 of 13 tons 15 cwt. − ·427125 of 1 ton.

42. 3·823 tons + 7·475 cwt. + 3·5 lbs.

43. 3·1625 of 7½ miles + ·8375 of 1 mile.

44. ·428 of 23 yds. 1 ft. + 5·29 of 5 yds. 2 ft. + 15·85 of 4 in.

45. Express ·425 of 29 tons 8 cwt. as the decimal of $6\frac{4}{5}$ of 6 tons 2 cwt. 50 lbs.

46. What decimal is $5\frac{1}{9}$ of £4. 5*s.* 6*d.* of $2\frac{5}{9}$ of 11*s.* 10½*d.*?

47. Reduce $\frac{1}{7}$ of $\frac{3}{10}$ of 40 yds. to the decimal of ¼ of 1 mi. 4 fur. 32 p.

48. Express $2\frac{7}{8}$ of 1 ton 19 cwt. 7 lbs. as the decimal of $5\frac{3}{4}$ of 125 cwt. 2·4 lbs.

MISCELLANEOUS EXAMPLES. LXXX.

1. A pendulum vibrates 1·375 times in a second; how many times will it vibrate in a week?

2. Add together 3·075 cwts. and 4·35 lbs., expressing the sum in ounces.

3. Express in yds. 1·03125 miles − 7·3125 fur.

4. Express ·50612 of 2 cwt. $8\frac{3}{4}$ lbs. in ounces.

5. What decimal of 4 yds. 6 in. is $\frac{1}{2800}$ of 10 yds. 2 ft. 6 in.?

6. Express ·05735 miles + 46·25 yds. as the decimal of 7 furlongs.

7. Three clocks all indicate 12 noon; one gains 90 sec. per hour, the second gains 80 sec. per hour, and the third loses 50 sec. per hour; how long will it be before the *large* hands all indicate the same minute?

8. A room is of length 22 ft. 2 in., breadth 20 ft. 7 in., height 12 ft. 8 in.; what is the greatest length that exactly measures each of these?

9. Express in terms of $4 as unit, the difference between $9 × $\frac{5}{48}$ and $$9\frac{5}{48}$.

10. Express 6 days 13 hr. 45 min. 37·92 sec. in weeks.

11. An English pint contains $34\frac{5}{8}$ cubic inches; what is the ratio of the cubic capacity of a pint to that of a cubic foot?

12. Two poles measure 4 yds. 2 ft. 1 in. and 8 yds. 11 in. respectively; find their greatest common measure, *i.e.* the longest length that each pole contains a whole number of times.

[N.B.—The highest common factor of two numbers is often called their **Greatest Common Measure.**]

13. Find the Greatest Common Measure of 8 days 10 hrs. 6 min. and 26 days 16 hrs. 42 min.

14. Find also their Least Common Multiple.

15. What is the ratio of $\{(\frac{1}{12}+\frac{1}{15}+\frac{1}{24})\sim\frac{1}{27}$ of $2\frac{1}{40}\}$ of 64 days 12 hrs. to 21 weeks 3 days 12 hrs.?

16. What decimals of a pound are 9·33 pence and ·78 of a farthing?

17. The circumferences of two wheels measure 336 and 812 tenths of an inch respectively; find the largest cogs which can be cut in each that they may work together.

18. A certain event is known to happen at regular intervals of time; it is *observed* to take place after an interval of 22 d. 11 hrs. 15 min. 3 sec., and again after an interval of 12 d. 20 hrs. 8 m. 36 sec.; what is the greatest possible interval at which the event regularly happens?

19. A man buys a certain number of articles for $100.10 and on another occasion a number of the same articles at the same price per article for $84.70; find the least possible number of articles bought.

20. For a certain number of articles I gave $147.77 and for a certain other number I gave $232.21; they each cost as much as possible under those circumstances; what was the price?

21. If $\frac{5}{8}$ of a ton is worth $28, what is the value of $\frac{3}{7}$ of a cwt.?

22. Subtract $\frac{1}{7}$ of $\frac{5}{8}$ of a guinea from ·4 of ·375 of £1, and express the difference as the decimal of a crown.

23. 10000000 acres are equivalent to 4046711 hectares; find to five places of decimals what fraction a hectare is of a square mile.

24. Express ·59685 acres in roods, poles, yds. etc.

25. A father divided a piece of land among his three sons thus: he gave $12\frac{1}{4}$ acres to the first, $\frac{3}{8}$ of the whole to the second, and to the third as much as to the other two together; how many acres did the third get?

26. How many francs are $5 worth, when one franc is worth 19·47 cents?

27. A metre being 39·371 in. express in metres 4 mi. 6 fur. 11 p.

28. How many metres (1 yd. = ·9144 metres) are there in 1 mile?

29. Find the value of 1 ton 10 cwt. $62\frac{1}{2}$ lbs. at $17.75 per ton.

30. How many cubic yards are there in 100 bushels?

31. From a rod a yard long, portions each ·057 of an inch in length are cut off; how many such portions can be cut off, and what will be the length of the remaining piece?

32. Into how many plots, each containing 1·45 poles, can a field of 10 acres be divided, and what part will be left over?

33. How many times can a length of $\frac{4}{15}$ths of an inch be cut off from a rod 1 yard 3 inches long; and what will be the length of the remaining piece?

34. A vessel holds $5\frac{3}{28}$ quarts; how many times can it be filled from a vessel holding $49\frac{5}{11}$ gallons? and how much will remain over?

35. The wheels of a bicycle are 3 ft. and 1 ft. respectively in diameter; how far will it have travelled when the smaller wheel has made 100 more revolutions than the larger (the circumference of a circle is 3·14159 times the diameter)?

36. If a litre is ·52 gallons, find to the nearest cent the value of a pint of liquid worth $5 per litre.

37. The circumference of a circle = 3·1416 times its diameter; how far will the extremity of the large hand of a clock which is 6 ft. long move over in 18 minutes?

38. When $4.90 is worth 25·05 francs what is gained by selling 3720 francs at $3.96 for 20?

39. If the weight of a cubic foot of water is 62·35 lbs. (Avoir.) find the error in calculating the weight of 1000 cubic ft. on the approximate supposition that a cubic foot of water weighs 1000 oz.

40. The length of the year is 365·242264 days, and in 400 years there are 97 leap years; how many 400 years must elapse before the error amounts to one day?

41. Which is the greatest, $\frac{1}{19}$ of 1 ton, $\frac{1}{20}$ of 1 ton 1 cwt. or $\frac{5}{8}$ of $\frac{1}{4}$ of 1 ton?

42. Which is the least, $\frac{7}{16}$ of 1 ton, $\frac{5}{8}$ of $10\frac{1}{2}$ cwt. or $\frac{1}{3}$ of 15 cwt. 78 lbs.?

43. What is the ratio of 1 cwt. 4 lbs. $2\frac{1}{2}$ oz. to $5\frac{4}{7}$ of 1 cwt. 20 lbs. $1\frac{1}{2}$ oz.?

44. Express in Troy weight $\frac{2}{3}$ of $\frac{5}{6}$ of $(\frac{1}{2}-\frac{1}{3}+\frac{1}{4})$ of 1 lb. Avoir.

45. What is the ratio of $\frac{3}{5}$ of $\frac{2}{7}$ of a foot to $\frac{4}{9}$ of $\frac{2}{2}\frac{1}{2}$ of a yard?

46. A wine merchant has 4 pipes, $2\frac{2}{9}$ pipes, $2\frac{5}{6}$ pipes, and $\frac{4}{90}$ of a pipe of four different wines respectively. He wishes to keep them, without mixing the wines, in equal sized casks, using as few casks as possible. What should be the size of the casks, and how many must he use? [N.B.—A pipe contains 126 gallons.]

47. Four bells commence tolling together; they toll at intervals of 1, $1\frac{1}{8}$, $1\frac{1}{12}$, $1\frac{3}{10}$ seconds respectively; after what interval will they toll together?

48. Express by whole numbers in terms of the same unit $\frac{4}{21}$ of 2 ft. 6 in., $\frac{5}{24}$ of 1 yd. 4 in., and $\frac{3}{28}$ of 2 yds. 2 ft. 5 in.

CHAPTER VI

PRACTICE

125. **Simple Practice** is a convenient method by which we can find the cost or measure of a given number of simple quantities, when the cost or measure of one of the quantities is given.

Example. Find the volume of 365 tons *of goods, the volume of* 1 ton *being* 2 cu. yds. 12 cu. ft. 147 cub. in.

Compound Practice is a method by which the cost or measure of a compound quantity can be found when the value is given of one of the units in terms of which the compound quantity is expressed.

Example. Find the volume of parcels weighing 365 tons 11 cwt. 14 lbs., *when the volume of* 1 ton *is* 2 cu. yds. 12 cu. ft. 147 cu. in.

SIMPLE PRACTICE.

126. The method is based upon the principles of vulgar fractions.

DEF. An **aliquot** part of a quantity is a quantity which is expressed in terms of that quantity as unit by a vulgar fraction whose numerator is one.

Example. 4 hrs. 48 min. (*which is* $\frac{1}{5}$ *of a day*) *is an aliquot part of* 1 day.

EXAMPLES. LXXXI.

Read off the value of the following aliquot parts—

1. $\frac{1}{2}$ of 1 yd. **2.** $\frac{1}{3}$ of 1 day. **3.** $\frac{1}{4}$ of 1 cwt. **4.** $\frac{1}{5}$ of $1.
5. $\frac{1}{6}$ of 1 yd. **6.** $\frac{1}{7}$ of 2 weeks. **7.** $\frac{1}{8}$ of 1 gal. **8.** $\frac{1}{9}$ of 1 cu. yd.
9. $\frac{1}{10}$ of 1 cwt. **10.** $\frac{1}{11}$ of 1 mile. **11.** $\frac{1}{12}$ of 1 yd. **12.** $\frac{1}{15}$ of 1 hr.
13. $\frac{1}{16}$ of 1 sq. ft. **14.** $\frac{1}{20}$ of $1. **15.** $\frac{1}{25}$ of $1. **16.** $\frac{1}{30}$ of 1 day.
17. $\frac{1}{40}$ of 1 ton. **18.** $\frac{1}{50}$ of $1. **19.** $\frac{1}{60}$ of 1 day. **20.** $\frac{1}{100}$ of 1 hr.
21. $\frac{1}{120}$ of 1 hr. **22.** $\frac{1}{8}$ of 1 lb. **23.** $\frac{1}{10}$ of 1 day. **24.** $\frac{1}{5}$ of 1 min.
25. $\frac{1}{2}$ of £1. **26.** $\frac{1}{3}$ of £1. **27.** $\frac{1}{4}$ of £1. **28.** $\frac{1}{5}$ of £1.

127. The method of Practice is as follows :—

Example i. *Find the weight of* 1234 *articles at* 12 lbs. 8 oz. *each.*

We notice that 12 lbs. 8 oz. is $\frac{1}{8}$ of 1 cwt.

Therefore the weight of a number of articles at 12 lbs. 8 oz. each is one-eighth of their weight at 1 cwt. each ; hence

	cwt.	lbs.	oz.	
The wgt. at 12 lbs. 8 oz. is $\frac{1}{8}$ of	)1234	0	0	the wgt. at 1 cwt. each ;
therefore dividing by 8	20) 154	25	0	is the wgt. at 12 lbs. 8 oz. each.
Dividing by 20, to get wgt. in tons, we have	tons 7	14	25 lbs ;	

that is, the weight of 1234 articles each weighing 12 lbs. 8 oz. is 7 tons 14 cwt. 25 lbs.

Thus, we write down the weight of the articles at 1 cwt. each, and dividing this by 8, obtain the required result.

Example ii. *Find the volume of* 365 tons *of goods, the volume of* 1 ton *being* 2 cu. yds. 10 cu. ft. 81 cu. in.

		c. yds.	c. ft.	c. in.	
Vol. at 2 c. yds. per ton.	2	365	0	0	vol. at 1 c. yd. p. ton.*
9 c. ft.	$\frac{1}{3}$			2	
		730	0	0	,, 2 c. yd. p. ton.
1 c. ft.	$\frac{1}{9}$	121	18	0	,, 9 c. ft. p. ton.
54 c. in.	$\frac{1}{32}$	13	14	0	,, 1 c. ft. ,,
27 c. in.	$\frac{1}{2}$		11	702	,, 54 c. in. p. ton.
			5	1215	,, 27 c. in. ,,
Hence adding,		865	22	189	=vol. at 2 c. yd. 10 c. ft. 81 c. in. p. ton.

EXAMPLES. LXXXII.

Find the value of each of the following sets of articles—

1. 1000 at 10c. each.
2. 372 at 25c. each.
3. 475 at $2\frac{1}{2}$c. each.
4. 500 at 20c. each.
5. 372 at 5c. each.
6. 365 at 20c. each.

Find the weight of each of the following sets of articles in cwt. lbs. oz.—

7. 421 at 4 oz.
8. 443 at 20 lbs.
9. 723 at 12 oz.
10. 230 at 60 lbs.
11. 3271 at 10 oz.
12. 489 at 15 oz.
13. 899 at 12 lbs. 8 oz.
14. 555 at 37 lbs. 8 oz.
15. 6721 at 14 oz.

---

* This line is to be read thus :—The volume (of 365 things) at 2 cu. yds. each is twice (365 cu. yds.) the volume at 1 cu. yd. each ; the volume at 9 cu. ft. each is $\frac{1}{3}$ of (365 cu. yds.) the volume at 1 cu. yd. each.

Find the volume of each of the following sets of parcels in cu. yds. cu. ft. cu. in.—

16. 400 at 864 cu. in.

17. 562 at 432 cu. in.

18. 201 at 1296 cu. in.

19. 76 at 1 cu. ft. 216 cu. in.

20. 26 at 21 cu. ft. 432 cu. in.

21. 90 at 2 cu. yds. 11 cu. ft. 1000 cu. in.

22. 71 at 24 cu. ft. 864 cu. in.

Find the value of each of the following—

23. 22 tons 5 cwt. 50 lbs. at $320 a ton.

24. 6 yds. 1 ft. 6 in. at $750 a yd.

128. The work in Practice may often be shortened by the exercise of a little ingenuity.

Example i. *Find the volume of* 365 tons *at* 2 cu. yds. 10 cu. ft. 81 cu. in. *per ton.*

		c. yds.	c. ft.	c. in.	
9 cu. ft.	$\frac{1}{3}$	365 .	0 .	0	vol. at 1 cu. yd.
				2	
		730 .	0 .	0	„ 2 cu. yds.
1 cu. ft.	$\frac{1}{9}$	121 .	18 .	0	„ 9 cu. ft.
81 cu. in.	$\frac{1}{192}$	13 .	14 .	0	„ 1 cu. ft.
			17 .	189	„ 81 cu. in.
Adding		865 .	22 .	189	=vol. at 2 cu. yds. 10 cu. ft. 81 cu. in.

Compare with Example ii. p. 120.

Example ii. *Find the volume of* 365 tons, *the volume of* 1 ton *being* 3 cu. yds. 25 cu. ft. 1701 cu. in.

Here we notice that 3 cu. yds. 25 cu. ft. 1701 cu. in. is 4 cu. yds. wanting 1 cu. ft. 27 cu. in. We therefore find the volume of 365 tons at 1 cu. ft. 27 cu. in. per ton, and subtract the result from the volume at 4 cu. yds. per ton; thus:

	c. yds.	c. ft.	c. in.	
	365 .	0 .	0	
	4			
	1460 .	0 .	0	volume at 4 cu. yds. per ton.
	13 .	19 .	1215	„ 1 cu. ft. 27 cu. in. per ton.
Subtracting	1446 .	7 .	513	3 cu. yds. 25 cu. ft. 1701 cu. in.

		c. yds.	c. ft.	c. in.	
1 cu. ft.	$\frac{1}{27}$	365 .	0 .	0	volume at 1 cu. yd.
27 cu. in.	$\frac{1}{64}$	13 .	14 .	0	,, 1 cu. ft.
			5 .	1215	,, 27 cu. in.
		13 .	19 .	1215	= vol. at 1 cu. ft. 27 cu. in.

NOTE.—The method of Art. 111 may be used to perform the above subtraction.

Example iii. *Find the weight of* 365¼ cu. yds. *at* 4 tons 7 cwt. 81 lbs. 4 oz. *per cu. yd.*

		tons	cwts.	lbs.	oz.	
5 cwt.	$\frac{1}{4}$	365 .	5 .	0 .	0	= weight of 365¼ at 1 ton each.
					4	
		1461 .	0 .	0 .	0	= wt. at 4 tons.
2 cwt. 50 lbs.	$\frac{1}{2}$	91 .	6 .	25 .	0	= 5 cwt.
25 lbs.	$\frac{1}{10}$	45 .	13 .	12 .	8	= ,, 2 cwt. 50 lbs.
6 lbs. 4 oz.	$\frac{1}{4}$	4 .	11 .	31 .	4	= ,, 25 lbs.
		1 .	2 .	82 .	13	= ,, 6 lbs. 4 oz.
By addition		1603 .	13 .	51 .	9	= wt. at 4 tons 7 cwt. 81 lbs. 4 oz.

EXAMPLES. LXXXIII.

Find the weight of each of the following sets of articles—

1. 300 at 3 tons 12 cwt. 75 lbs.
2. 400 at 2 tons 7 cwt. 75 lbs.
3. 821 at 5 tons 13 cwt. 75 lbs.
4. 177 at 8 tons 6 cwt. 76 lbs. 4 oz.
5. 901 at 12 tons 12 cwt. 95 lbs. 12 oz.
6. 374 at 10 tons 11 cwt. 90 lbs.
7. 475 at 21 tons 9 cwt. 31 lbs. 4 oz.
8. 777 at 40 tons 12 cwt. 80 lbs.
9. 897 at 61 tons 14 cwt. 12 lbs. 8 oz.
10. 95 at 101 tons 11 cwt. 81 lbs. 4 oz.
11. 479 at 31 tons 6 cwt. 95 lbs.
12. 139 at 432 tons 7 cwt. 98 lbs.

Find the volume of each of the following sets of packages—

13. 84 at 1 cu. yd. 7 cu. ft. 80 cu. in.
14. 100 at 3 cu. yds. 20 cu. ft. 864 cu. in.
15. 961 at 4 cu. yds. 24 cu. ft. 864 cu. in.
16. 72 at 12 cu. yds. 9 cu. ft. 432 cu. in.

Find the contents of each of the following sets of vessels—

17. 92 each containing 6 gals. 3 qts. 1 gi.
18. 764 each containing 16 gals. 2 qts. 1 pint.
19. 871 each containing 72 gals. 3 qts. 1 pt. 3 gi.
20. 1000 each containing 95 gals. 1 qt. 1 pt. 1 gi.

Find the amount of each of the following—

21. 900 sacks, each containing 25 bus. 2 pks. 3 qts.
22. 521 barrels, each containing 20 gals. 2 qts. 1 pt.
23. 432 lengths, each containing 1 mile 700 yds. 2 ft.
24. 465 periods, each containing 1 yr. 51 days 7 hours 30 min.

COMPOUND PRACTICE.

129. The method in Compound Practice is as follows:—

Example i. *Find the volume of* 365 tons 15 cwt. 75 lbs. *at* 2 cu. yds. 10 cu. ft. 81 cu. in. *per ton.*

First we find, as in Ex. i., p. 121 (or by multiplication), that the volume of 365 tons at 2 cu. yds. 10 cu. ft. 81 cu. in. per ton is 865 cu. yds. 22 cu. ft. 189 cu. in. Then

		cu.yds.	cu.ft.	cu.in.	
10 cwt.	½	2	10	81	=vol. of 1 ton.
5 cwt.	½	1	5	40·5	= ,, 10 cwt.
50 lbs.	$\frac{1}{10}$	0	16	20·25	= ,, 5 cwt.
25 lbs.	½	0	1	1038·825	= ,, 50 lbs.
		0	0	1383·4125	= ,, 25 lbs.
		1	23	754·9875	= ,, 15 cwt. 75 lbs.
		865	22	189	= ,, 365 tons.
		867	18	943·9875	=vol. of 365 tons 15 cwt. 75 lbs.

i.e. the volume of 365 tons 15 cwt. 75 lbs. *to the nearest cubic inch is*

867 cu. yds. 18 cu. ft. 944 cu. in.

Example ii. *Find the rent for* 7 months 3 weeks 4 days *at* $588 *per annum.*

6 mo. =	½ of	$588.00	=rent for 12 mo.
1 mo. =	⅙ of	$294.00	= ,, 6 mo.
2 wks. =	½ of	49.00	= ,, 1 mo.
1 wk. =	½ of	24.50	= ,, 2 wks.
3½ days =	½ of	12.25	= ,, 1 wk.
½ day =	⅐ of	6.125	= ,, 3½ days.
		.875	= ,, ½ day.
		$386.75	=rent for 7 mo. 3 wks. 4 days.

EXAMPLES. LXXXIV.

Find the value of each of the following—

1. 3 cwt. 76 lbs. 4 oz. at $1764 per cwt.
2. 9 tons 16 cwt. 50 lbs. at $988 per ton.
3. 85 yds. 1 ft. 10 in. at $130.68 per yd.
4. 71 yds. 2 ft. 9 in. at $9.12 per yd.
5. 73 mi. 1000 yds. at $654.72 per mile.
6. 19 lbs. 7 oz. of gold at $178.80 per lb.
7. 3 oz. 10 dwt. 15 gr. at $201.60 per lb.
8. 11 yds. 1 ft. 9 in. at $7.56 per yd.
9. 62 yds. 30 in. at $8.64 per yd.
10. 60 sq. yds. 8 sq. ft. 12 sq. in. at $36 per sq. yd.
11. 6 oz. 16 dwt. 18 grs. at $7.50 per oz.
12. 27 bus. 3 pks. 5 qts. at $5.75 per bus.
13. 11 gal. 3 qts. 3 gi. at $6.60 per gal.
14. 10 mo. 3 wks. 5 days at $53.25 per month.
15. 11 mo. 2 wks. 6 days at $45 per month.
16. 7 mo. 3 wks. 4 days at $140 per year.
17. 10 cwt. 10 lbs. 10 oz. at $640 per ton.
18. 6 cwt. 76 lbs. 9 oz. at $2560 per ton.
19. 10 mo. 1 wk. 6 days at $336 per year.
20. Find the produce of 14 a. 3 r. 25 p. at 46 bus. 2 pks. per acre.
21. Find the produce of 121 a. 1 r. 15 p. at 59 bus. 1 pk. per acre.
22. Find the produce of 37 a. 2 r. 88 sq. yds. at 14 cwt. 14 lbs. per acre.

130. The following examples may be noticed :—

Example i. *Evaluate* 76·5625 *of* 2 tons 18 cwt. 87 lbs. 8 oz.

10 cwt. =	½ of 1 ton	76·5625	·380859375 tons
		2	20
		153·1250	7·617187500 cwt.
5 cwt. =	½ of 10 cwt.	38·28125	100
2 cwt. =	$\frac{1}{10}$ of 1 ton.	19·140625	
1 cwt. =	$\frac{1}{10}$ of 10 cwt.	7·65625	61·71875 lbs.
50 lbs. =	$\frac{1}{10}$ of 5 cwt.	3·828125	16
25 lbs. =	½ of 50 lbs.	1·9140625	
12 lbs. 8 oz. =	½ of 25 lbs.	·95703125	11·50000 oz.
		·478515625	
		225·380859375 tons.	

The value required is 225 tons 7 cwt. 61 lbs. 11½ oz.

NOTE.—The work on the left hand gives the value of 2 tons 18 cwt. 87 lbs. 8 oz. at $76·5625 per ton. The value is $225.38.

Example ii. *Find to the nearest oz. the weight of* 471·325 cub. yds. *at* 4 tons 11 cwt. 70 lbs. 4 oz. *per cub. yd.*

10 cwt.	$\frac{1}{2}$	tons 471·325 = weight at 1 ton per cub. yd.		
		4		
		1885·300	=	4 tons
1 cwt.	$\frac{1}{10}$	235·6625	=	10 cwt.
50 lbs. =	$\frac{1}{2}$ cwt.	23·56625	=	1 cwt.
20 lbs. =	$\frac{1}{5}$ cwt.	11·783125	=	50 lbs.
4 oz.	$\frac{1}{80}$ of 20 lbs.	4·71325	=	20 lbs.
		·0589156	=	4 oz.

tons 2161·0840406 = wt. at 4 tons 11 cwt. 70 lbs. 4 oz.
and 2161·08404 tons = 2161 tons 1 cwt. 68 lbs. 13 oz.

131. These examples can, when convenient, be worked by reducing each compound quantity to a decimal.

Example. Find the cost of 6 miles 4 fur. 38 yds. 18 in. *of wire at* $55.32 *per mile.*

By process of Art. 123, 6 m. 4 fur. 38 yds. 18 in. = 6·521875 miles.

$$\begin{array}{r} 6\cdot521875 \\ 55\cdot32 \\ \hline 326\cdot09375 \\ 32\cdot609375 \\ 1\cdot9565625 \\ \cdot1304375 \\ \hline \$360\cdot790125 \end{array}$$

therefore the cost is $360.79.

But the work by this method is generally longer than by Practice.

MISCELLANEOUS EXAMPLES. LXXXV.

1. A bankrupt pays $3.76 in every $5 ; find the dividend on a debt of $17355.
2. Find the cost of 374 acres 80 poles at $93.25 per acre.
3. Find the value of 59 yds. 1 ft. 6 in. of silk at $1.41 per yd.
4. A bankrupt pays $57.10 in every $100. His debts being $17350, how much does he pay altogether?
5. Find the weight of 347 bales of cloth each weighing 7 cwt. 87 lbs. 8 oz.
6. Find the weight of 562 sacks of flour each weighing 1 cwt. 87 lbs. 8 oz.
7. Find the value of 4501 tons 15 cwt. at $91 per ton.

8. Find the value of 371 acres 120 poles at $184.83 per acre.

9. What is the value of the iron in a railway bridge which weighs 3575 tons at $17.75 per ton?

10. Find the value of 37 miles 682 yds. of telegraph wire at $58.72 per mile.

11. Find the cost of 5 yds. 3⅝ in. at $91.44 per yd.

12. Find the cost of fencing 3 miles 3 fur. 180 yds. 1 ft. 6 in. of road at $2398.75 per mile.

13. Find the value of 21 tons 5 cwt. 76 lbs. at $375 per ton.

14. Find the weight of 3741 things, ten of which weigh 37 tons 17 cwt. 76 lbs.

15. Find to the nearest cent the value of 375 yds. 1 ft. 10 in. at $3.62 per yd.

16. Find to the nearest cent the rent of 137 acres 41 sq. poles at $13.75 per acre.

17. A tradesman bought 451 articles at £3. 15*s.* 7*d.* each, paid £3. 13*s.* 4*d.* carriage for them, and sold them at £4. 7*s.* 7½*d.* each; how much did he gain?

18. A tradesman bought 151 bales of calico at £1. 12*s.* 11*d.* each, and sold 134 of them at £2. 1*s.* 10*d.* each, and was robbed of the rest; what did he gain or lose on the transaction?

19. Find the value of 220 miles 3 fur. 20 poles 2¾ yds. at $116.25 per mile.

20. Find the value of £1. 3*s.* 9*d.* when £1 is worth $4.90.

21. Find the value in dollars and cents of 1437526 gallons of water at £1. 3*s.* 9*d.* per 10000 gals. (£1 = $4.90.)

22. Find the number of quarters (1 quarter = 8 bus.) produced by 73·12475 acres at 7 qrs. 5 bus. 1 pk. per acre.

23. Find the value of 35 cub. yds. 3 cub. ft. 288 cub. in. at $16.72 per cub. yd.

24. If £1 = $4.87, find the value of £3. 5*s.* 9*d.*

25. Find to the nearest cent the value of a square mile of land at $1773.85 per hectare. (358·9895 hectares = 18 sq. miles.)

26. Find the value of 20 tons 11 cwt. 45 lbs. at $314.25 per ton.

132. The **Accounts** of a Business are writings which contain a record of every transaction entered into in the course of that Business.

The principal **Books** in the accounts of an ordinary Trade are the **Day Book** and the **Ledger.**

In the Day Book are entered each day's transactions in the order in which they occur.

At short intervals the entries in the Day Book are 'posted' in the

Ledger; the index of the Ledger contains a list of customers' names in alphabetical order, with a reference opposite each name to the page of the Ledger on which are collected all the transactions which have taken place with that particular customer.

An **invoice** is a detailed statement sent by a Seller to the Buyer, at the time of the delivery of goods, of their quality, quantity, and price.

An **account** (or, copy of an account) is said to be **rendered** by a Seller when he sends to the Buyer, at the end of a term of credit, a copy of the transactions between them as recorded in his Ledger.

SPECIMEN OF AN INVOICE.

INVOICE, New York, June 21, 1887.
From Messrs. Jones & Co. 42 West Street.

14 yds. Calico	at 11c.	$1.54
22 yds. Brussels Carpet. . .	at $1.14	25.08
2 Rugs, $5.20, $9.40 . . .		14.60
		$41.22

Sent to Henry Jenkins, Orange, N.J.

SPECIMEN OF A DETAILED ACCOUNT.*

Henry Jenkins,
Bought of J. Jones & Co., Midsummer, 1887.
42 West Street,
New York.

1887.				
May 21	40 yds. Irish Linens . .	at 64c.	$25.60	
	1 doz. Dusters . . .	at 14c.	1.68	
	25 yds. Towelling . . .	at 22c.	5.50	$32.78
June 4	23 yds. Flannel . . .	at 61c.	14.03	
	15 yds. Brown Holland . .	at 23c.	3.45	17.48
June 21	14 yds. Calico . . .	at 11c.	1.54	
	22 yds. Brussels Carpet . .	$1.14	25.08	
	2 Rugs, $5.20, $9.40 . .		14.60	41.22
				$91.48

---

* An account in which each separate item is given as above is called a *detailed* account, to distinguish it from an account in which the total only of each invoice is given.

EXAMPLES. LXXXVI.

Make out Invoices for the following entries in a Day Book, inserting names and other details of your own choosing.

1. 31 yds. sheeting at 34c. per yd., 37½ yds. Brussels carpet at 98c. per yd., 14¾ yds. muslin at 32c.

2. 21 lbs. tea at 78c. per lb., 14 lbs. sugar at 7c. per lb., 7 lbs. coffee at 27c.

3. 7¾ lbs. lamb at 26c. per lb., 17 lbs. 6 oz. sirloin beef at 21c. per lb., 5 lbs. suet at 15c. per lb.

4. 43 yds. tapestry carpet at 85c. per yd., 25½ yds. cocoa matting at 70c. per yd., 17¾ yds. linoleum at 93c. per yd., 3 rugs at $1.47, $3.17, $3.92 each.

5. 56 lbs. oatmeal at 3c. per lb., 3 dozen lbs. candles at 17c. per lb., 25 lbs. soap at $5.76 per cwt., 24 yds. house flannel at 11c. per yd.

6. 56 lbs. sugar at 5c. per lb., 30 lbs. rice at 7c. per lb., 17¾ lbs. cheese at 18c. per lb., 67 lbs. bacon at 17c. per lb.

7. 23 tons of coal at $5.68 per ton, 73 tons of coal at $4.68 per ton, 23 tons coke at $6.31 per ton.

8. 75 yds. huckaback at 27c. per yd., 53 yds. rough towelling at 21c. per yd., 13 dozen dusters at $1.96 per dozen.

9. 49 lbs. fresh butter at 34c. per lb., 213 lbs. salt butter at 30c. per lb., 149 lbs. cheese at 21c. per lb., 137 lbs. bacon at 19c. per lb.

10. 3 doz. pairs gloves at 73c. per pair, 1½ gross buttons at 9c. per dozen, 200 yds. braid at 42c. per doz. yds., 3½ doz. handkerchiefs at $2.92 per dozen.

11. Supposing that Questions **1**, **4**, **8**, and **10**, are entries in the Day Book at different dates of goods delivered to the same buyer, make out his account.

12. Assuming that **2**, **5**, **6**, and **9**, are entries in the Day Book at different dates of goods purchased by a buyer, make a copy of the entries against that buyer's name in the tradesman's ledger.

NOTE.—The Day Book and Ledger are used in every business. There are besides these other books such as the **Stock Book, Cash Book, Bank Book, Wages Book,** etc., which are used in nearly every business; but it will be found that in each particular kind of business there is some peculiarity in the system of accounts adopted.

A system of Accounts should be as simple and as clear as possible. It should be so arranged that it may not be difficult to ascertain from the accounts, half-yearly or yearly as the case may be, the exact state of the business.

Also, in very large businesses, each separate transaction should be entered in two different ways; so that each day's transactions may give rise to two separate totals. The advantage of this is, that these totals must be the same if the accounts have been accurately made up; so that in this way a test is obtained of the accuracy of the accounts. Such a system is called a system of **Double Entry.**

PART II

CHAPTER VII

RECURRING DECIMALS

133. It will be found in some cases, in the process of division, that, when the quotient is expressed as a decimal fraction, there is always a remainder; so that the quotient can be continued without limit.

Example i. *Divide* 1 *by* 3.

3)1·

·33333..., the dots indicating that the 3 is to be repeated without limit.

Example ii. *Express* $\frac{3}{7}$ *as a decimal fraction.*

$\frac{3}{7}$ of the unit = one-seventh of three units.

Process, 7)3·

·428571428571428... *Quotient.*

134. When this is the case, certain digits in the quotient are found to **recur** over and over again.

DEF. Decimal fractions in which certain digits recur are called **recurring** decimals.

They are called, by some writers, **circulating** decimals, and by others, **repeating** decimals.

135. The notation adopted is as follows :—

·33333... is written $\cdot\dot{3}$,

·123567567567... is written $\cdot 123\dot{5}6\dot{7}$.

The digits between and underneath the two dots (both included) are understood to recur in the same order continuously.

DEF. The digits which recur are called the **period.**

Example. In the decimal $\cdot 123\dot{5}6\dot{7}$ the **period** is 567.

EXAMPLES. LXXXVII.

Divide, expressing each quotient as a decimal,—

1. 2 by 3. **2.** 2 by 7. **3.** 3 by 7. **4.** 4 by 7.
5. 5 by 7. **6.** 6 by 7. **7.** 1 by 15. **8.** 7 by 11.
9. £1 by 9. **10.** 375 pence by 11. **11.** 456 lbs. by 7.
12. 47 in. by 6. **13.** 27 gallons by 7. **14.** 365 days by 7.

Express each of the following as a decimal fraction :—

15. $\frac{2}{3}$. **16.** $\frac{2}{7}$. **17.** $\frac{3}{7}$. **18.** $\frac{4}{7}$. **19.** $5\frac{1}{9}$.
20. $20\frac{3}{13}$. **21.** $2\frac{7}{11}$. **22.** $451\frac{6}{7}$. **23.** $50\frac{20}{75}$. **24.** $19\frac{11}{24}$.
25. $5\frac{5}{48}$. **26.** $200\frac{1}{81}$. **27.** $3\frac{1}{5}$ of $8\frac{7}{15}$. **28.** $4\frac{1}{3}\times 1\frac{1}{39}$. **29.** $\dfrac{4\frac{3}{8}}{7\frac{6}{7}}$.
30. $1\frac{1}{9}\times 10\frac{11}{90}$. **31.** $\frac{1}{9}$. **32.** $\frac{1}{99}$. **33.** $\frac{1}{999}$. **34.** $\frac{1}{99999}$.

136. A *recurring* decimal fraction is expressed in the form of a *vulgar* fraction as follows :—

Since by division we find that

$$\tfrac{1}{9}=\cdot\dot{1}\ ;\ \tfrac{1}{99}=\cdot\dot{0}\dot{1}\ ;\ \tfrac{1}{999}=\cdot\dot{0}0\dot{1}\ ;\ \tfrac{1}{9999}=\cdot\dot{0}00\dot{1}\ ;\ \text{and so on :}$$

And since

$$\cdot\dot{0}0\dot{1}\times 407=(\cdot 001001001\ldots)\times 407=\cdot 407407407\ldots=\cdot\dot{4}0\dot{7}:$$

therefore, (i) $\cdot\dot{7}=7$ times $\cdot\dot{1}=7\times\frac{1}{9}=\frac{7}{9}$,

(ii) $\cdot\dot{3}\dot{7}=37$ times $\cdot\dot{0}\dot{1}=37\times\frac{1}{99}=\frac{37}{99}$,

(iii) $\cdot\dot{4}0\dot{7}=407$ times $\cdot\dot{0}0\dot{1}=407\times\frac{1}{999}=\frac{407}{999}$,

and so on. Again,

(iv) $\cdot 23\dot{4}\dot{1}=\cdot 23+\cdot 00\dot{4}\dot{1}=\frac{23}{100}+\frac{1}{100}$ of $\cdot\dot{4}\dot{1}=\frac{23}{100}+\frac{41}{9900}$

$$=\frac{23\times 99+41}{9900}=\frac{23\times(100-1)+41}{9900}=\frac{2341-23}{9900}=\frac{2318}{9900}.$$

(v) $\cdot 034\dot{9}6\dot{3}=\cdot 034+\cdot 000\dot{9}6\dot{3}=\frac{34}{1000}+\frac{963}{999000}$

$$=\frac{34\times(1000-1)+963}{999000}=\frac{34963-34}{999000}=\frac{34929}{999000}.$$

The above results are included in the following statement :—

To express a recurring decimal as a vulgar fraction—

For the *numerator* take the whole decimal (*i.e.* the digits from the decimal point to the end of the first period), subtracting the number indicated by the digits which do not recur ;

For the *denominator*, take the number formed by as many 9's as there are digits recurring, followed by as many 0's as there are non-recurring digits.

The following is an alternative method :—

Example. Express $\cdot 4\dot{7}1\dot{1}$ *as a* vulgar *fraction.*

$$\begin{array}{lrl} & 10000 \text{ times } \cdot 4\dot{7}1\dot{1} & = 4711\cdot 711711711\ldots \\ & 10 \text{ times } \cdot 4\dot{7}1\dot{1} & = 4\cdot 711711711\ldots \\ \text{Subtracting,} & 9990 \text{ times } \cdot 4\dot{7}1\dot{1} & = 4711\cdot - 4. \end{array}$$

Hence,
$$\cdot 4\dot{7}1\dot{1} = \frac{4711-4}{9990} = \frac{4707}{9990}.$$

EXAMPLES. LXXXVIII.

Express as vulgar fractions—

1. $\cdot\dot{7}$. **2.** $\cdot 0\dot{1}\dot{1}$. **3.** $\cdot\dot{7}0\dot{0}$. **4.** $\cdot\dot{0}0\dot{7}$.
5. $2\cdot 0\dot{1}\dot{3}$. **6.** $10\cdot 0\dot{1}$. **7.** $14\cdot 013\dot{1}$. **8.** $9\cdot 6\dot{0}\dot{7}$.
9. $\cdot 10\dot{8}8\dot{2}$. **10.** $\cdot 006\dot{4}0\dot{3}$. **11.** $\cdot 0010\dot{6}\dot{4}$. **12.** $4\cdot 44\dot{0}\dot{3}$.

Express as vulgar fractions in their lowest terms—

13. $\cdot 0\dot{3}$. **14.** $\cdot 0\dot{8}\dot{1}$. **15.** $60\cdot 90\dot{6}\dot{3}$. **16.** $\cdot 009\dot{0}0\dot{9}$.
17. $\cdot 12\dot{3}\dot{6}$. **18.** $\cdot 09\dot{6}\dot{3}$. **19.** $\cdot 45\dot{9}0\dot{0}$. **20.** $\cdot 63\dot{0}9\dot{0}$.
21. $676\cdot 72\dot{0}7\dot{2}$. **22.** $30\cdot 90\dot{4}0\dot{5}$. **23.** $710\cdot 012\dot{3}\dot{6}$.
24. $\cdot 0012\dot{3}\dot{6}$. **25.** $60\cdot 9\dot{2}\dot{4}$. **26.** $10\cdot 02\dot{2}\dot{7}$.

27. Prove that
$$\frac{1}{7} = \frac{\cdot\dot{1}4285\dot{7}}{1} = \frac{\cdot\dot{2}8571\dot{4}}{2} = \frac{\cdot\dot{4}2857\dot{1}}{3} = \frac{\cdot\dot{5}7142\dot{8}}{4} = \frac{\cdot\dot{7}1428\dot{5}}{5} = \frac{\cdot\dot{8}5714\dot{2}}{6}.$$

28. Prove that $30\cdot 421\dot{2}8571\dot{4} = 30\cdot 421 + \frac{2}{7000}$.

137. A recurring decimal whose period is $\dot{9}$ can be expressed as a non-recurring decimal.

Example i. $\cdot 000\dot{9}$ *is equal to* $\cdot 001$.

This may be proved by the method of Art. 136 ; or directly by subtraction, thus :

$$\begin{array}{l} \cdot 001 \\ \underline{\cdot 0009999\ldots} \\ \underline{\underline{\cdot 0000000\ldots}} \end{array}$$

Example ii. $\cdot 345\dot{9} = \cdot 345 + \cdot 000\dot{9} = \cdot 345 + \cdot 001 = \cdot 346.$

A recurring decimal when transformed by multiplication to a non-recurring decimal appears in the form '9 recurring.'

Example i. $\cdot 1041\dot{6} \times \frac{3}{3} = \frac{\cdot 3124\dot{9}}{3} = \frac{\cdot 3125}{3} = \frac{2\cdot 5}{3 \times 8} = \frac{5}{48}$. [Art. 96.

Example ii. *Express* $4\cdot 012\dot{2}85714\dot{4}$ *as a vulgar fraction.*

$$4\cdot 012\dot{2}85714\dot{4} \times \tfrac{7}{7}\left[= 28\cdot 085\dot{9} \times \tfrac{1}{7}\right] = 4\tfrac{86}{7000} = 4\tfrac{43}{3500}.$$

Example iii. *Simplify* $\dfrac{\cdot 0605\dot{4}2857\dot{1}}{\cdot 8385\dot{2}\dot{7}}$.

$$\text{This fraction} = \frac{\cdot 0605\dot{4}2857\dot{1} \times 7 \times 11}{\cdot 385\dot{2}\dot{7} \times 11 \times 7} = \frac{\cdot 4238 \times 11}{4\cdot 238 \times 7} = \frac{11}{70}.$$

EXAMPLES. LXXXIX.

Express as non-recurring decimals—

1. $\cdot 0\dot{9}$. **2.** $30\cdot 47\dot{9}$. **3.** $8\cdot 1236\dot{9}$. **4.** $\cdot 005\dot{9}$. **5.** $3\cdot\dot{9}$.

6. $3\cdot 6\dot{9}\dot{9}$. **7.** $70\cdot\dot{9}9\dot{9}$. **8.** $\cdot 0000\dot{9}$. **9.** $1\cdot 678\dot{9}$. **10.** $1\cdot 24\dot{9}$.

Express as vulgar fractions in their lowest terms—

11. $4\cdot 123\dot{4}2857\dot{1}$. **12.** $7\cdot 017\dot{8}5714\dot{2}$. **13.** $\cdot 546\dot{1}5384\dot{6}$.

14. $\dfrac{4\cdot 37\dot{8}\dot{1}}{5\cdot 26\dot{7}\dot{2}}$. **15.** $\dfrac{\cdot 176\dot{2}8571\dot{4}}{\cdot 112\dot{1}\dot{8}}$. **16.** $\dfrac{47\cdot 286458\dot{3}}{1\cdot 351041\dot{6}}$. **17.** $\dfrac{\cdot 2291\dot{6}}{\cdot 344791\dot{6}}$.

138. The sum of any given recurring decimals may be found, correct to any required number of significant figures,* by addition in the usual way.

Example. Find the sum correct to 7 significant figures of

$$\cdot 3\dot{4}\dot{5},\quad \cdot 4\dot{1}0\dot{3},\quad \cdot 0\dot{5},\quad \cdot\dot{1}0\dot{1}.$$

$$\begin{array}{ll} \text{Process,} & \left.\begin{array}{l} \cdot 345454545\ldots \\ \cdot 410310310\ldots \\ \cdot 055555555\ldots \\ \cdot 101101101\ldots \end{array}\right\} \\ \hline \text{Result,} & \cdot 9124215\ldots \end{array}$$

We put down 9 significant figures, that we may *carry* the right figure to the seventh column.

---

* The first significant figure is the first figure on the left hand which is not a cypher. (See chap. viii.).

EXAMPLES. XC.

Find the sum, correct to 7 significant figures, of

1. $3\cdot4\dot{1} + 4\cdot5\dot{6}\dot{2} + \cdot0003\dot{1} + 365\cdot2$.
2. $4\cdot0\dot{3}\dot{6} + 2\cdot00\dot{1}2\dot{3} + \cdot0005 + 370\cdot10\dot{2}$.
3. $\cdot0\dot{7}2\dot{9} + 1\cdot3\dot{4}\dot{5} + \cdot0\dot{2}\dot{7} + 2\cdot008\dot{3}$.
4. $3\cdot2\dot{9}\dot{6} + 15\cdot0\dot{2}1\dot{3} + 2\cdot2\dot{8}0\dot{1} + 1\cdot\dot{1}$.
5. $36\cdot4\dot{0}\dot{1} + 407 + \cdot003\dot{7}\dot{1} + 5\cdot00\dot{3}70\dot{1}$.
6. $8001\cdot5 + 41\cdot37\dot{0}1\dot{6} + 4\cdot123456 + \cdot9\dot{0}\dot{1}$.

Find the value, correct to 7 significant figures, of

7. $4\cdot0\dot{0}1\dot{2} - \cdot\dot{9}12\dot{3}$.
8. $567 - \cdot\dot{0}9\dot{0}$.
9. $341\cdot1 - 44\cdot\dot{4}$.
10. $46\cdot\dot{2}13\dot{0} - \cdot9875$.
11. $\cdot4\dot{7}77\dot{1} - \cdot4\dot{1}77\dot{7}$.
12. $3\cdot\dot{1}101\dot{1} - 1\cdot\dot{3}303\dot{3}$.
13. $371 - 42\cdot\dot{3} + 5\cdot3\dot{6}6\dot{1}$.
14. $3\cdot14159 + 3\cdot\dot{3} - 6\cdot2\dot{4}1\dot{6}$.
15. $4\cdot3\dot{3}2\dot{1} + 3\cdot\dot{3}\dot{1} - 5\cdot\dot{1}234\dot{5} + \cdot00\dot{1}$.
16. $1\cdot\dot{1}\dot{0} - \cdot\dot{1}0\dot{1} + 10\cdot\dot{1}$.
17. $210\cdot45\dot{6} + 3\cdot2179 - \cdot4\dot{1}\dot{2}$.
18. $9 - \cdot03\dot{1} - \cdot7\dot{4}\dot{2}$.
19. $\cdot78\dot{9}6\dot{2} + \cdot\dot{3} - (\cdot79\dot{1}2\dot{3} - \cdot\dot{3})$.
20. $8\cdot1 + \cdot0\dot{2} - \cdot8\dot{9}$.
21. $1000 - 100\cdot\dot{0}\dot{1} + 20\cdot\dot{0}\dot{3} - 1\cdot00\dot{4}\dot{1}$.
22. $\cdot0\dot{3} - \cdot\dot{0}0\dot{5} + 1\cdot1\dot{0}\dot{1}$.

139. The sum of any given recurring decimals may be found in the form of a recurring decimal.

Example i. *Add together* $3\cdot1\dot{4}1\dot{6}$, $\cdot00\dot{7}1\dot{2}$, $\cdot4631\dot{4}\dot{2}$.

Process,

$$\begin{array}{r|r|l}
3\cdot1416 & 416416 & \ldots \\
\cdot0071 & 271271 & \ldots \\
\cdot4631 & 424242 & \ldots \\
\hline
\multicolumn{2}{r}{1111929} & \\
\multicolumn{3}{r}{1111929} \\
\multicolumn{1}{l}{3\cdot6018} & & \text{etc.} \\
\hline
\end{array}$$

Result, $\underline{3\cdot6019\dot{1}1193\dot{0}}$

We notice that the largest number of non-recurring decimal places to the right of the decimal point, in any of the numbers, is 4; therefore, the *columns* of digits *recur* after the fourth column. Also, the numbers of digits in the *periods* are 3, 3, 2 respectively; the **L.C.M.** of which is 6. Therefore the *columns* will recur in periods of 6[*] digits each; hence, the answer will recur in periods of 6 digits, commencing at the 5th place from the decimal point. Then we add separately the recurring columns and the non-recurring columns. The *sum* of the recurring columns itself recurs.

---

[*] A recurring decimal whose period contains 3 digits, may be said to have a period of 6 digits or of any multiple of 3; similarly it may happen that the sum, in an example in which the columns recur in periods of 12 digits, may have a period of 6, or of 4, or of 3, of 2, or of 1 digit. See Ex. xci. 19, 25.

Example ii. *Add* ${\cdot}\dot{0}01\dot{1}+{\cdot}\dot{7}\dot{8}+{\cdot}\dot{2}12\dot{0}+43{\cdot}\dot{7}+{\cdot}70\dot{2}$.

Process,

$$\begin{array}{r|l|l}
{\cdot}00 & 1100 & \ldots \\
{\cdot}78 & 7878 & \ldots \\
{\cdot}21 & 2021 & \ldots \\
43{\cdot}77 & 7777 & \ldots \\
{\cdot}70 & 2222 & \ldots \\
\hline
\multicolumn{2}{r}{20998} & \\
\multicolumn{3}{r}{20998\quad\quad} \\
\multicolumn{3}{r}{20998} \\
45{\cdot}46 & & \text{etc.} \\
\hline
\end{array}$$

Result, $\underline{45{\cdot}48\dot{1}00\dot{0}}$

EXAMPLES. XCI.

Find the recurring decimal which is equal to

1. $4{\cdot}0\dot{3}+1{\cdot}0\dot{2}+2{\cdot}01\dot{3}$. **2.** $3{\cdot}01\dot{2}\dot{3}+{\cdot}7\dot{8}+{\cdot}\dot{3}\dot{6}$.
3. ${\cdot}0312+{\cdot}02\dot{4}\dot{5}+1{\cdot}\dot{4}0\dot{2}$. **4.** $1{\cdot}\dot{1}+{\cdot}06\dot{1}\dot{2}+{\cdot}4\dot{1}2\dot{3}$.
5. $2{\cdot}45-2{\cdot}3\dot{6}\dot{7}$. **6.** $80-{\cdot}0\dot{8}$. **7.** $4{\cdot}\dot{3}-2{\cdot}0\dot{7}$.
8. $4{\cdot}5\dot{6}-3{\cdot}60\dot{6}$. **9.** $37{\cdot}4\dot{0}\dot{1}-{\cdot}26\dot{1}$. **10.** $4{\cdot}1\dot{2}0\dot{3}-2{\cdot}0\dot{6}\dot{1}$.
11. $5{\cdot}\dot{3}-3{\cdot}\dot{9}\dot{4}$. **12.** $6{\cdot}0\dot{3}11\dot{3}-5{\cdot}1\dot{9}\dot{3}$. **13.** ${\cdot}\dot{1}23\dot{4}-{\cdot}03\dot{7}4$.
14. $3-{\cdot}0\dot{1}\dot{2}-{\cdot}\dot{9}4\dot{1}$. **15.** $1-{\cdot}\dot{3}0\dot{6}-{\cdot}3\dot{7}$. **16.** $1-{\cdot}0\dot{9}-{\cdot}\dot{3}$.
17. ${\cdot}8-{\cdot}40\dot{7}\dot{1}-{\cdot}213\dot{8}\dot{2}$. **18.** ${\cdot}0\dot{1}\dot{2}+{\cdot}003\dot{7}+{\cdot}00\dot{1}2\dot{3}$.
19. $3{\cdot}4\dot{1}+{\cdot}0\dot{2}+{\cdot}\dot{1}\dot{2}+{\cdot}0\dot{0}5514\dot{6}$. **20.** $3{\cdot}0\dot{4}\dot{1}+{\cdot}67\dot{2}+{\cdot}25\dot{8}36\dot{3}$.
21. $1{\cdot}40201+{\cdot}\dot{3}+{\cdot}\dot{4}\dot{1}+{\cdot}03\dot{7}$. **22.** ${\cdot}\dot{4}5\dot{6}+{\cdot}0\dot{1}\dot{2}+{\cdot}000\dot{3}\dot{1}+{\cdot}82\dot{1}$.
23. $81+1{\cdot}4\dot{2}\dot{3}+2{\cdot}00\dot{5}\dot{1}+{\cdot}\dot{0}1\dot{2}$. **24.** ${\cdot}0\dot{1}2\dot{3}+{\cdot}\dot{0}12\dot{3}+{\cdot}1\dot{2}\dot{3}+{\cdot}\dot{1}2\dot{3}$.
25. ${\cdot}\dot{5}7142\dot{8}+{\cdot}\dot{2}8571\dot{4}+{\cdot}\dot{1}4285\dot{7}$. **26.** ${\cdot}0\dot{1}45\dot{1}+{\cdot}00\dot{1}4\dot{5}+14{\cdot}\dot{5}\dot{1}$.
27. ${\cdot}01\dot{0}1\dot{2}+10{\cdot}\dot{1}01\dot{2}+{\cdot}00010\dot{1}\dot{2}=1{\cdot}01212+10{\cdot}\dot{1}\dot{2}$.
28. $3{\cdot}6\dot{0}1\dot{1}+{\cdot}\dot{4}12\dot{1}-{\cdot}314\dot{2}-{\cdot}0\dot{1}2\dot{3}-{\cdot}1231\dot{2}\dot{3}$.
29. ${\cdot}00\dot{4}0\dot{1}+{\cdot}0006\dot{2}+72{\cdot}\dot{0}11\dot{1}+108{\cdot}\dot{0}\dot{1}$.
30. $300-2{\cdot}\dot{0}1\dot{4}-{\cdot}0\dot{4}11\dot{1}-{\cdot}03\dot{1}2\dot{2}-{\cdot}01\dot{2}1\dot{1}-{\cdot}\dot{1}4285\dot{7}$.

140. For the **multiplication** and **division** of **recurring** decimals we use the method of Arts. 50 and 51.

Example i. *Multiply* $4{\cdot}375$ *by* $5{\cdot}6\dot{7}2\dot{8}$.

We shall obtain the same result if we multiply $5{\cdot}6\dot{7}2\dot{8}$ by $4{\cdot}375$.

Process,

$$\left.\begin{array}{r|l}
5{\cdot}6728728 & 72 \\
4{\cdot}375\quad\quad\; & \\
\hline
22{\cdot}691\;491 & \mathbf{4914} \\
1{\cdot}701\;861 & 86\mathbf{18} \\
{\cdot}397\;101 & 101\mathbf{1} \\
28\;364 & 3643 \\
\hline
1817\quad & \\
24{\cdot}817\;\;\;1817\ldots & \\
\end{array}\right\}$$

We write down *two periods* of the multiplicand, and also the next two digits, in order to *carry* the correct number; when the multiplication is finished we fill in the vacant places (as has been done here with black type). *Product* $\underline{24{\cdot}\dot{8}1\dot{8}}$.

$\underline{24{\cdot}818\dot{8}1\dot{8}} = 24{\cdot}\dot{8}1\dot{8}$.

Example ii. *Divide* $11\cdot427\dot{5}5\dot{2}$ *by* $4\cdot375$.

$$
\begin{array}{l}
\text{Process,}\quad 4375)\underline{11427}\cdot552552\ldots(2\cdot6\dot{1}2\dot{0}12\ldots \\
\qquad\qquad\qquad\quad \underline{2677\ 5} \\
\qquad\qquad\qquad\qquad \underline{52\ 55} \\
\qquad\qquad\qquad\qquad\ \ \underline{8\ 802} \\
\qquad\qquad\qquad\qquad\qquad \underline{5255} \\
\qquad\qquad\qquad\qquad\qquad\ \ \underline{8802}
\end{array}
$$

The quotient is $\underline{2\cdot612\dot{0}}$

EXAMPLES. XCII.

Multiply—

1. $4\cdot2\dot{3}$ by 6.
2. $3\cdot0\dot{6}$ by 3.
3. $2\cdot4\dot{1}\dot{3}$ by $\cdot51$.
4. $20\cdot\dot{7}$ by $\cdot32$.
5. $\cdot00\dot{8}\dot{1}$ by 11.
6. $\cdot3\dot{2}\dot{7}$ by $5\cdot2$.
7. $31\cdot02\dot{6}\dot{3}$ by $4\cdot302$.
8. $\cdot00050\dot{7}\dot{2}$ by $\cdot009$.
9. $71\cdot\dot{3}4\dot{3}$ by $1\cdot012$.
10. $21\cdot01\dot{3}6\dot{0}$ by $41\cdot003$.
11. $3\cdot602$ by $2\cdot01\dot{2}\dot{3}$.
12. $84\cdot307$ by $4\cdot\dot{0}037\dot{1}$.
13. 2022 by $\cdot006\dot{1}0\dot{3}$.
14. $50\cdot307$ by $12\cdot001020\dot{5}$.
15. $301\cdot7$ by $4\cdot01\dot{0}20\dot{3}$.
16. $\cdot001\dot{0}6\dot{1}$ by $3\cdot418$.

Divide—

17. $3\cdot\dot{6}$ by 5.
18. $4\cdot372\dot{0}$ by 11.
19. $\cdot0\dot{3}0\dot{1}$ by $\cdot006$.
20. $2\cdot\dot{3}$ by $\cdot0021$.
21. $\cdot000\dot{4}$ by $\cdot0072$.
22. $40\cdot0\dot{1}0\dot{2}$ by $\cdot102$.
23. $41\cdot37\dot{0}\dot{6}$ by 130.
24. $8\cdot6\dot{5}$ by $\cdot0101$.
25. $\cdot\dot{2}8571\dot{4}$ by $\cdot07$.
26. $\cdot\dot{4}2857\dot{1}$ by $\cdot003$.
27. $\cdot036\dot{5}$ by $\cdot2\dot{9}$.
28. $37\cdot\dot{5}\dot{1}$ by $3\cdot44\dot{9}$.
29. $\cdot124\dot{9}$ by $1\cdot\dot{9}$.
30. $\cdot0\dot{7}1428\dot{5}$ by $\cdot006$.
31. $\cdot001\dot{9}$ by $\cdot3\dot{9}$.
32. $\cdot0\dot{3}9\dot{6}$ by $\cdot010\dot{9}$.
33. $\cdot\dot{2}8571\dot{4}$ by $2\cdot31$.
34. $99\cdot\dot{9}$ by $\cdot064$.

141. When a recurring decimal is to be multiplied (or divided) by a recurring decimal, it is generally convenient to express the multiplier (or divisor) as a vulgar fraction.

Example i. *Multiply* $12\cdot\dot{9}8\dot{1}$ *by* $5\cdot\dot{0}\dot{9}$.

Here $5\cdot\dot{0}\dot{9}=5\frac{9}{99}=5\frac{1}{11}=\frac{56}{11}$;

therefore we multiply $12\cdot981$ by 56 and divide the product by 11; thus,

$$
\begin{array}{r}
12\cdot981981\,|\,98\ldots \\
\underline{8\quad} \\
103\cdot855855\ 8\quad \\
\underline{7\quad} \\
11)\overline{726\cdot990990\ldots} \\
\underline{66\cdot090090}\ldots=\underline{66\cdot\dot{0}9\dot{0}}.
\end{array}
$$

Example ii. *Divide* $5\cdot4\dot{5}\dot{8}$ *by* $5\cdot\dot{4}\dot{5}$.

Here, $5\cdot\dot{4}\dot{5}=5\frac{45}{99}=5\frac{5}{11}=\frac{60}{11}$;

therefore we multiply $5\cdot4\dot{5}\dot{8}$ by 11, and divide the product by **60**; obtaining as result $\underline{1\cdot0\dot{0}7\dot{4}}$.

EXAMPLES. XCIII.

1. $\cdot0\dot{3}\times\cdot0\dot{6}$.
2. $\cdot0\dot{1}\dot{2}\times\cdot\dot{3}$.
3. $\cdot41\dot{3}\times\cdot\dot{2}\dot{7}$.
4. $31\cdot29\dot{8}\dot{1}\times\cdot0\dot{3}$.
5. $3\cdot\dot{9}8\dot{1}\times\cdot\dot{4}$.
6. $5\cdot0\dot{6}\dot{3}\times\cdot0\dot{5}$.
7. $15\cdot\dot{3}\times10\cdot\dot{5}$.
8. $21\cdot\dot{0}\dot{9}\times\cdot1\dot{4}$.
9. $43\cdot\dot{0}\dot{6}\times\cdot4\dot{3}$.
10. $26\cdot\dot{8}\dot{1}\times\cdot01\dot{2}$.
11. $3\cdot4\dot{6}\div2\cdot\dot{3}$.
12. $4\cdot\dot{5}\div3\cdot0\dot{4}$.
13. $12\cdot\dot{3}\div\cdot\dot{8}\dot{1}$.
14. $6\cdot0\dot{3}\dot{1}\div\cdot\dot{7}\dot{2}$.
15. $2\cdot3\dot{6}\div4\cdot0\dot{7}$.
16. $\cdot000\dot{5}\div12\cdot\dot{4}$.
17. $34\cdot0\dot{1}\times\cdot\dot{9}\div\cdot\dot{1}$.
18. $2\cdot\dot{4}\dot{5}\div\cdot000\dot{7}\dot{2}$.
19. $\cdot000\dot{3}\dot{6}\div2\cdot\dot{7}\dot{2}$.
20. $\cdot\dot{4}\dot{6}\times\cdot\dot{9}\div\cdot3\dot{7}\dot{8}$.
21. $\cdot0\dot{3}\dot{6}\times\cdot\dot{9}\div\cdot\dot{3}$.
22. $\cdot00\dot{3}6\dot{9}\div\cdot\dot{1}4285\dot{7}$.
23. $\cdot\dot{4}2857\dot{1}\div\cdot\dot{2}8571\dot{4}$.
24. $22\cdot\dot{2}\times\cdot0\dot{9}\div\cdot3\dot{7}$.
25. $(2\cdot\dot{3}-1\cdot\dot{5})\div(4\cdot\dot{1}+6\cdot0\dot{3})$.

142. **We can always tell whether the decimal form of any given vulgar fraction is a recurring decimal.**

Let the vulgar fraction be in its lowest terms; then if the **prime factors of the denominator** are each of them either **2**, or **5**, the equivalent decimal is *terminating*, that is does, not recur. In all other cases it is *recurring*.

Example i.

$\frac{12}{25}$, that is $\frac{12}{5\times5}$; $\frac{13}{50}$, that is $\frac{13}{2\times5\times5}$; $\frac{29}{64}$, that is $\frac{29}{2\times2\times2\times2\times2\times2}$

produce terminating decimals.

The reason is, that in each step in the continued process of division, the dividend is multiplied by 10, that is by 2×5; so that if the denominator contains those factors only the division will terminate.

$$\text{Thus, }\frac{29}{64}=\frac{29}{2\times2\times2\times2\times2\times2}=\frac{29\times10\times10\times10\times10\times10\times10}{2\times2\times2\times2\times2\times2\times1000000}$$

$$=\frac{29\times5\times5\times5\times5\times5\times5}{1000000}=\frac{453125}{1000000}=\cdot453125.$$

Example ii. $\frac{11}{24}$, that is $\frac{11}{2\times2\times2\times3}$; $\frac{4}{65}$, that is $\frac{4}{5\times13}$; $\frac{1}{7}$

produce recurring decimals.

The reason is, that neither of these numbers can be made whole numbers by multiplying repeatedly by 2×5.

Example iii. The fraction $\frac{7}{13}$ cannot have more than 12 figures in the period of its equivalent recurring decimal; for each remainder in the process of division by 13 must be less than 13, and cannot be 0; so that there are not more than 12 possible remainders which are all *different.*

Example iv. The fraction $\frac{5}{51}$ or $\frac{5}{3\times17}$ cannot have more than 17 digits in its period; for it is equal to $\frac{1}{17}$ of $1\cdot\dot{6}$, and in dividing $1\cdot\dot{6}$ by 17 each remainder must be less than 17 (it may be 0); so that there are not *more* than 17 *different* remainders.

NOTE.—It should be remembered that the division of a non-recurring decimal by 3 or by 9 gives a period of *one* digit; division by 11, a period of *two* digits; division by 7, or by 13, a period of *six* digits.

Example. $\cdot5\dot{9}2307\dot{6}\times5\cdot\dot{4}\dot{5}\times3\cdot\dot{7}1428\dot{5}$.

This is $\cdot5\dot{9}2307\dot{6}\times5\frac{5}{11}\times3\cdot\dot{7}1428\dot{5}\times\frac{7}{7}=\cdot5\dot{9}2307\dot{6}\times\frac{60}{11}\times\frac{26}{7}$
$=\cdot5\dot{9}2307\dot{6}\times13\times\frac{60}{11}\times\frac{2}{7}=7\cdot6\dot{9}\times\frac{60}{11}\times\frac{2}{7}=\underline{12}$.

EXAMPLES. XCIV.

State which of the following fractions, when expressed as decimals, are recurring; and state a limit to the number of figures in the period of each.

1. $\frac{3}{26}$. 2. $\frac{7}{35}$. 3. $\frac{9}{45}$. 4. $\frac{23}{15}$. 5. $\frac{21}{28}$.
6. $\frac{7}{24}$. 7. $\frac{9}{13}$. 8. $\frac{102}{120}$. 9. $\frac{49}{63}$. 10. $\frac{84}{105}$.
11. $\frac{33}{121}$. 12. $\frac{81}{96}$. 13. $\frac{101}{165}$. 14. $\frac{7}{81}$. 15. $\frac{124}{217}$.

EXAMPLES. XCV.

Simplify, giving each answer in decimals,—

1. $\dfrac{2+\cdot5-\cdot\dot{3}}{2\cdot\dot{3}-1\cdot25}$.

2. $\dfrac{2\cdot5+3\cdot\dot{3}-5\cdot25}{5\cdot1\dot{6}+7\cdot08\dot{3}}$.

3. $\dfrac{3\cdot5-1}{15\cdot\dot{3}+5\cdot5-3\cdot\dot{3}}\div\dfrac{1-\cdot75}{1+\cdot75}$.

4. $3\cdot\dot{3}\div\dfrac{1-\cdot9375}{\cdot2-\cdot125}$.

5. $\dfrac{3\cdot25-2\cdot\dot{3}}{3\cdot1\dot{6}+1\cdot5}\div1\cdot\dot{5}7142\dot{8}$.

6. $\cdot25\times\cdot875$ of $\left(\dfrac{2\cdot\dot{3}}{3\cdot5}+\dfrac{4\cdot5}{5\cdot25}\right)$.

7. $\dfrac{4+(\cdot\dot{1}\div\frac{2}{9})}{2+2\times\cdot25}\div\left(\dfrac{\frac{1}{2}\text{ of }5\cdot5}{2\div\frac{1}{3}}\times\cdot7\dot{4}\dot{5}\right)$.

8. $\dfrac{\cdot\dot{3}\text{ of }\frac{5}{6}}{\cdot5\times\cdot\dot{1}}+\dfrac{\cdot3\dot{5}}{\cdot0\dot{8}}$.

9. $\dfrac{14\cdot7\dot{3}\dot{2}+2\cdot\dot{9}\dot{8}}{20\cdot\dot{9}\dot{0}-1\cdot\dot{4}\dot{6}}$.

10. $\dfrac{(\frac{2}{3}-\frac{1}{4})(1\cdot\dot{4}-\cdot\dot{2})}{\frac{2}{3}\text{ of }2\cdot2\div\cdot5}$.

11. $\dfrac{2\cdot5+(3\cdot25\div\cdot8)\text{ of }\cdot65}{(2\frac{1}{3}+1\frac{3}{4})\times2\frac{4}{7}-5\cdot25}$. **12.** $\dfrac{2\cdot0\dot{3}+1\cdot\dot{2}\dot{1}}{2\cdot0\dot{3}-1\cdot\dot{2}\dot{1}}$.

13. $\dfrac{\frac{1}{5}\times2}{\frac{11}{15}-\frac{2}{5}}-\dfrac{\frac{3}{5}-\cdot\dot{0}9523\dot{8}}{1\frac{11}{35}-(\frac{1}{7}+\frac{2}{3})}$. **14.** $\cdot41\dot{6}$ of $\dfrac{\cdot015\times2\cdot1}{\cdot035}$ of $\dfrac{2}{70}$.

MISCELLANEOUS EXAMPLES. XCVI.

1. Express 3·645 pks. + ·0018 bus. − ·09 qts. in pints.

2. Express in inches 35 yds. − ·0045 mi. − ·052 fur. − ·153 p.

3. Express in yds. ·0049 mi. − (·5125 yds. + ·62734 p. + ·018325 fur.)

4. Express ·0029 ton + ·273 cwt. in lbs.

5. Reduce $\frac{7}{121}$ of 20 sq. rods to the decimal of $\frac{10}{11}$ of an acre.

6. Reduce $\cdot4097\dot{2}$ of \$3 + $\cdot2\dot{7}$ of \$8 to the dec. of \$20.

7. Reduce $\cdot23\dot{4}\dot{5}$ of 6 lbs. 14 oz. − $\cdot42708\dot{3}$ of 1 lb. to the dec. of 1 cwt.

8. Reduce $1\cdot\dot{2}$ year ~ $1\cdot3\dot{2}\dot{7}$ of 308 days to the dec. of ·5 year.

9. Express $\frac{3}{28}$ of 21 hrs. + $\cdot99791\dot{6}$ of 1 day as the dec. of 1 week.

10. Simplify $2\cdot74\dot{9}$ of $\cdot2\dot{5}\dot{4}$ of $\cdot\dot{1}9047\dot{6}$ of \$54.

11. Simplify $2\cdot2\dot{7}$ of \$4.50 + $7\cdot\dot{4}\dot{5}$ of \$16.50 + $\cdot3\dot{5}7142\dot{8}$ of \$3.50.

12. Prove that ·3 of 40 days + ·4 of 7 days 7 hrs. + ·5 of 4 hrs. = $\cdot\dot{6}$ of 22 days 12 hrs.

13. Simplify 2·5 of 2 tons 6 cwt. + 3·125 of 1 cwt. 16 lbs. + 3·75 of 448 oz.

14. Simplify ·856 of 2 cwt. 26 lbs. ~ $3\cdot2\dot{2}\dot{7}$ of 66 lbs.

15. Simplify $\dfrac{5\cdot1183}{\cdot0141}$ of 22·2 of $\cdot0\dot{9}$ of $\cdot2\dot{3}\dot{4}$.

16. Express in inches $\cdot03\dot{2}4\dot{9}$ of 10 yds.

17. If the unit 24 yds. be applied to $\cdot267\dot{1}87\dot{5}$, what does it express in yds. ft. in.?

18. Simplify $1\cdot7\dot{2}$ of $\cdot\dot{2}7\dot{6}$ of 15.

19. Simplify $\cdot\dot{6}\dot{5}$ of $4\cdot\dot{1}\dot{1}$ of $\dfrac{3\frac{2}{3}}{13}$ of $2\cdot\dot{4}3\dot{2}$ of \$13.50.

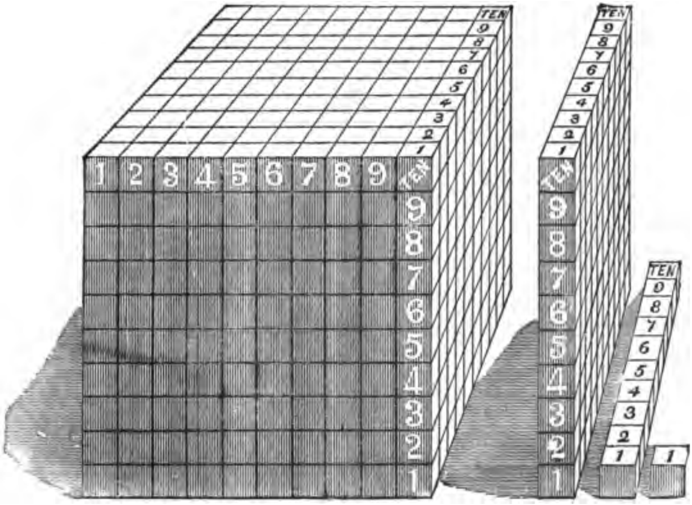
20. Reduce $\cdot\dot{4}\dot{5}$ of $2\cdot1\dot{6}$ of 3c. to the fraction of $\cdot\dot{0}2\dot{7}$ of $1\cdot\dot{1}\dot{8}$ of \$1.20.

21. When a fraction in its lowest terms is reduced to a decimal, show that if the decimal is not finite it must *recur*.

22. Show that $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ expressed as decimals may be arranged in a square, so that the sum of each column and of each row is the same.

CHAPTER VIII

APPROXIMATION



143. The above diagram is intended to illustrate the **relative** importance of two digits, in a decimal expression which are **three** decimal places apart.

Example (i). In each of the following numbers the left-hand 1 may represent the large cube, the right-hand 1 the small cube

100100; 1001; 10·01; ·0001001; etc.

The *comparative* importance of the digits is independent of the position of the decimal point.

Example (ii). 6758 small cubes = 6 of the large cubes + 7 of the next block of cubes + 5 of the next + 8 small cubes.

There is not space in the page to give a pictorial illustration on the same scale of the relative importance of digits which are more than three decimal places apart.

The student who wishes to realise in some way the relative importance of digits which are **four** decimal places apart, is advised to go through the labour of counting 10,000 counters, as suggested in Art. 7.

He will in this way understand how **insignificant** is the quantity represented by right-hand digits, in the following expressions, when **compared** with the quantity represented by the left-hand digit, no matter what may be the unit to which the number is applied.

3000300; 10001; ·070007; ·000020002; etc.

144. The **first significant figure** in a number is the first figure on the left which is not a cypher.

Example. In each of the following numbers the 3 is the **first** significant figure: 37054000; 37·005; ·00037504; ·0000304.

145. The **significant figures** of a number are those digits which indicate actual measurement.

Example (i). It is stated that the population of England and Wales on a certain day in 1881 was 25,974,000. Here then are five significant figures. The expression may be read as '25974 *thousands* of people.'

Example (ii). It is stated that the average distance of the earth from the sun is about 92,000,000 miles; here there are **two** significant figures; it may be read 92 *million* miles.

Example (iii). The average distance of the moon from the earth is about 238,000 miles; here there are three significant figures.

In the expression '*about* 92,000,000 *miles*' the cyphers do **not** indicate the *absence* of a digit; they only indicate that the 92 stands for 92 **millions**, and they are **not** '*significant figures.*'

NOTE.—A cypher may be a '*significant figure.*'

Example (iv). The distance from Grantham to King's Cross (London) is 105·25 miles.

The 0 in this expression is a significant figure because it indicates actual measurement. Thus 0 **is** a significant figure when it indicates the absence of a power of ten in a number; 0 is **not** a significant figure when it is to the left of the *first* significant figure, **nor** when it is merely used as in Ex. (i), (ii), (iii) above, to indicate for other figures their proper power of ten. [See Art. 13.]

146. The following illustration will explain the importance of the term *significant figures.*

Let the pupil attempt to make some measurement correct to four significant figures. Let him take the distance on a board between two carefully marked lines (about 30 or 40 inches apart) and measure the length of the line joining them; and let him estimate this length to the hundredth part of an inch.

Let him repeat the measuring on another occasion ; he will have to use the very greatest care in making each measurement if he wishes the two results to agree to four significant figures.

147. The statement that the measure of a quantity is **correct to a number**, say 4, **of significant figures,** is understood to assert that the *fourth* figure is the *nearest* to the actual value. [See Example, p. 100.]

Example (i). An imperial gallon of Great Britain contains 277·274... cubic inches. To 3 significant figures, a gallon contains 277 cub. in., to 4 significant figures, a gallon contains 277·3 cub. in.*

Example (ii). The statement, *a line contains* 39·14 inches *correct to* 4 *figures*, asserts that the actual distance differs from 39·14 in. by less than ·005 in.; the actual distance may be greater than 39·14 in. or it may be less, it is not stated which.

148. The **degree of approximation** to which a measurement is carried may be measured roughly by the number of significant figures known to be correct, in the number indicating the quantity.

Example. The distance of the moon is given as 238000 miles, correct to 3 figures ; that is, 238 thousand miles. The wave-length of light of a particular colour is given as ·000688 inches correct to 3 figures ; that is, 688 millionths of an inch. In the first case the '*error*' is less than 500 miles [see Art. 147], in the second, less than ·0000005 in.: that is, in the first case, less than $\frac{1}{400}$th, in the second less than $\frac{1}{1200}$th of the whole. The **degree of approximation** in both cases is about the same.

The degree of approximation indicated by the expression correct to a certain number of significant figures does *not* depend on the *magnitude* of the *unit.*

NOTE.—It should be carefully noticed that the asking for a calculation to be carried to the '*sixth place of decimals*' meaning '*to the sixth negative power of ten*' without any indication of the magnitude of the unit employed gives no idea of the degree of approximation required.

Example. When asking for the expression of the distance of the moon correct to the '*sixth decimal place,*' it is important to state whether the unit is a million miles, or a mile, or an inch.

---

* NOTE.—When it is known that the number is too small it is convenient to indicate it thus : '*a gallon contains* 277 + ... c. in. ;' when it is known to be too large, thus : '*a gallon contains* 277·3 − ... c. in.'

149. The pupil will now be prepared for the statements, (i) that in practical work, measurements are not often correct to more than *three* significant figures; (ii) that when a quantity has been measured with the greatest degree of accuracy attainable, under the most favourable conditions, by the most skilful and accurate observers, the degree of accuracy attained very rarely exceeds that indicated by the expression *correct to* six *significant decimal figures.*

150. We may also put the above statement thus: (i) in ordinary practice we are satisfied if the magnitude of any quantity is known to about the *thousandth* part of itself;

Examples. A man whose income is five or six hundred dollars a year would not think of stating its value more accurately than to the nearest dollar.

When the expenditure of the nation is mentioned no one thinks of attempting more than two significant figures. For example, the expenditure of Great Britain for 1883-84 was *about* 87 *millions of pounds.*

(ii) in careful scientific work it is very rarely possible to obtain with certainty the measurement of any quantity nearer than the *millionth* part of itself.

As regards intervals of **time**, it is said that Sir George Airy was able with his instruments to estimate one part in 8,640,000. As regards determination of **weight**, Professor Clerk Maxwell considered that one part in 5,000,000 *could* be detected. The measurement of **length** is most difficult; in very careful measurements the degree of accuracy attained is considered to be about 1 part in 60,000 [Jevons, *Principles of Science*, page 304. Macmillan].

Example. The length of the base line, on Salisbury Plain, used in the Ordnance Survey of England is 'about 36578 – ...' feet correct to five significant figures, that is, it is considered to differ from 36578 ft. by less than 5 tenths of a foot. [See Art. 147.]

151. Absolute accuracy in practical work is thus obviously an impossibility. This, however, does not render **accuracy in Arithmetic** less necessary or less important.

For a *mistake* in Arithmetic, if it occur at all, is just as likely to be made in the first or second significant figure as in any other.

152. In the **Metric System of weights and measures** the units are as follows. [See Art. 120.]

The unit of **Length** is the **Metre** = $39{\cdot}37_{079}$...inches.
The unit of **Capacity** is the **Litre** = $\cdot 26_{41}$...gallons.
The unit of **Weight** is the **Gram** = $15{\cdot}43_{235}$...grains.
= $\cdot 00002204_{621}$...cwts.
The unit of **Surface** is the **Are** = $119{\cdot}6_{033}$...sq. yds.
= $\cdot 02471_{143}$...acres.

The names of the multiples and sub-multiples are—

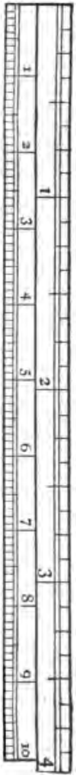
$$\left.\begin{array}{l}\text{milli-}\\ \text{centi-}\\ \text{deci-}\\ \text{one}\\ \text{deka-}\\ \text{hecto-}\\ \text{kilo-}\\ \text{myria-}\end{array}\right\} \left.\begin{array}{l}\text{-metre}\\ \text{-litre}\\ \text{-gram}\\ \text{-are}\end{array}\right\}, \text{ indicating } \left\{\begin{array}{c}\frac{1}{1000}\\ \frac{1}{100}\\ \frac{1}{10}\\ 1\\ 10\\ 100\\ 1000\\ 10000\end{array}\right\} \text{ times } \left\{\begin{array}{l}\text{a metre,}\\ \text{a litre,}\\ \text{a gram,}\\ \text{an are.}\end{array}\right.$$

All the other units are derived from the **metre**, thus:

A litre is the content of a cubic decimetre.

A gram is the weight of a cubic centimetre of pure water.

An are is a square measuring a dekametre each way.



The following facts may be noticed—

1 **Metre** = $39\frac{3}{8}$ in. *nearly*. 70 yds. = 64 metres *nearly*.
25 millimetres = 1 inch *nearly*.*

1 **Kilometre** = 1100 yds. *nearly*.
8 kilometres = 5 miles *roughly*.

1 **Are** (= 100 sq. metres) = *rather less than* 4 sq. rods.

1 **hectare** = *rather less than* $2\frac{1}{2}$ acres.

1 **Litre** = *nearly* $1\frac{1}{18}$ liquid quarts, or $\frac{9}{10}$ of a dry quart.
4 litres *are a little more than* a gallon.

1 **Gram** = $15\frac{1}{2}$ grains *nearly*.
28 grams = 1 oz. Avoir. *roughly*.

1 **Kilogram** = $2\frac{1}{5}$ pounds Avoir. *nearly*.

Metric ton (= 1000 kilos) = 22 cwt. *roughly*.

---

* At the side are two scales; the longer contains 4 inches, and is subdivided into inches and eighths of an inch. The shorter contains 1 decimeter and is subdivided into 10 centimetres and into 100 millimetres.

153. Since, in any number, the quantity indicated by all the significant figures after the sixth, is so small (*compared with the quantity indicated by the first two or three significant figures*) as to be imperceptible to our senses, it follows that the working out, by arithmetic, of any result correct to *more than six or seven significant figures*, is often rather a matter of curiosity than of practical utility; hence methods of **contraction** are often useful.

Example i. *Multiply* 431·26725 *by* 3·9425631... *obtaining the product correct to* six *figures.*

$$\begin{array}{r|l} 431\cdot2\cancel{6}\cancel{7}\cancel{2}\,|\,\cancel{5} & \\ 3\cdot9425\ 631 & \\ \hline 1293\cdot8017+ & \\ 388\cdot1405+ & \\ 17\cdot2507- & \end{array}$$

We multiply first by 3. It is generally safer to retain *two* places more than the number required to be correct; we therefore cross out the 5 in the multiplicand, and commence multiplying 2 by 3; at the same time observing that we *carry* 1 from the 5. Having finished the multiplication by 3, we cross out the 2 and commence the multiplication by 9 at the figure 7, carrying 2 from the 22; and so on.

The process above is left unfinished; the work is at the stage when the multiplication by 2 is to be begun. An example completely worked will be found on page 146.

EXAMPLES. XCVII.

Obtain the products to five significant figures of

1. 3·14159 × 39·1393. **2.** 1·41421 × 1·760773.

3. 1·44225 of 39·37079. **4.** 365·2422 of 1·968412.

5. $14372\cdot\dot{1}$ of $\cdot790\dot{3}21$. **6.** $43\frac{5}{13} \times 1\cdot4\dot{2}3\dot{7}$.

7. $4\dot{7}\cdot120\dot{3}$ of 361·402. **8.** 37·4123 of $115\cdot0\dot{3}\dot{7}$.

9. $\cdot01234\dot{5}$ of $49\cdot\dot{3}\dot{6}$. **10.** $4\cdot\dot{3}7\dot{1}$ of 90·625.

11. $(3\cdot14159)^2$ × 3 m. 231 yds. 2 ft.

12. $(1\cdot41421)^2$. **13.** $(1\cdot44225)^3$.

Express

14. 452 pints in litres. **15.** 1760 metres in yds.

16. The distance of the moon in kilometres to 3 figures.

17. 84123 metres in feet. **18.** 4840 sq. metres in sq. yds.

Example ii. *Divide* 1700·306 *by* 431·26725 etc. *correct to* five significant figures. Process,

$$\left.\begin{array}{r}
431\cancel{2}\cancel{6}\cancel{7}\cancel{2}|\cancel{5})\underline{17003060}(\underline{3\cdot94258}\ldots \\
\underline{4065042} \\
\underline{183637} \\
\underline{11130} \\
\underline{2505} \\
\underline{349}
\end{array}\right\}$$

Since 5 significant figures are required, keep (5+2) figures (if there are as many) in the divisor, and cross out a digit in the divisor after each multiplication; remembering in multiplying to *carry* from the last digit crossed out.

N.B.—Should there be less than 7 digits in the divisor, then make up the 7 digits with ciphers (to be successively crossed out).

EXAMPLES. XCVIII.

Obtain the following quotients to 5 significant figures:—

1. 27·3782 ÷ 4·3267. **2.** 4·07012 ÷ 37·776. **3.** 571·013 ÷ ·067412.

4. 1 ÷ 3·14159. **5.** 10 ÷ 1·41421. **6.** 372·1 ÷ 1·44225.

7. Express $\frac{4273}{5737}$ as a decimal to 4 figures.

Prove the following statements: **8.** 1 in. = ·0253995... metres.

9. 1 mile = 1609·315... metres. **10.** 1 gram = 15·4... grains.

11. 1 quintal (100000 grams) = 2·2046... cwt.

12. 1 litre = ·2642... gallons. **13.** 1 hectare = 2·47114... acres.

14. 1 sq. ft. = ·092899... sq. met. **15.** 1 cub. yd. = ·764513... cub. met.

16. How many metres are there in 39·1393 in.?

17. A year contains 365·242216... mean solar days, or 366·242216... sidereal days; show that a sidereal day = ·99727 solar days.

154. We may note the following facts:—

A **solar year** contains 365·242 + ... days.

The **diagonal** of a SQUARE = 1·41421 + ... times a side.

The **diagonal** of a CUBE = 1·73205 + ... times a side.

The **circumference** of a CIRCLE = 3·1416 − ... times its diameter.

The **area** of a CIRCLE = 3·1416 − ... × the square on its radius.

The **surface** of a SPHERE = 4 × 3·1416 − ... × the sq. on its radius.

The **volume** of a SPHERE = $\frac{4}{3}$ of 3·1416 − ... times the cube having the radius of the sphere for an edge.

EXAMPLES. XCIX.

1. Find to an inch the diameter of a square whose side is 100 yds.

2. Find the side of a square whose diameter is a mile.

3. Find how many times a bicycle-wheel, whose diameter is 50 inches, will turn in travelling one mile.

4. Find to the 100th part of an inch the diameter of a cubic yard.

5. Find the area of a circle whose radius is 10 ft.

6. Find the diameter of a wheel whose circumference is 15 ft.

7. Find the circumference of a wheel 4·3825 ft. in diameter.

8. How many revolutions will a carriage wheel 5 ft. in diameter make in going a mile?

9. The circumference of the dome of St. Paul's is 339·4286 ft., find the area of its base in sq. ft.

10. How many yards are there in the diameter of a square each of whose sides measures 1 mile?

11. What must be the length of the side of a square that the diagonal may measure 1 mile?

12. What must be the length in inches of the side of a cube that its diagonal may measure 1 yard?

155. The following examples illustrate the subject:—

Example i. *Find the value of* 365 yds. 32 in. *at* $14·829 *per yd.*
Here 365 yds. 32 in. = 365·8888 yds.

$$\begin{array}{lrl}
\text{Multiplying} & 365\cdot\cancel{8}\cancel{8}\cancel{8} & \\
 & 14\cdot829 & \\
\cline{2-2}
 & 3658\cdot888 & \\
 & 1463\cdot555 & \\
 & 292\cdot711 & \\
 & 7\cdot318 & \\
 & 3\cdot293 & \qquad\text{[See Art. 156.} \\
\cline{2-2}
\text{The value is} & \$5425\cdot665 & \\
 & =\underline{\$5425\cdot67} & \text{nearly.}
\end{array}$$

Example ii. *A tradesman's debts are* $47916.98, *and his assets are* $19436.79; *how much can he pay in* $100, *and how much does a creditor for* $3799.62 *receive?*

Here the debtor pays $19436.79 for $47916.98,

therefore he pays $\$\frac{19436.79}{47916.98}$ for $100.

$$\begin{array}{r}
479\cancel{1}\cancel{6}\cancel{9}\cancel{8})194367900\cdot(40\cdot5634 \\
2699980 \\
304131 \\
16630 \\
2255
\end{array}$$

Hence the debtor pays $40.56 in $100, and a creditor for $3799.62 receives $3799.62 × ·405634.

That is, $1541.25 nearly.

156. It is most important to notice that when one of two factors is correct to, say, **4** places, while the other is correct to, say, **7** places, their **product** is correct only to the smaller number **4** of places.

For 10,000 × 10,000,000 = 100,000,000,000,
and 10,000·5 × 10,000,000·5 = 100,005,005,000·25.

EXAMPLES. C.

The answers expressed in $ to be given to the nearest cent; other answers to be given correct to five significant figures.

1. Find the value of 347·25 articles at $17932.50 each.

2. Find the value of $4712\frac{3}{4}$ articles at $109.30 each.

3. Find the value of 437·325 cwt. at $26.97 per cwt.

4. Find the value of 4 m. 2 f. 11 p. of wire fencing at $218.60 per mile.

5. A bankrupt owes $47510, and his assets* are $37414.50. What can he pay in $100?

6. A bankrupt pays $3.99 in $5, and his assets are $21798.75. What are his debts?

7. Find the value of 3·14168 tons at $77.65 per ton.

8. Find the value of 13 tons 11 cwt. $97\frac{1}{2}$ lbs. at $79.18 per cwt.

9. Find the value of $469\frac{15}{21}$ articles at $21.97 each.

10. If 347·562 cwt. cost $18708.70, what will 1 cwt. cost?

11. If 4·3012 tons cost $423.45, what will 1 ton cost?

12. If 3 tons 11 cwt. $31\frac{1}{4}$ lbs. cost $2187.35, what will 1 cwt. cost?

13. Find the value of 40 yds. 2 ft. $7\frac{5}{8}$ in. at $50.74 per yd.

14. A bankrupt fails for $33098, and his assets are $13896.21; how much does he pay in $100, and how much will a creditor for $450.95 receive?

15. 3764 sq. yds. 7 sq. ft. of building land is sold for $6208.50; find the price per sq. yd. and per acre.

16. A bankrupt's debts are $11503.43, and he pays $49.48 in $100; what are his assets?

17. If 347·123 cwt. of goods cost $11797.30, how much will 41·325 cwt. cost?

---

* By a bankrupt's **assets** is meant the value of his property which is available for the payment of his creditors.

*MISCELLANEOUS EXAMPLES. CI.

1. Find the difference between $\frac{15}{16}$ of 16 hrs. 7 min. and ·0125 of 67 days 18 hrs. 45 min.

2. Multiply $\cdot 3\dot{1}4285\dot{7}$ by $\cdot 2\dot{2}8571\dot{4}$ to five significant figures, and verify by fractions.

3. Find a decimal that is within $\frac{1}{1000000}$ of $\frac{355}{113}$.

4. Divide $\cdot 2\dot{4}2857\dot{1}$ by $\cdot 6\dot{8}5714\dot{2}$ to five significant figures, and verify by fractions.

5. Standard gold contains $\frac{11}{12}$ of its own weight of pure gold; how much pure gold is there in an English sovereign? [A sovereign contains 123·274 grains of standard gold.]

Find the value to five significant figures of

6. $$\frac{1}{1}+\frac{1}{2}+\frac{1}{2\times 3}+\frac{1}{2\times 3\times 4}+\frac{1}{2\times 3\times 4\times 5}+\text{etc}$$

7. $$\frac{1}{1}+\frac{1}{3}+\frac{1}{3\times 5}+\frac{1}{3\times 5\times 7}+\text{etc.}$$

8. $$16\times\left(\frac{1}{5}-\frac{1}{3\times 5^3}+\frac{1}{5\times 5^5}-\frac{1}{7\times 5^7}\right)-\frac{4}{239}.$$

9. $$1+\frac{1}{1}\times\frac{1}{7}+\frac{1}{2}\times\frac{1}{7^2}+\frac{1}{2\times 3}\times\frac{1}{7^3}+\frac{1}{2\times 3\times 4}\times\frac{1}{7^4}+\cdots$$

10. Find the area of a circular estate enclosed by a ring fence a mile long. [See Art. 154.]

11. Find the cost of gravelling, at 20c. per yard, a circular road 10 yds. wide, the inner radius of the circle being 400 yds.

12. Find the average speed of a train which travels $87\frac{11}{16}$ miles in 1 hr. 21 m. 9 sec.

13. Prove to five significant figures that

$$\left(2+\frac{1}{2}+\frac{1}{2\times 3}+\frac{1}{2\times 3\times 4}+\text{etc.}\right)\times\left(\frac{1}{2}-\frac{1}{2\times 3}+\frac{1}{2\times 3\times 4}-\text{etc.}\right)=1.$$

14. Find the cost of 87·6 metres, at 2.31 fr. per metre.

15. Find the weight of 234·5 litres at 63 grams per litre.

16. How many litres are there in a cubic yard?

17. How many kilograms of water will fill a cistern whose capacity is $6\frac{1}{2}$ cubic yards?

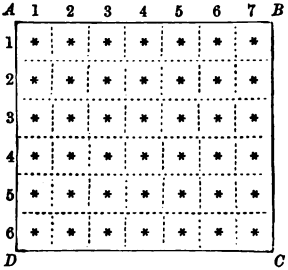
CHAPTER IX

THE MEASUREMENT OF SURFACE AND OF CUBIC-CONTENT

157. The surfaces which are considered in Arithmetic are bounded by four straight lines forming a figure in which every angle is a right angle.

The figure thus formed is called a **rectangle**.

The opposite sides are equal; hence, when we know the lengths of two of the sides which meet at one of the corners, we also know the lengths of the other two sides.



The figure $ABCD$ represents a rectangle; the angles at A, at B, at C and at D are all right angles; the side $AB=DC$; the side $BC=AD$.

The length of AB is called the **length** of the rectangle; the length of AD is called the **breadth** of the rectangle.

The number of square yards, square feet, etc., in a surface is called its area.

158. In Art. 98 the unit of surface is defined to be a square surface measuring a linear foot each way called a **square foot.**

We define a **square inch** to be a square surface measuring a linear inch each way.

We define a **square yard** to be a square surface measuring a linear yard each way.

Example. To prove that 144 square inches *make* 1 square foot.

Take a square surface a foot each way and rule on it parallel straight lines an inch apart (as in the figure on p. 162). The square foot is thus divided into twelve rows, each row containing twelve square inches. Therefore a square foot contains 12×12 square inches.

159. The length and breadth of a rectangle are called its **dimensions.**

Examples. Each of the following is a *rectangle*; the *floor*, each *wall*, the *ceiling*, of an ordinary room; each *side* of a rectangular box; a lawn-tennis *court*; the *sheet* of a newspaper.

Example i. Let the figure *ABCD* on the last page represent the surface of a table 7 ft. long, 6 ft. wide; let parallel lines be drawn on the table a foot apart, as in the figure. Then the surface of each square thus made is called a square foot. [Art. 98.]

There are six rows each containing seven square feet; hence, as in Art. 27, there are $\underline{6 \times 7 \text{ square feet}}$ in the surface of the table.

Example ii. Suppose the length of a table to be 7 ft. 9 in. and the breadth to be 4 ft. 4 in.

Here, 7 ft. 9 in. = 93 in.; 4 ft. 4 in. = 52 in.

Therefore, if we rule on the surface of the table, lines an inch apart, each little square will be a square surface an inch each way.

And there will be 52 rows, each row containing 93 square inches.

So that there will be in all, $\underline{52 \times 93 \text{ square inches}}$ in the surface of the table.

160. The *number* of square feet in any *rectangular* surface is the product of the *number* of the linear feet in the length × the *number* of the linear feet in the breadth of the rectangle.

NOTE.—Instead of *feet*, we may put *yards* throughout the above statement, or we may put *inches.*

This is what is meant by the abbreviated statement; *length* × *breadth* = *area*;

whence, *breadth* = *area* ÷ *length*;

also, *length* = *area* ÷ *breadth*.

It is convenient to read the symbol =, in statements like the above, as *numerically equals.*

Example. The length *of a rectangle is* $9\frac{3}{4}$ ft., *and the* breadth $5\frac{1}{3}$ ft.; *find the* number of square feet *in its surface.*

Answer. The required number of sq. ft. is

$$5\tfrac{1}{3} \times 9\tfrac{3}{4}\text{; which} = \tfrac{16}{3} \times \tfrac{39}{4} = 4 \times 13 = \mathbf{52}.$$

That is, the area contains 52 square ft.

[For, $9\frac{3}{4}$ ft. = 9 ft. 9 in. = 117 in.
$5\frac{1}{3}$ ft. = 5 ft. 4 in. = 64 in.

∴ by Example (ii) Art. 159, the area is 117 × 64 sq. in.;

that is, $\dfrac{117 \times 64}{144}$ sq. ft. $= \left(\dfrac{117}{12} \times \dfrac{64}{12}\right)$ sq. ft. $= \left(\dfrac{16}{3} \times \dfrac{39}{4}\right)$ sq. ft.

$$= (9\tfrac{3}{4} \times 5\tfrac{1}{3}) \text{ sq. ft.} = 52 \text{ sq. ft.}]$$

EXAMPLES. CII.

Find the area of the six rectangles whose dimensions are given, drawing a figure to verify the result in each case:—

1. 3 ft. long, 3 ft. broad. **2.** 10 in. long, 9 in. broad.

3. 1 ft. 4 in. long, 1 ft. 1 in. broad.

4. 4 yds. 2 ft. long, 3 yds. 1 ft. broad.

Find the number of sq. yds. sq. ft. and sq. in. in each of the six following rectangles:—

5. A room 26 ft. long, 15 ft. broad.

6. A courtyard 55 yds. 2 ft. long, 36 yds. 1 ft. broad.

7. A hall 97 ft. 6 in. long, 40 ft. broad.

8. The lid of a chest 5 ft. 3 in. long, 3 ft. 4 in. wide.

9. A picture 2 ft. 5 in. long, 1 ft. 11 in. broad.

10. The sheet of a newspaper 2 ft. 9 in. long, 2 ft. 4 in. broad.

How many square yards of matting will be required for each of the rectangular floors having the following dimensions?—

11. 16 ft. by 14 ft. 3 in. **12.** 21 ft. by 15 ft. 9 in.

13. $29\frac{1}{4}$ ft. by $17\frac{1}{3}$ ft. **14.** 5 yds. 2 ft. 6 in. by 4 yds. 2 ft.

15. 32 ft. by 5 yds. 1 ft. 3 in. **16.** $7\frac{1}{2}$ yds. by 3 yds. 1 ft.

Find the breadth of each of the following rectangles, given their area and length:—

17. Area 63 sq. ft., length 9 ft.

18. Area 28 sq. yds., length 21 ft.

19. Area 5 sq. yds. 7 sq. ft., length 9 ft. 9 in.

20. Area 16 sq. yds. 4 sq. ft. 72 sq. in., length 6 yds. 2 ft. 3 in.

21. How many acres of land are there in a street 1320 yds. long, 11 yds. wide?

22. How many square yards of York stone would be required for a pavement 7 ft. wide down one side of the above street?

23. How many square yards of carpet would be required for a square room, each side of which is 17 ft. 6 in.?

24. A garden roller is 2 ft. 6 in. wide, and its circumference is 6 ft. 6 in.; how many sq. ft. of turf does it pass over in one complete revolution?

25. The perimeter of (*i.e.* the length of a line drawn all round) a field is a third of a mile; how many square feet of brickwork will there be in a wall 7 feet 6 inches high which surrounds it?

26. What is the difference between '3 *ft. square*' and '3 *square feet*'?

27. A cricket field 100 yds. long, 57 yds. wide, has a path 5 ft. wide going all round it; what is the area (i) of the field, (ii) of the turf, (iii) of the path?

28. A room 21 ft. long, 16 ft. 6 in. broad, has a stained border all round it 3 ft. 6 in. deep; what is the area (i) of the floor, (ii) of the unstained part, (iii) of the stained part?

29. A field 100 yds. by 62½ yds. contains 3 cinder tennis courts, each 78 ft. by 36 ft., the rest of the field being turf; find the cost of the turf at ½c. per sq. ft.

30. Find the cost of polishing a marble slab 7 ft. 6 in. long, 4 in. wide, at 20c. per sq. ft.

31. Find the cost of carpeting a room 17 ft. 6 in. long, 12 ft. wide, at $1.25 per sq. yd.

32. Find the cost of glazing 6 windows, each 7 ft. 6 in. high, 4 ft. wide, at 18c. per sq. ft.

33. A gravel path 4 ft. wide runs round a square field 75 yds. long; find the cost of making it at 28c. per sq. yd.

34. How many sq. yds. etc. of linoleum will remain out of 50 sq. yds. after covering a floor 21 ft. 9 in. by 16 ft. 4 in.?

35. The area of a rectangular floor is 64 sq. yds., its length is 27 feet; what is its breadth?

36. The area of a rectangular courtyard is 144 sq. yds., and its length is 36 ft.; what is its breadth?

37. What length of a road 24 ft. wide will have an area of half an acre?

161. The carpet which will cover the floor of a room has the same area as the floor.

Example. Find the length *of carpet* ¾ yd. wide *required for a room* 21 ft. long 16 ft. broad.

[The area of the carpet before it is cut up into strips is equal to the area after it has been so cut up; that is, it is equal to the area of the floor. Hence

the required number of feet × $\frac{9}{4}$ (which is the width of carpet in ft.)
= no. of square feet in the floor
= 21 × 16; so that]

the req. number of feet is $21 \times 16 \div \frac{9}{4}$ or, $\frac{21 \times 16 \times 4}{9}$ or, $\frac{7 \times 16 \times 4}{3}$,
that is, 149⅓.

Hence, the required length of carpet is 149⅓ ft.

162. Thus, to find the *number* of feet of carpeting required for a room, we multiply the *number* of feet in length of the room by the *number* of feet in the breadth of the room, and divide the product by the *number* of feet in the width of the carpet.

[Instead of *feet* we may put *yards* throughout the above statement; or we may put *inches*.]

This is what is meant by the abbreviated statement:—*To find the length of carpet required for a room we multiply the length of the room by its breadth, and divide by the width of the carpet.*

Or, Length of carpet *numerically equals*
length of room × breadth ÷ width of carpet.

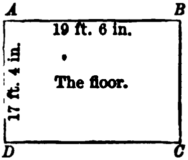
EXAMPLES. CIII.

Find the number of yards of carpet required for rooms having the following dimensions:—

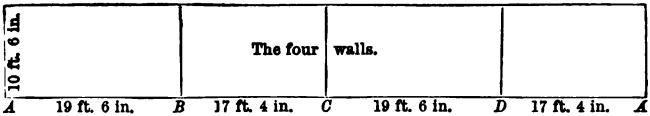
1. Room, 18 ft. long, 14 ft. broad; carpet, 1 yd. wide.
2. Room, 15 ft. long, 12 ft. broad; carpet, 2 ft. 3 in. wide.
3. Room, 17 ft. 6 in. long, 16 ft. broad; carpet, 2 ft. 6 in. wide.
4. Room, 21 ft. long, 14 ft. 3 in. broad; carpet, 2 ft. 3 in. wide.
5. Room, 15 ft. 4 in. long, 14 ft. 3 in. broad; carpet, 3 ft. 6 in. wide.
6. Room, 9 yds. long, 5 yds. 2 ft. 4 in. broad; carpet, 3 ft. wide.

Example. A room is 19 ft. 6 in. long, 17 ft. 4 in. wide, *and* 10 ft. 6 in. high; *find the* number of sq. ft. *in the surface of its* walls.

[Note.—In questions like this it is **understood** that the *floor* of the room is a *rectangle*.



Let *ABCD* be the rectangular floor of the room. Suppose the walls to be four screens each 10 ft. 6 in. high; and suppose these four screens placed in a row as in the figure below.



The four walls would then form a rectangle whose length is 19 ft. 6 in. + 17 ft. 4 in. + 19 ft. 6 in. + 17 ft. 4 in. and whose width is 10 ft. 6 in.

Hence, *the area of the walls of a room numerically equals twice the sum of the length and breadth multiplied by the height.* See Arts. 160, 162.

Thus, the process is as follows:]

The area of the walls in square ft. is $(19\frac{1}{2}+17\frac{1}{3})\times 2\times 10\frac{1}{2}$,

that is $36\frac{5}{6}\times 21$,

that is $110\frac{1}{2}\times 7$ or, $\underline{773\frac{1}{2}}$ (sq. ft.)

EXAMPLES. CIV.

Find the number of sq. yds. sq. ft. etc. in the walls of the following rectangular rooms:—

1. Length 16 ft. breadth 14 ft. height 10 ft.
2. Length 5 yds. 2 ft., breadth 5 yds. 1 ft., height 3 yds. 2 ft.
3. Length 14 ft. 3 in., breadth 12 ft., height 9 ft. 6 in.
4. Length 20 ft. 8 in., breadth 13 ft. 4 in., height 10 ft. 3 in.
5. Length 20 yds. 2 ft., breadth 18 yds. 2 ft., height 15 ft. 9 in.
6. Length 17 ft. 5 in., breadth 15 ft. 5 in., height 12 ft.

NOTE.—In papering the walls of a room, it is usual to use rolls of paper, each roll consisting of a strip of paper 12 yards long, and 21 inches wide. The problem of finding how many linear yards of these strips of paper are required for the walls of a room, is like the problem of finding how many yards of a roll of carpet are required for the floor of a room : *the length of paper* numerically equals *the area of the walls divided by the width of the paper.*

[It may however be remarked that a certain amount of paper and of carpet is usually wasted through the necessity of matching the patterns as the strips are placed side by side, while a certain amount of paper is saved by the space occupied by windows, doors, etc., for which paper is not required.]

Example. How many yards of paper 21 in. wide *are required for a room* 18 ft. long, 12 ft. wide, 10 ft. high ?

Here, the number of sq. ft. in the walls is $2(18+12)\times 10$ or, 600; the width of the paper is $\frac{7}{4}$ ft. ; hence,

the no. of (linear) yards required is $\frac{600\div\frac{7}{4}}{3}$ or, $200\times\frac{4}{7}$, that is, $\underline{114\frac{2}{7}}$.

EXAMPLES. CV.

Find the length of wall paper required for the following rooms :—

1. 24 ft. long, 21 ft. wide, 12 ft. high ; paper 21 in. wide.
2. 20 ft. long, 15 ft. wide, 14 ft. high ; paper 21 in. wide.
3. 40 ft. long, 20 ft. 6 in. wide, 12 ft. high ; paper 22 in. wide.
4. $38\frac{1}{2}$ ft. long, $16\frac{1}{2}$ ft. wide, $13\frac{1}{2}$ ft. high ; paper 22 in. wide.
5. 33 ft. 4 in. long, 12 ft. 6 in. wide, 15 ft. high ; paper 25 in. wide.
6. 17 ft. 3 in. long, 13 ft. 4 in. wide ; 13 ft. 6 in. high ; paper 27 in. wide.

Example. How many paving stones, *each stone* 2 ft. long, 17 in. wide, *will cover a pavement* 340 yds. long, 6 ft. wide ?

[The number of square feet in all the paving stones together must be equal to the number of square feet in the pavement.

Therefore the required number of stones is the number of times the area of one stone is *contained in* the area of the pavement.]

Each stone contains $2\times\frac{17}{12}$ sq. ft. ;
the pavement contains $340\times 3\times 6$ sq. ft.

Hence, the req. number of stones is $\frac{340\times 3\times 6}{\frac{17}{6}}=20\times 3\times 36=\underline{2160}.$

EXAMPLES. CVI.

1. How many floor-boards, each 12 ft. 6 in. long, 5 in. wide, will cover the floor of a room 25 ft. long, 15 ft. wide?

2. How many stone slabs, each 3 ft. long, 2 ft. 6 in. wide, will pave a court 50 yds. long, 31 yds. 2 ft. wide?

3. How many pieces of India matting, each 6 ft. long, 3 ft. 4 in. wide, will cover the floor of a room 13 ft. 4 in. long, 12 ft. wide?

4. How many pieces of paper, each 12 yds. long, 21 in. wide, will cover the walls of a room 28 ft. long, 13 ft. 6 in. wide, 14 ft. high?

5. How many pieces of French wall paper, each 9 yds. long, 25 in. wide, will cover the walls of a room 18 ft. 9 in. long, 18 ft. wide, 12 ft. high?

6. What will be the expense of papering a room 18 ft. 9 in. long, 14 ft. 3 in. wide, and 14 ft. 3 in. high, with paper 19 in. wide, at 84c. per piece of 12 (linear) yards?

7. Find the cost of painting the outside of a rectangular box 4 ft. long, 3 ft. wide, 2 ft. high, at 2c. per sq. ft.

8. Find the cost of painting the walls and ceiling of a room 16 ft. long, 15 ft. wide, 13 ft. 4 in. high, at $1.08 per sq. yard.

9. Find the cost of papering the walls of a room 18 ft. long, 14 ft. wide, 12 ft. high, with paper 18 in. wide, at 60c. per piece of 12 (linear) yds.

10. Find the cost of carpeting a room 15 ft. 6 in. long, 12 ft. 9 in. broad, with carpet 2 ft. wide, at $1.87½ per yard.

11. Find the cost of covering the floor of a room 40 ft. 8 in. long, 24 ft. 6 in. wide, with carpet 1 ft. 6 in. wide, at $1.12½ per yd.

12. Find the cost of painting the walls and ceiling of a room 16 ft. 8 in. long, 15 ft. 9 in. broad, 14 ft. high, at 27c. per sq. yd.

13. The floor of a room which is 18 ft. long contains 27 sq. yds.; what is its breadth?

14. It costs $60 to carpet a room 12 ft. wide at $1 per sq. yard; what is the length of the room?

15. It costs $60 to carpet a room 15 ft. wide with carpet 27 in. wide, at $1.12½ per yard; what is the length of the room?

16. A lawn 100 ft. long is turfed with sods 3 ft. long, 1 ft. wide, at $1.50 per hundred sods, at a cost of $30; what is the width of the lawn?

17. A room 18 ft. long, 14 ft. wide, has a stained border 2 ft. wide; the rest of the floor is to be covered with Turkey carpet at $4.44 per sq. yard; the staining costs 20c. per sq. yard; what will be the total expense?

18. A garden 120 ft. long, 60 ft. broad is to have a path 4 ft. wide running round it, and the remainder is to be turfed; the path is to cost 50c. for 3 sq. yds., and the turfing $2.50 per hundred sods, 3 ft. by 1 ft., including labour; find the total cost.

19. Find the cost of covering the inside of an open cistern 5 ft. long, 3 ft. wide, 4 ft. deep, with lead weighing 7 lbs. to the sq. foot, at $2.50 per cwt.

20. How many bricks, each 9 in. long, 4½ wide, will pave the floor of a cellar 13 ft. 6 in. long, 6 ft. 9 in. wide?

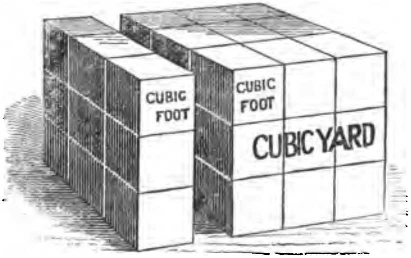
CUBIC MEASURE.

163. A **rectangular solid** is bounded by six rectangular faces, each opposite pair of faces being equal and parallel to each other.

164. In Art. 98 the standard unit of solid measurement is defined to be a rectangular solid measuring a linear foot each way, called a **cubic foot.**

DEF. A **cubic yard** is a rectangular solid measuring a linear yard each way. A **cubic inch** is a rectangular solid measuring a linear inch each way.

Example i. *To prove that* a cubic yard *contains* 27 cubic feet.



It will be seen from the figure that a cubic yard can be divided into **3** parallel slices, each slice a foot thick. The largest surface of each slice is a *square yard;* this can be divided into **9** square feet; under each square foot in each slice is a cubic foot; so that in a cubic yard there are **3** × **3** × **3** cubic feet.

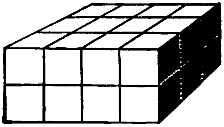
Example ii. *Prove that* a cubic foot *contains* 1728 cubic inches.

By the aid of the figure on p. 162 the student will be able to prove, as above, that a cubic foot contains 12 × 144 cubic inches.

165. A rectangular solid is said to have **three dimensions,** viz. **length, breadth,** and **thickness.**

The length and breadth are the length and breadth of the largest rectangle; the thickness (or depth) is the distance between these two largest rectangles.

Example. The dimensions *of a rectangular solid are* length 4 ft., breadth 3 ft., thickness 2 ft.; *find the number of* cubic feet *in it.*



The solid can be cut into *two* slices each 1 ft. thick, by a plane parallel to the largest rectangle, because it is 2 ft. thick. [If it were 3 ft. thick it could be cut into *three* slices each 1 ft. thick, and so on.]

The surface of each slice contains 3 × 4 square feet (because it is 4 ft. long, 3 ft. broad); underneath each square foot is a cubic foot. So that the solid contains $\underline{2 \times 3 \times 4 \text{ cubic feet.}}$ Hence

166. The *number* of cubic feet in a rectangular solid is the *continued product* of the *number* of feet in the length by the *number* of feet in the breadth, by the *number* of feet in the thickness.

[Instead of *feet* we may put *yards* throughout the above statement; or we may put *inches.*]

This is what is meant by the abbreviated statement *cubic content = length × breadth × thickness.* [Art. 160.]

EXAMPLES. CVII.

Find the cubic contents of the rectangular solids (proving the result in each case by a figure) having the following dimensions:—

1. Length 5 ft., breadth 4 ft., thickness 3 ft.
2. Length 5 yds., breadth 3 yds., depth 3 yds.
3. Length 2 yds., breadth 1 yd. 1 ft., depth 1 yd.
4. Length 2 yds. 1 ft., breadth 2 yds., depth 4 ft.
5. Length 1 ft., breadth 10 in., depth 4 in.
6. Length 3 yds. 1 ft., breadth 1 yd. 2 ft., depth 1 yd.

167. A cubic foot of water weighs about 1000 oz. (Avoir.) [See p. 82.]

Example i. A cistern, *whose interior* dimensions *are* 2 ft. 3 in., 2 ft., 1 ft. 6 in., *weighs* 5 cwt. 4 lbs. *when empty; what does it weigh when full of water?*

Its dimensions are $2\frac{1}{4}$ ft., 2 ft., $1\frac{1}{2}$ ft.

Therefore it contains $\frac{9}{4}\times\frac{2}{1}\times\frac{3}{2}$, that is $\frac{27}{4}$ cubic ft.

Therefore the water weighs $\frac{27\times 1000}{4}$ oz. $=6750$ oz.

$=4$ cwt. 21 lbs. 14 oz.

Therefore the total weight is 9 cwt. 25 lbs. 14 oz.

Example ii. A wall 15 yds. long, 7 feet high, *and* 9 inches thick, *is to be built of* bricks, *each of which with its share of the mortar measures* 9 in. by 5 in. by 3 in.; *how many* bricks *are required?*

[N.B.—The principle to be employed in solving such questions is this: The *number* of the bricks is the *number* of times which the cubic content of one brick is contained in the cubic content of the whole wall.]

The number of cubic feet in the whole wall is $(15\times 3)\times 7\times\frac{3}{4}$.

The number of cubic feet in a brick is $\frac{3}{4}\times\frac{5}{12}\times\frac{1}{4}$.

Therefore the number of bricks required is

$\frac{15\times 3\times 7\times\frac{3}{4}}{\frac{3}{4}\times\frac{5}{12}\times\frac{1}{4}}$, that is, $3\times 7\times 4\times 12\times 3$, or 3204 (bricks).

EXAMPLES. CVIII.

1. Find the weight of water in a cistern 11 ft. by 7 ft. by 8 ft.

2. How many bricks each 9 in. by $4\frac{1}{2}$ in. by $3\frac{1}{2}$ in. are required for a wall 27 yds. by 7 ft. by 15 in.?

3. How often can a bucket holding $1\frac{1}{2}$ cub. ft. of water be filled from a square tank 6 ft. long, 4 ft. 6 in. deep?

4. How many times can a barrel of 36 gallons be filled from a tank 7 yds. 2 ft. 1 in. by 1 yd. 2 ft. by 1 yd. 10 in. deep? (1 gal. $=231$ cub. in. say).

5. In how many minutes can a cistern 12 ft. by 10 ft. by 8 ft. be emptied by a pipe which discharges 15 cubic ft. of water per minute?

6. In what time can a cistern 3 yds. 2 ft. $6\frac{1}{2}$ in. by 1 yd. by 18 in. be emptied by a pipe which discharges a gallon (231 cub. in.) per second?

168. Using the abbreviated statement of Art. 166, we have, *cubic content* = *length* × *breadth* × *depth*.

Therefore, *cubic content* ÷ *length* = *breadth* × *depth*
= *surface* of one rectangle;

also, *cubic content* ÷ (*length* × *breadth*) = *depth*.

The student should write out in full the exact meaning of the above statements. [See Arts. 160, 162, 166.]

Example. A cistern is 8 ft. long, 6 ft. broad; *what is the depth of the water in it when it contains* 1000 gallons?

A gallon contains 231 cubic inches [Def. p. 82], therefore 1000 gallons contain 231000 cubic inches.

The length of the cistern is 96 in.
The breadth is 72 in.

Therefore the req. depth of the water is $\frac{231000}{96 \times 72}$ in. or 33·42... in.

EXAMPLES. CIX.

1. There are 2400 cub. ft. of air in a room 12 ft. long, 10 ft. high; how wide is the room?

2. A rectangular tank 1 ft. 3 in. deep contains 2100 cub. ft. of water; what is the area of its surface?

3. There are 1152 cub. ft. of air in a room 9 ft. high; find the cost of carpeting the room at 66c. per sq. yd.

4. A cistern 32 ft. long, 25 ft. wide, 11 ft. deep, is emptied in 20 minutes by a pipe whose sectional area is 44 sq. in.; how fast does the water flow in the pipe?

5. The water from a cistern 16 ft. long, 5 ft. wide, 3 ft. deep, just fills a pipe whose section is 15 sq. in.; find the length of the pipe.

6. A rectangular log of mahogany is 2 ft. broad, 1 ft. 8 in. thick, and contains 4 cub. yds.; what is its length?

7. Find the value at 54c. per cub. ft. of a block of stone 2 yds. 1 ft. 3 in. long, 3 ft. 4 in. broad, 1 ft. 6 in. thick.

8. A rod of brickwork contains 306 cub. ft.; find the cost at $60 per rod of a brick wall 34 yds. by 7 ft. 6 in. by 14 in. thick.

9. The water in a cistern 5 ft. long, $2\frac{1}{2}$ ft. wide, weighs 1 ton 5 cwt. 39 lbs. 1 oz.; find the depth of the water.

10. A box is 4 ft. 6 in. long, 3 ft. 3 in. broad, 2 ft. 9 in. deep; find the cost of filling it with packets of note-paper each 9 in. by 6 in. by $\frac{1}{2}$ in. at 32c. per packet.

11. Find the value of a stack, 30 ft. by 16 ft. 6 in. by 9 ft. 3 in. of bricks, each 9 in. by 4½ in. by 3 in., at $25 per 1000 bricks.

12. A solid stack, 41 ft. 8 in. by 16 ft. 8 in. by 14 ft. 7 in., contains 125000 bricks, each 10 in. long, 3½ in. thick; find the width of each brick.

13. A pond of 3 acres is frozen to a depth of 6 in.; find the weight of the ice at 896 oz. (Avoir.) per cubic ft.

14. A river 14 ft. deep, 182 yds. wide, flows at the rate of 3 miles an hour; (i) how many tons, (ii) how many gallons of water pass a certain point per minute?

15. Find the weight in tons per acre of a rainfall of ½ in.

16. Find the weight in tons per sq. mile of a rainfall of 3 in.

17. A room 14 ft. 7 in. long, 12 ft. 5 in. wide, contains 2037 cub. ft. 189 cub. in.; find the cost of papering it with paper 2 ft. wide at 1c. per yard.

18. A cistern containing 720 gals. measures externally 7 ft. by 2 ft. 11 in. by 5 ft. 6 in.; the sides are 1½ in. thick; find the thickness of the bottom, supposing 7½ gals. = 1 cub. ft.

* DUODECIMALS.

169. In the 'Duodecimal' method (used by glaziers and others) the following definitions are necessary.

A *linear* **prime** (1′) is the twelfth part of a foot.
A *superficial* **prime** (1′) is the twelfth part of a square foot.
A *cubic* **prime** (1′) is the twelfth part of a cubic foot.

A **second** (1″) is the twelfth part of a prime.
A **third** (1‴) is the twelfth part of a second.
A **fourth** (1⁗) is the twelfth part of a third,
and so on.

Taking Fig. I. on the next page to represent a cubic foot, Fig. II. represents a cubic prime, Fig. III. a cubic second, Fig. IV. a cubic third.

Taking the shaded face of Fig. I. to represent a sq. ft., the shaded face of Fig. II. is a superficial prime, the shaded face of Fig. III. is a superficial second.

Thus a cubic third = a cubic inch; a superficial second = a square inch; and a linear prime = a linear inch.

Example i. *Express* 22 ft. 10¼ in. *as a* duodecimal.

22 ft. 10¼ in. = 22 ft. 10′.3″.

Example ii. *Express* 327 sq. ft. 10′.11″.6‴ *in* sq. yds. etc.

327 sq. ft. 10′.11″.6‴ = $(327 + \frac{10}{12} + \frac{11}{144} + \frac{6}{12}$ of $\frac{1}{144})$ sq. ft.

= 36 sq. yds. 3 sq. ft. 131½ sq. in.

EXAMPLES. CX.

Express in feet and inches—

1. 4 ft. 3′. **2.** 21 ft. 11′.3″. **3.** 3 sq. ft. 3′.5″.6‴.

4. 8 sq. ft. 11′.0″.4‴. **5.** 37 sq. ft. 10′.9″.7‴.

6. 3 c. ft. 3′.4″.6‴. **7.** 5 c. ft. 10′.8″.8‴. **8.** 0 c. ft. 4′.3″.7‴.8⁗.

Express as duodecimals— **9.** 41 ft. 9 in. **10.** 3 ft. 11½ in.

11. 3 sq. ft. 36¼ sq. in. **12.** 4 c. ft. 108¼ c. in.

Add together (i) as duodecimals, (2) as feet and inches—

13. 12 ft. 7′.8″, 15 ft. 3′.5″, 17 ft. 9′.7″.

14. 4 sq. ft. 10′.9″, 5 sq. ft. 10′.11″.6‴, 28 sq. ft. 9″.9‴.

15. 1 c. yd. 20 c. ft. 3′.10″.11‴, 4 c. yds. 26 c. ft. 3′.4″.8‴, 0 c. ft. 10′.11″.11‴.6⁗.

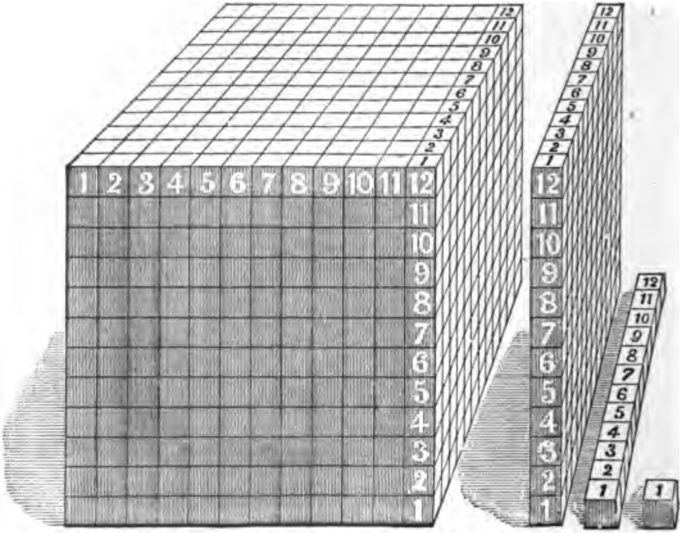


Fig. I. II. III. IV.

170. The *number* of primes in the breadth of a rectangle multiplied by the *number* of feet in the length is the *number* of (superficial) primes in the surface of the rectangle.

For $7' \times 3$ ft. $= \frac{7}{12}$ ft. $\times 3$ ft. $= \frac{21}{12}$ sq. ft. $= 21'$.

Hence we say, (linear) *primes* × (linear) *feet* give *primes* (superficial)
Similarly, (linear) *primes* × (linear) *primes* give *seconds* (superficial).
(linear) *primes* × (linear) *seconds* give *thirds* (superficial).
primes × *primes* × *feet* give *seconds* (cubic).

Example i. *Find the area of a rectangle* 8 ft. 6′. 6″ long, 11 ft. 4′. 9″ broad.

$$\begin{array}{lrrrrr}
\text{Process,} & 8\,. & 6'\,. & 6'' & & \\
 & 11\,. & 4\,. & 9 & & \\ \cline{2-4}
 & 93\,. & 11\,. & 6 & & \\
 & 2\,. & 10\,. & 2\,. & 0''' & \\
 & & 6\,. & 4\,. & 10\,. & 6'''' \\ \cline{2-6}
\textit{Answer} & \underline{97\,.} & \underline{4'\,.} & \underline{0''\,.} & \underline{10'''\,.} & \underline{6''''} \\
\text{or,} & \multicolumn{5}{l}{\underline{\text{97 sq. ft. } 48\tfrac{21}{24}\text{ sq. in.}}}
\end{array}$$

We first multiply by 11 ft., remembering that 12″ = 1′ and 12′ = 1 sq. ft. We then multiply by 4′, remembering that 1′ × 1″ = 1‴ (superficial), and then we multiply by 9″, remembering that 1″ × 1″ = 1⁗ (superficial).

Example ii. *The* breadth *of a rectangle which contains* 4 sq. ft. 7′. 3″. 9‴ *is* 2 ft. 1′; *find its* length.

$$\begin{array}{lrrrrrl}
 & \text{ft.} & \text{sq. ft.} & & & & \text{ft.} \\
\text{Process,} & 2\,.\,1') & 4\,. & 7'\,. & 3''\,. & 9''' & (\underline{2\,.\,2'\,.\,6''}\text{ etc.} \\
 & & 4\,. & 2 & & & \\ \cline{3-4}
 & & & 5\,. & 3 & & \\
 & & & 4\,. & 2 & & \\ \cline{4-6}
 & & & 1\,. & 1\,. & 9 & \\
 & & & 1\,. & 0\,. & 6 & \\ \cline{4-6}
 & & & & 1\,. & 3 &
\end{array}$$

The length required is 2 ft. 2½ in. nearly.

EXAMPLES. CXI.

Find by duodecimals the areas of the following rectangles—

1. 6 ft. 3 in. by 11 ft. 8 in.
2. 12 ft. 8′. 2″ by 21 ft. 11′. 1″.
3. 8 ft. 11′. 10″ by 2 yds. 1 ft.
4. 8 ft. 8′. 1″ by 36 ft. 4′. 10″.

Find the volumes of the following rectangular solids—

5. 6 ft. 4′ long, 5 ft. 11′. 1″ broad, 4 ft. 6′. 6″ deep.
6. 15 ft. 1′. 3″ long, 8 ft. 7′. 4″ broad, 6 ft. 3′ deep.
7. 8 ft. 7′. 2″ long, 8 ft. 7′ broad, 6 ft. 1′. 3″ deep.
8. A floor 13 ft. 10′. 2″ long contains 150 sq. ft., find its breadth.

NOTE.—The student may also work many of the other examples in this chapter by duodecimals.

CHAPTER X

THE UNITARY METHOD *of solving* PROBLEMS, *formerly called* RULE OF THREE

171. *DEF.* A quantity is said to **vary** directly as another quantity when they are so related, that when one quantity is *doubled*, or *trebled*, or *multiplied by* any number whatever, the other quantity is *doubled*, or *trebled*, or *multiplied by* the same number.

It follows also that when one of the quantities is *divided* by any number the other must be *divided* by the *same* number.

Example. The quantity of goods purchased varies directly as the amount of money spent, provided that equal quantities of the goods cost equal sums of money; for then, if the goods be doubled, the money must be doubled also; and so on.

The quantity of goods purchased does *not* vary directly as the amount of money spent, when equal quantities do *not* cost equal sums. As for example, when some allowance in price is made to purchasers of large quantities.

172. In questions stated in the following form **it is always understood that the quantities named vary as one another.**

Example i. *If* 1 lb. *of tea costs* 75c., *what will* 5 lbs. *cost?*

The answer required consists of a number of cents,
also, the more lbs., the *more* cents [*]; hence,
since the number of cents which 1 lb. costs is 75,
therefore the number of cents which 5 lbs. cost is 5 times 75,
that is 375 (cents), *i.e.* $3.75.

---

[*] We shall use this expression '*the more lbs. the more cents,*' as an abbreviation for '*when the number of lbs. is multiplied by any number,*

Example ii. *If* 12 horses *can plough* 75 acres *in a certain time, how many* acres *can* 28 horses *plough in the same time?*

The answer required consists of a number of **acres**,
also the more horses, the *more* acres ; hence,
since the number of acres which 12 horses plough is 75,
therefore the number of acres which 1 horse can plough is $\frac{1}{12}$ of 75,
and the number of acres which 28 horses can plough is 28 times $\frac{1}{12}$ of 75,
that is $\underline{175}$ (acres).

EXAMPLES. CXII.

1. If 1 lb. of sugar costs 6c., how much will 11 lbs. of the same sugar cost?

2. If 8 lbs. of coffee cost $3.20, how much will 1 lb. of the same coffee cost?

3. If 4 lbs. of sugar cost 24c., how much will 14 lbs. cost?

4. If the railway fare for 96 miles is $2.88, what is the railway fare for 1 mile, and what for 20 miles?

5. If it costs $10 to send 8 tons a certain distance, how much will it cost to send 10 tons the same distance?

6. If it takes 6 hours to pump 5400 gallons of water with a certain pump, how many gallons can be pumped in 1 hour? how many can be pumped in 7 hours?

7. If 20 men can reap 15 acres in one day, how many acres can 32 men reap in one day?

8. If 8 horses can plough 56 acres in a certain time, how many acres can 11 horses plough in the same time?

9. A man walks 65 miles in 13 hours; in how many hours will he walk 45 miles at the same pace?

10. If 840 sovereigns weigh 21 lbs., how many lbs. will 480 sovereigns weigh?

173. *DEF.* A quantity is said to **vary inversely** as another quantity, when they are so related, that when one quantity is *doubled*, or *trebled*, or *multiplied by* any number whatever, the other quantity is *halved*, or *divided by three*, or *divided by* the same number.

---

the number of cents must be multiplied by the same number.' The student must notice that the converse is also true, *i.e.* that '*the less pounds, the less cents.*'

Example. The **time** (*i.e.* **interval** of time) required for a piece of work varies inversely as the number of men employed, provided every man always does the same amount of work in the same time. For when the number of men is doubled, the time occupied is consequently halved; and so on.

Example i. *If* 9 men *do a piece of work in* 7 days, *in how many* days *will* 1 man *do the same ?*

The answer consists of a number of **days**,
also, the less men, the *more* days ;* hence,
since the number of days in which 9 men can do the work is 7,
therefore the no. of days in which 1 man can do the work is 9 times 7,
that is, 63 (days).

Example ii. *If* 9 men *can do a piece of work in* 20 days, *how many* men *could do it in* 15 days?

The answer required is a number of **men**,
also, the more days, the *less* men ; hence,
since the number of men who do the work in 20 days is 9,
therefore the number of men who could do it in 1 day is 20 times 9,
and the number of men who could do it in 15 days is $\frac{1}{15}$ of 20 times 9,

that is, $\frac{5 \times 4 \times 3 \times 3}{5 \times 3}$ or 12 (men).

EXAMPLES. CXIII.

1. If 7 men can mow a field in 3 days, in how many days could 1 man mow the same field?

2. A ship performs a voyage in 75 days, sailing 5 knots an hour; how many days would she take to perform the same voyage sailing 1 knot an hour?

3. If 5 horses can plough a field in 7 days, in how many days could 7 horses plough the same field?

4. A man walks from a certain distance in 14 hours, walking 4 miles an hour; how long would he take if he rode at the rate of 7 miles an hour?

5. A trains performs a journey in 7 hours, travelling at the rate of 35 miles an hour; how long would it take to do the same distance, travelling at the rate of 49 miles an hour?

6. A garrison of 2500 men has provisions for 11 months; how long would they last if the garrison were reduced to 100 men?

7. If 12 men can mow a field in 22 days, how long would 33 men take to mow the same field?

---

* We shall use the expression '*the less men, the more days*,' as an abbreviation for '*when the number of men is divided by any number, the number of days must be multiplied by the same number.*'

8. In what time will 39 men do a piece of work, which it takes 30 men 26 days to accomplish?

9. If 20 bushels of corn feed 24 horses for 15 days, for how many days will they feed 9 horses?

10. If 18 horses plough $111\frac{1}{2}$ acres in 15 days, how many horses can plough the same number of acres in 27 days?

174. In proceeding to solve the following problems we must in each case consider whether, from the *nature* of the two quantities, to an increase in the one corresponds an *increase* or a *diminution* in the other.

Example i. *If a loaf at* 12c. *weighs* 64 oz. *when wheat is* $13.50 *per quarter, what should it weigh when wheat is* $18 *per quarter?*

The answer required is a number of **ounces**,
also, the more dollars per qr., the *less* the number of oz.; hence,
since the number of oz. when wheat is $13.50 per qr. is 64,
therefore, the number of oz. when wheat is $1 per qr. is $13\frac{1}{2} \times 64$,
and the number of oz. when wheat is $18 per qr. is $\frac{13\frac{1}{2}}{18} \times 64$,
that is, $\underline{48}$ (oz.)

Example ii. *If* 75 bushels *feed* 27 horses *for a week, how many* horses *will* 125 bushels *feed for the same time?*

The answer required is a number of **horses**,
also, the more bushels, the *more* horses; hence,
since the number of horses fed by 75 bushels is 27,
therefore, the number of horses fed by 1 bushel is $\frac{1}{75}$ of 27,
and the number of horses fed by 125 bushels is $\frac{125}{75}$ of 27, or $\underline{45}$ (horses).

N.B.—It is convenient to arrange the work as above.

We consider, (i) which quantity is *like the answer*; (ii) whether an increase in the other quantity *increases* or *diminishes* that like the answer. Then we arrange the supposition so that the measure of the quantity like the answer comes last. For example, in Ex. ii. p. 168, the supposition

'*If* 9 men *can do a piece of work in* 20 days' is put thus:
'*The number of* men *who do the work in* 20 days *is* 9.'

EXAMPLES. CXIV.

1. If a workman earn $100 in 50 working days, how many days will he take to earn $210?

2. If 12 men reap a field in 20 hours, in how many hours will 16 men reap the same field?

3. If a carriage of 3 cwt. for 72 miles cost a certain sum, how much will be carried 27 miles for the same money?

4. If a 2 cent loaf weighs 10 oz. when wheat is $3 per bushel, what is the price per bushel when a 2 cent loaf weighs 6 oz.?

5. If the rent of 42 acres of land is $1260, how many acres of the same land should be let for $810?

6. If 13 cwt. cost $169, what will 50 cwt. cost?

7. How many men can do a piece of work in 16 hrs. which 4 men can do in 24 hrs.?

8. In how many days can 20 men do a piece of work which it takes 125 men to do in 64 days?

9. If from a sack of flour I can make 48 loaves weighing 3 lbs. each, how many 4 lb. loaves can I make from the same?

10. If 40 bushels feed 60 horses for a week, how many horses would they keep for 35 days?

11. If 40 bushels feed 60 horses for a week, how many bushels will be required to feed 27 horses for the same time?

12. If 40 bushels feed 60 horses for a week, for how many days would they feed 12 horses?

13. If 40 bushels feed 60 horses for a week, for how many days would 280 bushels feed the same number of horses?

14. If 40 bushels feed 60 horses for a week, how many will be required to feed the same number of horses for 35 days?

15. How many yards of cloth worth $1.25 per yard must be given for 35 yards at $1.75 per yard?

16. If for the loan of a sum of money for 5 months, I pay $221, what ought I to pay for a year?

17. If 63 yds. of cloth cost $225, how many yds. cost $150?

18. A certain amount of sugar costs $168 at 7c. per lb., how much will it cost at 8c. per lb.?

19. A man gives away 3 cents for every dollar of his income; what is his income, if he gives away $84?

20. A bankrupt pays $3 in every $5, and his assets are $2250; what are his debts?

175. We work out two more examples.

Example i. *How much will* 1 cwt. 25 lbs. *cost, at* $28.35 *for* 5 cwt. 67 lbs.?

Here 1 cwt. 25 lbs. = 125 lbs., and 5 cwt. 67 lbs. = 567 lbs.; and less weight, *less* cost.

Therefore since 567 lbs. cost $28.35,
therefore 1 lb. costs $\frac{1}{567}$ of $28.35,
and 125 lbs. cost $\frac{125}{567}$ of $28.35; or, $\underline{\$6.25}$.

Example ii. *What are a man's* debts *when his* assets *are worth* \$2262.50, *and he pays* \$2 *in* \$3? [See Note, p. 147.]

Here, more payment, *more* debt; and since

a payment of \$2 cancels a debt of \$3,
therefore a payment of \$1 cancels a debt of ½ of \$3,

therefore a payment of \$2262.50 cancels a debt of $\frac{2262.50}{2}$ times \$3,

that is, a debt of \$$\frac{2262.50 \times 3}{2}$ or, $\underline{\underline{\$3393.75.}}$

N.B.—*No answers are given to the next* 20 *questions.*

21. If 36 articles cost \$279, what will 37 articles cost?

22. If the 8c. loaf weigh 48 oz. when wheat is \$12.50 per quarter, what should it weigh with wheat at \$8 per quarter?

23. How many yards of cloth at \$2.20 per yard should be given for 121 yds. at \$2 per yard?

24. Find the price of 2 tons 16 cwt. of coal when 1 ton 4 cwt. cost \$4.50.

25. If 7 houses are built in 1 year 9 months, how many can be built in 1 year 3 months at the same rate?

26. If 21 men can build a wall in 35 hours, how many could build it in 5 days of 9 hours each?

27. What is the value of 464 bushels of wheat at the rate of \$7.25 for 5 bushels?

28. How many yards of silk can be bought for \$10.75 when 444 yds. cost \$397.75?

29. Find the value of 8½ tons at the rate of \$48.75 for 3 tons 5 cwt.

30. What will be the rent of 43¾ acres when 18¾ acres are let for \$288.75?

31. How many dinners can be provided for \$52.80 at the rate of 37 dinners for \$2.59?

32. I borrow \$8000 for 3 months; how long should I lend \$6000 to repay the obligation?

33. Find the tax on \$2160 at the rate of 16c. for every \$21.60.

34. If the tax on a rental of \$950 is \$25, what should it be on a rental of \$494?

35. What are a man's debts when his property is worth \$2571.25, and he pays at the rate of 68c. for each \$1?

36. Having given away at the rate of 16c. for every \$1 of his income, a man has \$2419.41 left; what is his income?

37. A clock gains at the rate of 7 min. in 360 hours; how much will it gain in 7 days?

38. How many days should 36 men take to finish a work of which 27 men have done half in 30 days?

39. If 20 men reap a field of 144 acres in 5 days, how long will they take to reap a field of 96 acres?

40. If 70 yds. of matting 2 ft. wide cost $47.25, what should be the price of 70 yds. of similar matting 7 ft. wide?

*176. When two quantities vary directly as each other, and one of them is multiplied by any fraction, the other must be multiplied by the *same* fraction.

For when one of the quantities is multiplied by the numerator of the fraction and divided by the denominator, the other quantity must be so too.

When two quantities vary inversely as each other, and one of them is multiplied by any fraction, the other must be multiplied by the *inverse* of that fraction.

We may express the same thing by the use of the word **ratio** [see Arts. 118, 119]. A quantity is said to be **changed in a given ratio**, say the ratio of 37 to 35, when it is multiplied by $\frac{37}{35}$.

A quantity is said to be changed in the **inverse** of a given ratio, say in the inverse of the ratio of 37 to 35, when it is multiplied by $\frac{35}{37}$; hence

Art. 171 may be stated thus: Two quantities are said to *vary* as one another which are so related, that when one of the quantities is changed, the other is changed in the *same* ratio.

Art. 173 may be stated thus: Two quantities are said to *vary inversely* as one another which are so related, that when one of them is changed, the other is changed in the *inverse of the same* ratio.

Example. If 17 men *can do a piece of work in* 187 days, *in how many* days *could* 11 men *do it?*

That is: If 17 men can do a piece of work in 187 days, in how many days could $\frac{11}{17}$ of 17 men do it?

Here, less men, *more* days;
therefore the required number is $\frac{17}{11}$ times 187; or, $\underline{289}$ (days).

177. We work out three more examples.

Example i. *If* 25 men *reap a field in* $3\frac{1}{5}$ hours, *how many* men *could reap the same field in* $6\frac{2}{3}$ hours?

That is: If 25 men reap a field in $3\frac{1}{5}$ hours, how many men will reap the same field in $\frac{6\frac{2}{3}}{3\frac{1}{5}}$ times $3\frac{1}{5}$ hours?

Here, more hours, *less* men ; therefore the required number of men is $\frac{3\frac{1}{5}}{6\frac{2}{3}}$ times 25 ; that is, $\frac{16 \times 3 \times 25}{20 \times 5}$ or $\underline{12}$ (men).

Example ii. *If the carriage of* 13·6875 cwt. *cost* $9.49, *what will the carriage of* 1·6875 cwt. *cost for the same distance ?*

That is: If the carriage of 13·6875 cwt. cost $9.49, what will the carriage of $\frac{1{\cdot}6875}{13{\cdot}6875}$ times 13·6875 cwt. cost ?

Here, less weight, *less* cost ; therefore the required cost is $\frac{1{\cdot}6875}{13{\cdot}6875}$ times $9.49 ; that is, $\frac{9}{73}$ of $9.49, or $1.17.

Example iii. *If* 31 bush. 2 pks. *of corn cost* $51.66, *how much* corn *can be purchased for* $102.50 ?

That is: If 31 bush. 2 pks. of corn cost $51.66, how much corn can be purchased for $\frac{102{\cdot}50}{51{\cdot}66}$ times $51.66 ?

Here, more money, *more* corn ; therefore the required amount of corn is $\frac{102{\cdot}50}{51{\cdot}66}$ times 31 bush. 2 pks., or $\frac{5125}{2583} \times 126$ pks., or $\underline{\text{250 pks.}}$

EXAMPLES. CXV.

1. If the carriage of $54\frac{3}{4}$ cwt. cost $18.98, what will the carriage of $13\frac{1}{2}$ cwt. cost for the same distance ?

2. If a household consumes $2\frac{5}{8}$ bushels in a week, how many will be consumed in 3 days ?

3. How many yards of cloth at $5.50 per yard must be given for 30 yards at $3.75 per yard ?

4. If $\frac{2}{9}$ of an article costs $1.80, how much would $1\frac{3}{10}$ articles cost ?

5. If 1·25 articles cost $7.50, what would $1\frac{7}{11}$ articles cost ?

6. If 1·5 tons of goods cost $17.50, what will 8·75 tons cost ?

7. If ·441 lbs. cost $15.75, how much will 45·346 lbs. cost ?

8. If a man walk 3·67 miles in an hour and a quarter, how long will he take to walk 132·12 miles at the same rate ?

9. If $3\frac{1}{2}$ cwt. cost $29.68, what will be the cost of 9 cwt. ?

10. A bankrupt whose assets are $4842.41 pays 47c. in the $1 ; what must his assets be to pay 64c. in the $1 ?

11. If 10 bush. 2 pks. of corn cost $5.25, how much can be purchased for $205 ?

12. What is the rent of 17 a. 25 p. when 3 a. 69 p. are let for $103.14?

13. If the carriage of 2 tons is $2.08, how much will be the carriage of 2 tons 1 cwt. 25 lbs. for the same distance?

14. A watch which gains 3 min. 12 sec. per day is put right at noon on Monday; what time will it indicate at 3 P.M. on the following Tuesday?

15. The circumference of a circle varies as its diameter; the circumference of a circle whose diameter is 1 yd. is 3·1416 yds.; find the circumference of the circle whose diameter is 25 ft.

16. A man working $3\frac{3}{4}$ hours per day does a piece of work in $3\frac{1}{5}$ days; how many hours per day must he work to do the same work in $1\frac{5}{7}$ days?

17. If it requires $57\frac{1}{2}$ yds. of carpet $\frac{3}{4}$ yd. wide to cover a room, how many yards of carpet $\frac{5}{8}$ of a yard wide will be necessary to cover the same room?

18. Two clocks, one of which gains $5\frac{1}{4}$ minutes per day, and the other gains $3\frac{3}{4}$, are set right at noon on Monday; in what time will there be a difference of half-an-hour in the times denoted by them?

19. Two clocks, one of which gains 1 minute and the other loses $2\frac{3}{4}$ minutes per day, differ by 15 minutes at noon on Monday; how long will it be before they differ by 30 minutes?

20. 2370 men have provisions for 57 days, and after 17 days 120 men go away; how long will the remaining provisions last the men who are left?

21. Taking 64 metres as equal to 70 yards, how many metres are there in 39·37079 inches?

22. It costs $161.16 to carpet a room 39 ft. 6 in. long, 25 ft. 6 in. wide; what would it cost if the room were 10 ft. longer at the same price per sq. yd.?

23. If 865 bush. 1 pk. of barley cost $969.08, what will 602 bush. 2 pks. cost?

24. Find the value of $7\frac{3}{7}$ things, when $\frac{4}{7}$ of one of them cost $95.92.

25. If a bankrupt pays $3.85 in $6, how much does he owe a creditor who receives $2401.63?

26. Find the cost of 7 cwt. 25 lbs. at $43.12 per cwt.

27. If 6 cwt. 52 lbs. of tea cost as much as 61 cwt. 94 lbs. of sugar, how much sugar should be given for 15 lbs. of tea?

28. If 3 cwt. 26 lbs. of tea cost as much as 30 cwt. 97 lbs. of sugar, how much tea should be given in exchange for 95 lbs. of sugar?

29. A train going $33\frac{3}{5}$ miles per hour takes $2\frac{7}{24}$ hours to travel a certain distance; how long would a train going $38\frac{1}{2}$ miles an hour take to do the same distance?

30. A wall that is to be built to the height of 27 ft. is raised 9 ft. high by 18 men in 4 days; how many men can finish it in 4 days?

31. In a certain time the population of a town increased from 15786 to 19670, find by how many a town of 100,000 inhabitants increases at the same rate in the same time.

32. What dividend is paid on a debt of $626.30 by a bankrupt who pays 64 cents in the dollar?

33. In a two-mile race between *A* and *B*, *A* wins by 150 yards; how much start could *A* give *B* in a mile race?

34. If 11 oxen are worth 85 sheep and 13 oxen are worth $1657.50, what is one sheep worth?

35. If 518 men can build a house in 243 days, in how many days will 567 men build a house three-fourths of the size?

36. If 8 miles 2 furlongs of railway cost $825143.15, find to the nearest cent the cost of 22 miles 7 furlongs.

37. It costs $36.12 to paint a room 16 ft. long, 8 ft. 4 in. broad, 7 ft. 6 in. high; what would it cost if the room were $2\frac{1}{2}$ ft. higher at the same rate?

38. How many men would be required to finish in 12 days some work, of which 4 men can do one-third in 10 days?

39. What length of a strip of ground 363 yds. wide should be given in exchange for a piece measuring 907 yds. 1 ft. 6 in. long, 78 yds. 1 ft. wide?

40. A garrison of 5000 men has provisions for 55 days, after 10 days 500 men are killed; how long can the garrison now hold out at the same rate?

41. A garrison of 4000 men has provisions for 128 days, and after 80 days is reinforced by 800 men; how long will the provisions now last at the same rate?

42. If 365 men can dig a field in 21 days of 10 hours each, in how many days of $10\frac{1}{2}$ hours can twice as many men dig it?

43. If 55 men can reap a field in $7\frac{1}{2}$ days, how many days will 25 men take to reap a field half as large again?

44. A bankrupt can pay $1.38 in $5; if his assets were $2250 more he could pay $1.56 in $5; find his debts.

45. A bankrupt can pay 41c. in the $; if his assets were $3333.33 less he could only pay 38c.; find his assets.

46. A garrison has provisions for a certain number of days; after 12 days, $\frac{1}{5}$ of the men desert, and it is found that the provisions will now last just as long as at first; how long was that?

178. In the following **Complex Problems** each quantity **varies as more than one** other quantity. A Complex Problem is reduced to two simple problems by considering *each* quantity to vary *separately*.

Example i. *If* 4 men *earn* \$36 *in* 15 days, *in how many* days *would* 5 men *earn* \$66?

The answer required consists of a number of **days**;
and the number of days in which 4 men earn \$36 is 15.

Now, since the *more* men, the *less* days, to earn a given sum,

$$\left.\begin{array}{l}\text{therefore the number of days in which 1 man earns \$36 is } 4 \times 15,\\ \text{and the number of days in which 5 men earn \$36 is } \frac{4}{5} \times 15.\end{array}\right\}$$

Again, since *more* money, *more* days (the no. of men being unaltered),
the number of days in which 5 men earn \$1 is $\frac{1}{36} \times \frac{4}{5} \times 15$,
and the no. of days in which 5 men earn \$66 is $\frac{66}{36} \times \frac{4}{5} \times 15$,

$$\text{that is, } \frac{6 \times 11 \times 4 \times 15}{6 \times 6 \times 5} \text{ or } \underline{22} \text{ (days).}$$

After sufficient practice the method of **ratio** may be adopted, and the work arranged as in Art. 176.

Example ii. *If* 65 men *reap a field of* 10 acres *in* 3 days, *how many* acres *will* 9 men *reap in* 26 days?

That is: If 65 men reap 10 acres in 3 days, how many acres will $\frac{9}{65}$ of 65 men reap in $\frac{26}{3}$ of 3 days?

Here, less men, *less* acres; more days, *more* acres; therefore the required number is $\frac{9}{65} \times \frac{26}{3}$ times 10; that is, $\underline{12}$ (acres).

Example iii. *If* 21 men *in* 25 days *dig a* trench 363 yds. long, 5 ft. wide, 26 in. deep, *how long a* trench 2 yds. 1 ft. wide, 22 in. deep, *could* 8 men *dig in* 49 days?

That is: If 21 men in 25 days dig a trench 363 yds. long, 5 ft. wide, 26 in. deep, how long a trench $\frac{7}{5}$ of 5 ft. wide, $\frac{22}{26}$ of 26 in. deep, could $\frac{8}{21}$ of 21 men dig in $\frac{49}{25}$ of 25 days?

Here, more width, *less* length; less depth, *more* length; less men, *less* length; more days, *more* length; therefore the required length is $\frac{5}{7} \times \frac{26}{22} \times \frac{8}{21} \times \frac{49}{25}$ times 363 yds., or $\underline{228\frac{4}{5}\text{ yds.}}$

EXAMPLES. CXVI.

1. If 3 men earn \$28 in 12 days, in how many days will 9 men earn \$56?

2. If 5 men earn \$135 in 21 days, how many dollars will 30 men earn in 7 days?

3. If 11 men earn \$275 in 40 days, how many men will earn \$125 in 8 days?

4. If 7 men can live on \$1400 for 56 weeks, how long will \$225 last 3 men?

5. If 7 men can live on $1400 for 56 weeks, how many men can live on $600 for 3 weeks?

6. If 7 men can live on $1400 for 56 weeks, how much will keep 56 men for 35 weeks?

7. If 12 furnaces consume 12 tons of coal in 10 hours, how long will 7 furnaces take to consume 35 tons?

8. A man earns $17 in 7 days of 9 hours each; in how many days of 7 hours can he earn $25.50?

9. If 70 men dig 4 acres in 8 days, how many men can dig 11 acres in 44 days?

10. If 48 tons can be carried 120 miles for $106, how much should be charged for carrying 35 tons 144 miles?

11. If 10 men dig a trench 15 yds. long in 21 days, how many men can dig a similar trench 40 yds. long in 35 days?

12. If 14 men mow 76 acres in 6 days, how many men can mow 114 acres in 9 days?

13. 25 men can do a work in 3 days of 8 hours each; how many hours per day must 24 men work to do the same in 5 days?

14. What is the interest on $500 for 1 year if the interest on $2500 for 7 years is $700?

15. If 16 pumps can raise 1000 tons of water in 8 hours, how many pumps can raise 1250 tons of water in 20 hours?

16. If the 4 lb. loaf cost 12c. when corn is $10 per bushel, how much ought the 3 lb. loaf to cost when corn is $8 per bushel?

17. If $12 is charged for the carriage of 8 cwt. for 128 miles, how far ought 4 cwt. to be carried for $1.50?

18. A train travelling 33 miles an hour does a certain journey in 90 minutes; how long will an express travelling 54 miles an hour take to do double the distance?

19. If 1500 copies of a book of 352 pages require 66 reams of paper, how many reams will be required for 10000 copies of a book of 396 pages?

20. If a man walk 96 miles in 5 days walking 6 hours a day, in how many days will he walk 480 miles walking 5 hours a day?

21. If 2 horses can plough 7 acres in a day, how many horses can plough 161 acres in 11½ days?

22. A man travels 1056 miles in 12 days, travelling 11 hours a day; in how many days would he travel 480 miles at the same rate, travelling 6 hours a day?

23. If 16 tons 5 cwt. of provisions serve 2100 men for 13 days, on what weight could 4340 men live for 6 weeks?

24. The railway fares in Prussia are two-thirds of those in England; if it costs $5.40 to travel 180 miles in England, what will it cost to travel 160 miles in Prussia?

25. If pasturage for 28 oxen for 11 days can be obtained for $37.20, what should be paid for 33 oxen for 63 days?

26. A garrison of 1000 men has supplies for 10 weeks, allowing 8 oz. per man per day, but is reinforced by 250 men; what must be the daily allowance that they may now last out 12 weeks?

27. If 25 men can do some work in 13½ days of 10 hours each, in how many days of 9 hours will 30 men do twice as much?

28. A garrison of 1800 men has provisions for 12 weeks; how long will the provisions last if 600 more men are introduced, but the daily allowance per man diminished by a third?

29. If 45 men in 28 days of 10 hours excavate 2790 tons of earth, in how many days will 35 men, working 12 hours a day, do twice that amount of work?

30. If 7 horses draw 5½ tons along 56 yards on a road in 10 minutes, how many horses will be required to draw 11 tons along 50 yards in 5 minutes?

31. If 5 men can hoe 10 acres of turnips in 4 days of 10 hours each, and 3 men hoe as much as 5 boys in the same time, in how many days of 12½ hours will 8 boys hoe 12 acres?

32. If 7 men mow 37 acres in 3 days, working 8 hours a day, how many men could mow a field of twice the size in 7 days, working 4 hours longer each day?

33. If 15 men can build a wall 7 ft. long, 15 ft. high, 14 in. thick in 16 days, in how many days will 10 men build a wall 35 ft. long, 14 ft. high, 9 in. thick?

34. If 7 men, working 16 days, can mow a field 132 yds. long, 880 yds. wide, how long is a field which is 660 yds. wide which 4 men can mow in 42 days?

35. If 20 men can mow a field of 9½ acres in 8½ days of 10 hours each, how many fields of twice the area can 255 men mow in 10 days of 8 hours each?

36. If 5 men or 10 women can reap a field of 12½ acres in 3½ days working 16 hours a day, in what time can 2 men and 10 women reap a field of 15 acres working 12 hours a day?

37. If the gas for 5 burners 5 hours each evening for 10 days costs $1.02, what will be the cost of 75 burners which are lighted 4 hours every evening for 15 days, supposing that 5 of the first set of burners consume as much gas as 6 of the second?

38. If 22 men can dig a trench 420 yds. long, 5 wide, 3 deep, in 350 days of 9 hours each, in how many days of 11 hours each will 252 men dig a trench 210 yds. long, 3 wide, 2 deep?

39. If 72 cwt. of lead will cover a roof 48 ft. long, 32 wide, with lead $\frac{1}{12}$ of an inch thick; what is the length of a roof 25 ft. wide which requires 75 cwt. of lead ⅛ of an inch thick to cover it?

40. If 9 men can mow a square meadow in 4 days, in what time can 7 men mow a square meadow one-sixth as long again as the former?

41. If 6 men and 9 boys, working 12 hrs. a day, make a drain 225 yds. long in 9 days, in how many days can 10 men and 12 boys, working $10\frac{1}{2}$ hrs. a day, make a drain 175 yds. long, the work of a boy being $\frac{2}{3}$ that of a man?

42. 160 men in 6 days of 11 hours each dig a trench 230 yds. long, $5\frac{1}{2}$ wide, $1\frac{1}{2}$ deep; in how many days of 8 hours each could 96 men dig a trench 220 yds. long, $3\frac{1}{2}$ wide, and 1 deep, when the hardness of the ground in the first case is to that in the second case as 5 to 7, and 4 men of the latter company can do the work of 5 men in the former?

179. The following examples in '**Exchange**' are Complex Problems.

Example. When exchange at New York *on* Paris *is* 516 francs per 100 dollars, *and exchange at* Paris *on* Hamburg *is* 123 francs per 100 marks; *what will a draft for* 2580 marks *at* Hamburg *cost in* New York?

That is, 100 dollars at New York will buy an order for 516 francs to be paid in Paris; 123 francs in Paris will buy an order for 100 marks to be paid in Hamburg; we are to find how much should be paid in New York for a draft for 2580 marks payable in Hamburg.

Hence we have,

$$\begin{aligned} 2580 \text{ marks} &= 2580 \text{ times } 1 \text{ mark} \\ &= 2580 \times \tfrac{123}{100} \text{ times } 1 \text{ franc} \\ &= 2580 \times \tfrac{123}{100} \times \tfrac{100}{516} \text{ times } 1 \text{ dollar} \\ &= \underline{615 \text{ dollars.}} \end{aligned}$$

N.B.—In practice, one line only need be used, the reducing ratios being introduced successively.

The following method is called **Chain Rule.**

We write down the statements in the question thus,

$$\left.\begin{aligned} \text{required no. of dollars} &= 2580 \text{ marks,} \\ 100 \text{ marks} &= \ \,123 \text{ francs,} \\ 516 \text{ francs} &= \ \,100 \text{ dollars,} \end{aligned}\right\}$$

[Here each denomination occurs once on each side of the sign =; we then equate the product of the numbers on each side.]

so that (req. number of dollars) × 100 × 516 = 2580 × 123 × 100,

therefore req. no. of dollars is $\dfrac{2580 \times 123 \times 100}{100 \times 516}$, that is, $\underline{615}$.

EXAMPLES. CXVII.

1. If on Exchange £5 is worth 34 German thalers and 51 francs are worth £2, how many thalers will be given in exchange for 50 francs?

2. If 100 dollars are worth 514 francs, and 100 marks worth 126 francs, how many dollars will be exchanged for 2056 marks?

3. If £1 is exchanged for 25·6 francs, and 1 franc for 172 Portuguese reis, how many reis should I get for £6?

4. £1 is exchanged for 12·15 Dutch florins and 9·5 Dutch florins for 20 francs; find the value of £10. 9*s.* in francs.

5. If for £5 I receive 127 francs and for 20 francs 22 lire 5 centesimi, how many lire and centesimi should I receive for £26. 16*s.* 8*d.* (1 lira =100 centesimi)?

6. 117 lire are given for £4 and 94·2 lire for 88 francs; how many francs will be given for £26. 3*s.* 4*d.*?

7. If at Philadelphia exchange on Liverpool is 4·895 dollars per £1, and at Paris on Liverpool 24·965 francs per £1, what is the exchange on Philadelphia at Paris per dollar?

8. If in London exchange on Paris is 25·25 francs per £, and at Paris exchange on Lisbon is 172 reis per franc, what is the exchange at London on Lisbon per £?

9. If 4 chickens are worth 3 ducks, 7 ducks worth 2 geese, and 9 geese worth 7 turkeys, what is the price of a chicken when a turkey costs $4.50?

10. If on exchange 48 lbs. of tea are given for 55 gals. of ale, and 63 gals. of ale for 24 bottles of wine, and 11 bottles of wine for 9 pairs of gloves, how many pounds of tea must be given for 20 pairs of gloves?

11. If $1\frac{3}{5}$ yds. of cloth are worth $\frac{5}{12}$ of a bushel of corn and 12 yds. of cloth cost $4\frac{1}{2}$ dollars, what is the value in English money of 40 bushels of corn, a dollar being worth 4*s.* 2*d.*?

12. On exchange £1 is worth 4 dollars 7 cents, and 25 francs 2 centimes in French money; how many cents do I lose in exchanging £15 at the rate of £1 for 25 francs?

13. A merchant in London is indebted to one at St. Petersburg 15000 roubles; exchange between St. Petersburg and London is 50*d.* per rouble; between St. Petersburg and Amsterdam 91*d.* (Dutch) per rouble, and between Amsterdam and London 36*s.* 3*d.* (Dutch) per £1; what difference will it make if the London merchant is drawn upon through Amsterdam instead of direct?

14. A New York merchant owes 1500 marks in Frankfort; should he remit direct from New York or through London? Exchange at New York on Frankfort being 23·5 dollars per 100 marks, on London 4·875 dollars per £, and at London on Frankfort 20·75 marks per £.

CHAPTER XI

ON PROBLEMS CONCERNING TIME

I. TIME AND DISTANCE

180. The average **speed** of a moving object varies directly as the distance traversed and inversely as the interval of time occupied in traversing that distance.

The average speed of a train going 30 miles in an hour is three times as great as that of a train going 10 miles in an hour; and half as great as that of a train going 30 miles in half an hour.

Example i. *One man* A *starts from London to walk to Cambridge, a distance of* $52\frac{1}{4}$ miles, *at a uniform speed of* $3\frac{1}{4}$ miles in one hour; *an hour later another man* B *starts from Cambridge for London, walking* $3\frac{3}{4}$ miles in one hour; *when will* A *meet* B?

Here, *A* has already gone $3\frac{1}{4}$ miles before *B* starts.

Hence, when *B* starts there are 49 miles to be traversed: of this, in one hour *A* walks $3\frac{1}{4}$ and *B* walks $3\frac{3}{4}$; thus of the 49 miles $(3\frac{1}{4}+3\frac{3}{4})$, or 7 miles are passed over in each hour.

Thus, between them they will have walked the whole distance, and will therefore meet in 7 hours after *B* started.

Example ii. *An ordinary train leaves a place at* 10 A.M. *and travels at the rate of* 30 miles per hour; *an express train leaves the same place at* 11.15 A.M., *and travels on another line of rails at the rate of* 45 miles per hour; *where will the second train overtake the first?*

To solve questions of this kind we must consider how much one train **gains** on the other in an hour.

The first train has started $\frac{5}{4}$ of an hour and is therefore $\frac{5}{4}$ of 30 miles or $37\frac{1}{2}$ miles away, when the second train starts;

therefore the express has to *gain* $37\frac{1}{2}$ miles on the other.

The express *gains* 15 miles while it goes 45 miles;

therefore it *gains* $37\frac{1}{2}$ miles while it goes $\dfrac{37\frac{1}{2}}{15}$ of 45 miles; that is, while it goes $\frac{75}{30} \times 45$, or $112\frac{1}{2}$ miles.

That is, it overtakes the other $\underline{112\frac{1}{2}\text{ miles from the starting place.}}$

Example iii. *At what time between* 5 *and* 6 o'clock *will the hands of a clock be at right angles to each other?*

Let the circumference of the dial be divided into 60 (minute) **spaces**. The large hand passes 60 *spaces* in one hour, the hour hand passes 5 *spaces* in one hour.

The large hand therefore **gains** 55 spaces in one hour.

Now at 5 P.M. the large hand is 25 spaces behind the hour hand, and it is at right angles to it (i) when it is 15 spaces behind, and (ii) when it is 15 spaces ahead, of the hour hand. It will be at right angles therefore between 5 and 6, (i) when it has gained (25 – 15) or 10 spaces, and (ii) when it has gained (25 + 15) or 40 spaces.

The large hand *gains* 55 spaces in an hour; therefore,

(i) it gains 10 spaces in $\frac{10}{55}$ of an hour,
that is, in $\frac{2}{11}$ of 60 min., that is, in $10\frac{10}{11}$ min.

(ii) it gains 40 spaces in $\frac{40}{55}$ of an hour,
that is, in $\frac{8}{11}$ of 60 min., that is, in $43\frac{7}{11}$ min.

Hence the hands are at right angles (i) $\underline{\text{at } 10\frac{10}{11}\text{ min. past 5,}}$ (ii) $\underline{\text{at } 43\frac{7}{11}\text{ min. past 5.}}$

Example iv. *In a mile race* A *beats* B *by* 80 yards *and he beats* C *by* 87 yards; *by how much could* B *beat* C *in a mile race?*

Here, B runs (1760 – 80) yards while C runs (1760 – 87) yds., hence

B runs 1 yard while C runs $\frac{1673}{1680}$ yds.
and B runs 1760 yards while C runs $\frac{1673}{1680} \times 1760$ yds.
that is, $\dfrac{22 \times 1673}{21}$ or $1752\frac{2}{3}$ yards.

Thus, B beats C by $7\frac{1}{3}$ yds. in a race of 1760 yds.

EXAMPLES. CXVIII.

1. A train going 30 miles an hour leaves Bristol for London at 1 P.M., another going 50 miles an hour leaves London for Bristol at the same time; when and where will they pass each other? [London to Bristol is 120 miles.]

2. A train going 25 miles an hour leaves Bristol at 1 P.M. for London, another going 30 miles an hour leaves London for Bristol at 2.12 P.M.; when and where will they meet?

3. A mail train leaves London for Edinburgh (400 miles) at 10.20 A.M., and travels 36 miles an hour; an express leaves London for

Edinburgh at noon and travels 45 miles an hour; when will it overtake the mail, and how much sooner will it reach Edinburgh?

4. *A* starts from Ely for London (68¼ miles) at 9 A.M., walking 3½ miles an hour, *B* starts an hour later from Ely and reaches London at the same time as *A*; what was his speed?

5. One clock gains 3 minutes in 24 hours, and a second gains 3 minutes in 15 hours; the first is put right on Jan. 1st, the second on Jan. 2nd; when will they indicate the same time?

6. A postman has 10 miles to walk in 3 hours; he walks the first 4 miles at a speed of 3¾ miles an hour, the next 2 miles at a speed of 3⅓ miles an hour, the next mile at a speed of 3 miles an hour; in what time must he walk the last three miles?

7. Two clocks are at 12 noon; one loses 7 seconds and the other gains 8 secs. in 24 hours; when will one clock be half an hour before the other, and what time will each show?

8. At what times will the hands of a clock be at right angles (i) between 7 and 8, (ii) between 2 and 3, (iii) between 12 and 1, (iv) between 3 and 4?

9. When are the hands of a clock (i) over one another, (ii) opposite one another, between 1 and 2? also between 4 and 5?

10. A watch which is 10 minutes too fast at noon on Monday loses 3 m. 10 sec. per day (24 hrs.); what time will it indicate at a quarter past 10 A.M. on the following Sunday?

11. If the hands of a clock coincide every 65 minutes, how much does the clock gain or lose in a day?

12. Two columns of troops, one 1000 yds. long, the other 5100 yds. long, are marching in opposite directions, the first at 4 miles in an hour, the other at 3½ miles in an hour; how long would they take to pass each other marching in opposite directions?

13. If the above columns (*Question* 12) were marching in the same direction, how long would they take to pass each other?

14. *A* and *B* run a mile race; *A* runs the whole course at a uniform rate of 320 yds. per minute; *B* runs the first half mile at 300 yds. per minute, the second half at 340 yds. per minute; which wins, and by how many seconds?

15. In a mile race *A* starts at 270 paces of 48 inches per minute, *B* at 300 paces of 44 in. per minute; after 4 minutes *B* quickens his speed to 320 paces per minute? which wins? by how much? and in what time?

16. *A* can beat *B* by 20 yds. in a mile race, *B* can beat *C* by 20 yds. in a mile race; how many yards start can *A* give *C* that there may be a dead heat?

17. In a mile race *A* beats *B* by 20 yds. and he beats *C* by 40 yds.; by how much could *B* beat *C* in a mile race?

18. *A* can give *B* 20 yds. and *C* 41 yds. start in a quarter of a mile;

B can give *C* a start of 3 seconds over the same distance; how long does *C* take to run a quarter of a mile?

II. TIME AND WORK.

181. In the following questions the **amount of work** done in a certain interval of time by a number of men is always understood to vary *directly* as the number of men and *inversely* as the interval of time.

Example i. *Two men* A *and* B *working separately can finish a piece of work in* 5 hours *and* 6 hours *respectively; in how many hours could they finish it working together?*

A can do the work in 5 hours; therefore *A* can do $\frac{1}{5}$ of the work in 1 hour; similarly *B* can do $\frac{1}{6}$ of the work in 1 hour.

Together, they can do $\frac{1}{5}+\frac{1}{6}$ or $\frac{11}{30}$ of the work in 1 hour;
therefore they can do 11 times the work in 30 hours,
and they can do the work in $\frac{30}{11}$, that is, in $2\frac{8}{11}$ hours.

Example ii. A *and* B *undertake to do* a piece of work *for* $28; A *could do it alone in* 7 days; B *could do it alone in* 8 days; *with the help of a* boy *they do it in* 3 days; *how should the money be divided?*

Let the piece of work be the *unit work;* then since *A* could do the work in 7 days, he does $\frac{1}{7}$ in 1 day; similarly *B* does $\frac{1}{8}$ in 1 day.

Therefore, in the 3 days, *A* does $\frac{3}{7}$ of the work, *B* does $\frac{3}{8}$, and the boy does the rest, which is $1-\frac{3}{7}-\frac{3}{8}$, that is $\frac{11}{56}$ of the work.

Hence *A* should have $\frac{3}{7}$ of $28, or $12; *B* should have $\frac{3}{8}$ of $28, or $10.50; the boy should have $\frac{11}{56}$ of $28, or $5.50.

Example iii. *A cistern can be filled from either of* 3 pipes *A, B, and C; A and B together fill the cistern in* 20 min., *B and C together fill it in* 18 min.; *and A and C together fill it in* 15 min.: *in how many* minutes *could B alone fill it?*

Let the cubic content of the cistern be the *unit* of cubic content,
then the pipes *A* and *B* together pour in $\frac{1}{20}$ of the unit in 1 min.
and the pipes *B* and *C* together pour in $\frac{1}{18}$ in 1 min.; therefore
4 pipes, namely 1 of *A*'s size, 2 of *B*'s and 1 of *C*'s, together, could pour in $\frac{1}{20}+\frac{1}{18}$, that is $\frac{19}{180}$ of the unit in 1 min.

But *A* and *C* together pour in $\frac{1}{15}$ in 1 min.; therefore (subtracting) 2 pipes of *B*'s size could pour in $\frac{19}{180}-\frac{1}{15}$, that is, $\frac{7}{180}$ in 1 min.; therefore *B* pours in $\frac{1}{2}$ of $\frac{7}{180}$, that is, $\frac{7}{360}$ in 1 min.

Therefore *B* could fill the cistern in $\frac{360}{7}$ min. or $51\frac{3}{7}$ min.

EXAMPLES. CXIX.

1. *A* can do a piece of work in 10 days which *B* can do in 15 days; how long would they take to do it working together?

2. *A* and *B* together can do a piece of work in three days which *A* alone can do in 6 days; in what time could *B* alone do it?

3. *A*, *B*, and *C* together can do a piece of work in 5 days which *B* alone can do in 12 days; in how many days can *A* and *C* together do it?

4. Two taps separately fill a cistern in 9 and 12 min. respectively; the waste pipe empties it in 8 min.; in what time will the two taps together fill the cistern when the waste pipe is open?

5. A cistern is filled by two spouts in 15 and 25 minutes respectively; it is emptied by a tap in half an hour; what portion of it will be filled in 10 minutes when they are all left open?

6. Two taps can separately fill a cistern in 10 and 12 min., and when the waste pipe is open they can together fill it in 15 min.; how long does the waste pipe alone take to empty the cistern?

7. *A* and *B* working together can do a piece of work in 7 days: *B* alone can do it in 13 days; supposing *B* works at it for 3 days, in how many days could *A* alone finish it?

8. After a certain number of men had been employed on a certain work for 24 days and had half finished it, 16 more were set on, and the remaining half was completed in 16 days; how many men were employed at first?

9. 35 men begin a piece of work which together they could complete in 45 days, but at the end of 15 days 7 men are taken off; in how many days will the work be completed?

10. *A* and *B* undertake to do a piece of work for $5.60; *A* can do it alone in 6 days, *B* can do it in 7 days; with the help of a boy they finish it in 3 days; how much ought they to give the boy?

11. *A*, *B*, and *C* can together do a piece of work in 5 days; after working at it with *B* and *C* for 2 days, *A* leaves off and *B* and *C* finish it in 5 days more; in what time can *A* do it?

12. *A* does half as much work again as *B* in the same time, and *B* does one-third as much again as *C*; working together they can do a certain work in 5 days; if after working at it alone for 2 days *A* leaves off, how long will *B* and *C* take to finish it?

13. Of a piece of work 2 men and 5 boys can do a half in 5 days; 3 men and 4 boys can do a third in 3 days; in how many days could 9 men do it?

CHAPTER XII

PERCENTAGE

182. **Per cent.** (that is, *per centum*) means '*for every hundred.*'

Example. '*In an examination a certain candidate obtained* 73 per cent. *of full marks,*' means, that he obtained on the whole 73 out of every hundred, or $\frac{73}{100}$ of the total number of marks given : Hence

73 *per cent.* expresses *the fractional part* $\frac{73}{100}$.

73 *per cent.* is often written 73 %.

In many cases the **importance** of a quantity depends more on *what part* it is of some other quantity than on its actual size ; for example, suppose that in a certain town the number of deaths in a given year is 4000. This would be a startling fact if the town contained only say 10,000 inhabitants, but it would be nothing unusual if the town contained, say 200,000 inhabitants. In the first case the number of deaths is $\frac{4000}{10000}$ or $\frac{40}{100}$ of 10,000, that is, '40 *per cent.*' of the population; in the second case it is $\frac{4000}{200000}$, or $\frac{2}{100}$, of 200,000, that is, '2 *per cent.*' of the population.

Since then it is often important to know what fractional part one quantity is of another, and since fractions having a common denominator are easy of comparison, quantities are often expressed as *so many per cent.* of another quantity.

Example i. *What is* 4 per cent. *of* 5150 ?

4 per cent. means 4 *hundredths*, hence

4 per cent. of 5150 is $\frac{4}{100}$ of 5150, that is, $\frac{20600}{100}$, or $\underline{206}$.

Example ii. *What is* 5 per cent. *of* $522.80 ?

5 per cent is $5 out of every $100,
that is $\frac{5}{100}$, that is, $\frac{1}{20}$ of the whole.

Hence the percentage required is
$\frac{1}{20}$ of $522.80, that is $26.14.

Example iii. *What is* 7 per cent. *of* \$563?

The required percentage is $\frac{7}{100}$ of \$563,
that is $\frac{1}{100}$ of \$3941, that is \$39.41.

Example iv. *What is* $\frac{1}{4}$ *per cent. of* \$560?

$\frac{1}{4}$ per cent. is \$$\frac{1}{4}$ out of every \$100,
that is, \$1 out of every \$400,
that is, $\frac{1}{400}$ of the whole.

Hence the percentage required is
$\frac{1}{400}$ of \$560, that is \$1.40.

EXAMPLES. CXX.

Find the value of

1. 3 per cent. of 150.
2. 4 per cent. of 1000.
3. $7\frac{1}{2}$ % of \$90.
4. $3\frac{3}{4}$ % of 7 tons.
5. $22\frac{1}{2}$ % of \$900.
6. 6 % of 150 men.
7. $13\frac{1}{3}$ % of 15 acres.
8. $\frac{1}{3}$ % of \$150.
9. $\frac{1}{19}$ % of \$95.

Write down the value to the nearest cent of 5 per cent. of

10. \$27.56.
11. \$323.75.
12. \$979.92.
13. \$2397.12.
14. \$872.13.
15. \$7642.37.

Write down the value to the nearest cent of $2\frac{1}{2}$ per cent. of

16. \$1823.65.
17. \$17107.90.
18. \$2258.25.

19. My income is \$4325; if I save 15 per cent. of it, how much do I save?

20. Having 150 sheep, I sell 20 per cent. of them, how many have I left?

21. The salaries in a bank are increased 3 per cent.; how much more does the manager receive whose salary is \$6375?

22. A tradesman allows 5 per cent. reduction for ready money; how much cash should I have to pay for an article marked \$6?

23. Of 150 eggs $16\frac{2}{3}$ per cent. are broken, how many are not broken?

24. A train which runs 45 miles an hour is to be quickened $6\frac{2}{3}$ per cent.; how far will it then run per hour?

25. I pay $\frac{1}{10}$ per cent. for insuring my house for \$8750; how much is the premium (*i.e.* what do I pay altogether)?

26. I pay $2\frac{1}{2}$ per cent. for insuring my life for \$10000; what is the premium?

27. A man pays $\frac{3}{40}$ per cent. for insuring his house for \$15000, and $\frac{1}{8}$ per cent. for insuring his stables for \$2500. What is the total premium paid?

Example v. *How much per cent. is* 24 oz. *in the* 1 cwt. ?

1 cwt. contains 16×100 oz.

24 oz. in every 1600 oz. is $\frac{24}{1600}$, that is, $\frac{3}{200}$ of the whole; hence it is $\frac{3}{200}$ of 100 in every hundred,
that is, $1\frac{1}{2}$ per cent.

Example vi. *The population of a town increased from* 5125 *to* 5935 *in* 10 years; *what is the* gain per cent. ?

The total gain is $5935 - 5125$, or 810.

Hence, the gain is 810 on 5125, the *original* population,
that is, at the rate of $\frac{810}{5125}$ of 100 in every hundred,
that is, $\frac{81000 \times 8}{5125 \times 8}$, or $\frac{648}{41}$, or $\underline{15\frac{33}{41}}$ per cent.

Example vii. *A man obtains* $3\frac{1}{2}$ per cent. *for his money, and his* total income *is* \$1400; *what is his* property *worth* ?

He obtains $\$3\frac{1}{2}$ income for every \$100 of property;
therefore he has as many \$100 of property as the number of times that $3\frac{1}{2}$ is contained in 1400;
that is, his property is worth $\frac{1400}{3\frac{1}{2}}$ times \$100, or $\underline{\$40000}$.

EXAMPLES. CXXI.

How much per cent. is each of the nine following ?

1. 1 cwt. in the ton. **2.** $1\frac{1}{2}$ in 20. **3.** $\frac{3}{8}$ of the whole.

4. 6 hrs. in the day. **5.** 2 in. in the ft. **6.** $\frac{2}{3}$ in 5.

7. $\frac{1}{2}$ of the whole. **8.** $\frac{7}{12}$ of the whole. **9.** ·075 of the whole.

In each of the six following questions, what per cent. is

10. 2 of 15 ? **11.** 75 lbs. of 1 ton ? **12.** 5 p. of 6 acres ?

13. ·35 of 1 ? **14.** ·375 of 8 ? **15.** £1. 2*s*. of £11 ?

16. From a cask of 36 gals., 15 gals. are lost by leakage; what is the loss per cent. ?

17. A tradesman sells $\frac{4}{15}$ of his stock; what per cent. of his stock did he sell ?

18. An army loses $16\frac{1}{2}$ per cent. of its total, viz. 396 killed and wounded; find the number of men in the army.

19. A city loses $2\frac{1}{4}$ per cent. by deaths, and gains $3\frac{1}{2}$ per cent. by births in a year; it increases 250 in that year; what was the population ?

20. A man spends 76 per cent. of his income, and saves \$960 every year; what is his income ?

21. I insure my house at the rate of $\frac{3}{40}$ per cent., and pay $5.85 premium ; for what is my house insured ?

22. An agent charges $180 *commission* for selling a house, which is $2\frac{1}{2}$ % on the price of the house ; for what did he sell it ?

23. The population of a town of 64000 inhabitants increases at the rate of $2\frac{1}{2}$ per cent. in each year, find its population (i) 1, (ii) 2, and (iii) 3 years hence.

24. The population of a town increases at the rate of 2 per cent. yearly, it has now 132651 inhabitants, how many had it, (i) 1, (ii) 2, and (iii) 3 years ago ?

[*This subject is continued in* Chapter XIV.]

INTEREST.

183. **Interest** is money paid, by a borrower, for the use of money.

The money lent is called the **Principal** ; the Interest and Principal added together is called the **Amount**.

Interest is always calculated at so much for the use of each $100 for a stated interval of time (generally for one year).

The **number** *of dollars paid for the use of each* $100 *for one year* is called the '**rate** *per cent. per annum.*'

N.B.—In Questions on Interest the percentage is understood to be *per annum*, unless it is expressly stated to be otherwise.

Example i. *Find the* Interest *on* $375 *for* 1 year *at* 4 per cent. per annum.

Here, the Interest is 4 per cent. (that is, $\frac{4}{100}$) of the principal.

Therefore the required Interest is $\frac{4}{100}$ of $375, or $\underline{\$15}$.

Example ii. *Find the* Interest *for* 1 year *on* $61085 *at* 5 per cent. per annum.

The required Interest is $\frac{5}{100}$ of $61085, that is $3054.25.

Otherwise, since $\frac{5}{100}$ is ·05, the interest may be found by multiplying the principal by ·05.

The interest on $61085 for 1 year at 5 per cent. is $61085 × ·05, that is, $3054.25.

When the Interest for a number, say 3, of years is simply 3 times the Interest for one year, it is called **Simple Interest.**

Thus, the Simple Interest for 3 years is the interest for 1 year multiplied by 3; hence the Simple Interest for a given number of years is the Principal × the no. of years × the rate per cent. ÷ 100.

N.B.—Since we have to multiply by the number of years, and by the rate per cent., it is often convenient first to find the product of these two.

Example iii. *Find the* Simple Interest *on* $4567.50 *for* 2⅔ years *at* 5¼ per cent. per annum.

Here we have to multiply $4567.50 by 2⅔ and by 5¼, and divide the product by 100.

Now $$2\tfrac{2}{3} \times 5\tfrac{1}{4} = \frac{8}{3} \times \frac{21}{4} = \frac{2 \times 4 \times 7 \times 3}{3 \times 4} = 2 \times 7.$$

Hence we multiply by 2 and by 7, and divide the result by 100.

$$\begin{array}{r} \$4567.50 \\ 2 \\ \hline \$9135.00 \\ 7 \\ \hline \$63945.00 \end{array}$$

Hence the Interest is $639.45.

EXAMPLES. CXXII.

Find the Simple Interest on

1. $200 for 1 year at 5 per cent.
2. $1750 for 1 year at 4 %.
3. $2250 for 3 years at 4 %.
4. $4000 for 15 years at 3½ %.
5. $5250 for 5½ years at 3⅓ %.
6. $666 for 6 years at 4 %.
7. $6240 for 5⅓ years at 3¾ %.
8. $8000 for 5¼ years at 3⅓ %.
9. $4795 for 6⅔ years at 2¼ %.
10. $1375 for 6¼ years at 2⅖ %.
11. $8525 for 3⅕ years at 3¾ %.
12. $981 for 3⅓ years at 5½ %.
13. $2992.80 for 4 years at 3¾ %.
14. $6344 for 3 months at 2½ %.
15. $7945 for 4 months at 3 %.
16. $170 for 8 months at 4½ %.
17. $738 for 6 yrs. 8 mths. at 1½ %.
18. $2510.00 for 11 m. at 7⅕ %.
19. $6872.10 for 73 days at 3½ %.
20. $149.00 for 3 yrs. at 3 %.
21. $2200 for 146 days at 3 %.
22. $725 for 1 yr. 146 d. at 5⅓ %.
23. $469 for 2½ months at 7½ %.
24. $4848 for 3½ years at 4½ %.
25. $8640 for 7½ years at 3¾ %.

184. When interest is calculated from one given day of the year to another, it is customary to *include one only* of the days named.

Example. Find the Interest *on* \$2438.80 *from* January 3 *to* May 26, *at* 4 per cent (per annum).

Excluding January 3, including May 26, giving February 28 days, this interval contains 143 days; which is $\frac{143}{365}$ of an ordinary year.

Hence for 143 days the Interest required is

$$\frac{143}{365} \times \frac{4}{100} \text{ of } \$2438.80.$$

\$2438.80 × ·04 = \$97.552.

$$\begin{array}{r}
97{\cdot}552\\
143\\
\hline
9755{\cdot}2\\
3902{\cdot}08\\
292{\cdot}656\\
\hline
365)\overline{13949{\cdot}936}(\$38.219\\
\overline{2999}\\
\overline{79{\cdot}9}\\
\overline{6{\cdot}93}\\
\overline{3{\cdot}286}\\
\overline{1}
\end{array}$$

\$38.22 Interest.

N.B.—The factors of 365 are 5 and 73.

EXAMPLES. CXXIII.

Find to the nearest cent the Simple Interest on

1. \$2437 for 4 years at 3 per cent.
2. \$2675.47 for 2 years at 3½ per cent.
3. \$2807.50 for 73 days at 2¾ per cent.
4. \$4395.50 for 219 days at 5½ per cent.
5. \$8967.70 for 1 year 151 days at 6¼ per cent.
6. \$20178.60 from July 18 to Sept. 19 at 5 per cent.

Find to the nearest cent the Amount at Simple Interest of

7. \$4238.75 from Jan. 13 to June 5 at 3 per cent.
8. \$3675 for 7 months at 3¼ per cent.
9. \$9430 for 100 days at 4 per cent.
10. \$38756.25 for 200 days at 3½ per cent.
11. \$43875 from Jan. 1 to April 10 at 4 per cent.
12. \$84123 from March 18 to July 31 at 4½ per cent.

NOTE on **Simple Interest.** Let £P = the Principal, £I = the Simple Interest on £P for n years at £r per cent. per annum, then,

$$I=\frac{n\times r\times P}{100}\ldots \text{ [Art. 183.]} \qquad P=\frac{100\times I}{n\times r}\ldots \text{ [Art. 186, Ex. (iii)],}$$

$$r=\frac{100\times I}{n\times P}\ldots \text{ [Art. 186, Ex. (ii)]} \qquad n=\frac{100\times I}{r\times P}\ldots \text{ [Art. 186, Ex. (i)].}$$

185. When money is lent for a number of years, the interest is usually paid by the borrower at regular intervals, yearly, or half-yearly, or quarterly, as the case may be. Suppose, however, it is agreed between the lender and borrower that instead of the interest being actually *paid* at the end of each year (or half-year) the interest due is *lent* in addition to the principal. In such a case the principal would be increased at the end of each year by that year's interest; hence,

When the Interest due is added to the principal at the end of stated periods of time (as at the end of each year) it is called **Compound** Interest.

Example i. *Find the Compound Interest on* $75030 *for* 3 years *at* 4 per cent. (per annum).

	$75030		=1st Principal.
$4\% = \frac{4}{100}$	3001	·20	=1st year's Interest.
	$78031	·20	=2d Principal.
	3121	·248	=2d year's Interest.
	$81152	·448	=3d Principal.
	3246	·09792	=3d year's Interest.
	$84398	·54592	=Amount.
	75030		=Principal.
	$9368	·54592	=Compound Interest (exact),

therefore the required compound interest, to the nearest cent, is $9368.55.

We write down the principal; take $\frac{4}{100}$ of this by multiplying by 4 and placing each digit of the product 2 places to the right; this gives us the first year's interest. The second principal is the result of adding the 1st principal to the interest for the 1st year. Similarly the interest for the 2d year is obtained from the 2d principal, and adding this to the 2d principal, we have the 3d principal; and so on. The amount at the end of the 3 years is thus found to be $84398·545, and subtracting from this the original principal we have $9368·544 for the Compound Interest.

N.B.—In questions of Compound Interest the interest is understood to become due *annually*, unless it is expressly stated to be otherwise.

EXAMPLES. CXXIV.

Find the Compound Interest on

1. $5000 for 3 years at 4 per cent
2. $8750 ,, 3 ,, 4 ,,
3. $16000 ,, 3 ,, 5 ,,
4. $7620.50 ,, 2 ,, 6 ,,
5. $7862.50 ,, 2 ,, 3 ,,
6. $7690.25 ,, 2 ,, 3 ,,
7. $4251 ,, 3 ,, 2 ,,
8. $4251 ,, 2 ,, 3 ,,
9. $5012.94 ,, 2 ,, 3 ,,
10. $5012.94 ,, 3 ,, 2 ,,
11. $7632.75 ,, 2 ,, 5 ,,
12. $9325.00 ,, 3 ,, 4 ,,

Example ii. *Find the Compound Interest on* $3751.27 *for* 2 years *at* $4\frac{3}{4}$ per cent.

$4\% = \frac{4}{100}$
$\frac{1}{2}\% = \frac{1}{8}$ of 4%
$\frac{1}{4}\% = \frac{1}{2}$ of $\frac{1}{2}\%$

$$\begin{array}{r|l l}
\$3751{\cdot}27 & & =\text{1st Principal.} \\
150{\cdot}05 & 08 & \\
18{\cdot}75 & 63 & \left.\right\} =\text{1st year's Interest.} \\
9{\cdot}37 & 81 & \\
\hline
\$3929{\cdot}46 & & =\text{2d Principal.} \\
157{\cdot}17 & 84 & \\
19{\cdot}64 & 73 & \left.\right\} =\text{2d year's Interest.} \\
9{\cdot}82 & 36 & \\
\hline
\$4116{\cdot}11 & & =\text{Amount at end of 2 years.} \\
3751{\cdot}27 & & \\
\hline
\underline{\underline{\$364{\cdot}84}} & & =\text{Compound Interest for 2 years.}
\end{array}$$

Example iii. *Find the Compound Interest on* $3751.27 *for* $2\frac{1}{2}$ years *at* $4\frac{3}{4}$ per cent.

The interest for a half-year is ½ of 4¾, *i.e.* 2⅜ per cent. As in Example ii, we find

	$4116·11		= Amount at end of 2 years.
2% = $\frac{2}{100}$	82·32	22	
¼ = ⅛ of 2%	10·29	02	= Interest for the half-year.
⅛ = ½ of ¼%	5·14	51	
	$4213·87		= Amount at end of 2½ years.
	3751·27		
	$462·60		= Compound Interest for 2½ years.

EXAMPLES. CXXV.

Find the Compound Interest on

1.	$22400	for	2	years at	3¾	per cent.
2.	$7642	,,	2½	,,	3	,,
3.	$9321	,,	3½	,,	4	,,
4.	$1234.56	,,	2½	,,	3½	,,
5.	$1234·56	,,	3½	,,	2½	,,
6.	$5000	,,	2¾	,,	3	,,
7.	$84123.75	,,	2	,,	3⅓	,,
8.	$8432.51	,,	3	,,	3½	,,

Example iv. *Find the amount at Compound Interest on* $4370 *for* 1½ years *at* 3 per cent. per annum, payable half-yearly.

3 per cent. is 1½ per cent. per half-year.

Hence we add to the principal $\frac{1}{100}$ of itself, then ½ this, and repeat the process for each half-year.

$4370		= 1st Principal.
43·70		= 1st half-year's Interest
21·85		
$4435·55		= 2d Principal.
44·35	55	= 2d half-year's Interest.
22·17	775	
$4502·08		= 3d Principal.
45·02	08	= 3d half-year's Interest.
22·51	04	
$4569·61		= Amount at end of 1½ years.

NOTE.—The amount at Compound Interest for 12 years on $131.12 at 3% per cent. per annum, payable half-yearly, is $\$131.12 \times (1\cdot015)^{24}$; this may be obtained to 5 significant figs. by contracted multiplication thus,

$(1{\cdot}015)^2=1{\cdot}0302$; squaring, $(1{\cdot}015)^4=1{\cdot}0613$; squaring again, $(1{\cdot}015)^8=1{\cdot}1244$; and again, $(1{\cdot}015)^{16}=1{\cdot}2643$. The product of the last two, *i.e.* $(1{\cdot}015)^{24}$, will be found to be $1{\cdot}4216$; the product of this by $131.12 is $\underline{\$186.40}$.

EXAMPLES. CXXVI.

Find to the nearest cent the Compound Interest on

1. $3721 for 3 years at 4 per cent.
2. $4017.50 ,, $2\frac{1}{2}$,, 5 ,,
3. $8751.75 ,, 3 ,, $3\frac{1}{3}$,,
4. $347156 ,, 2 ,, $4\frac{3}{4}$,,
5. $41875.50 ,, $2\frac{1}{2}$,, 4 ,,
6. $375.25 ,, 2 ,, $2\frac{1}{2}$,,

7. Find the Amount of $7035 for 1 year at 3 per cent. per annum, the interest being due half-yearly.

8. Find the Interest on $8491 for $1\frac{1}{2}$ years at 3 per cent. per annum, the Interest being due half-yearly.

9. Find the Interest on $9000 for $1\frac{1}{2}$ years at 4 per cent., the interest being due quarterly.

Find the Amount at Compound Interest of

10. $1485.36 for 3 years at 5 per cent.
11. $84123.84 ,, 2 ,, $4\frac{1}{2}$,,
12. $1829.25 ,, 3 ,, $3\frac{1}{2}$,,
13. 14275 fr. ,, 2 ,, 3 ,,
14. 76321 fr. ,, 3 ,, $2\frac{3}{4}$,,
15. $4375.25 ,, 3 ,, 5 ,,
16. $7642.01 ,, $2\frac{1}{2}$,, $2\frac{1}{2}$,,
17. $9375.24 ,, $3\frac{1}{3}$,, $3\frac{1}{3}$,,
18. 457123 fr. ,, 3 ,, $3\frac{1}{4}$,,

NOTE on **Compound Interest.** Let £A be the amount on £P for n years at r per cent. per annum Compound Interest (payable yearly);

then $$A=\left(\frac{100+r}{100}\right)^n\times P;\ I=A-P;\ P=\left(\frac{100}{100+r}\right)^n\times A.$$

186. **Inverse** questions on Interest.

Example i. *At what* **Rate** per cent. per annum *will the Simple Interest on $660 amount to $88 in $2\frac{1}{2}$ years?*

The rate per cent. *per annum* is the total rate per cent. divided by the number of years;

Here the total rate per cent. is $\frac{88}{660}$ of \$100 (p. 186); hence the rate per cent. per annum is

$$\frac{1}{2\frac{1}{2}} \times \frac{88}{660} \text{ of \$100, that is } \frac{2 \times 88 \times 100}{5 \times 660},$$

$$\text{or } \frac{2 \times 4 \times 22 \times 5 \times 10 \times 2}{5 \times 3 \times 22 \times 10} \text{ or } \frac{16}{3},$$

$$\text{that is } \underline{\$5\tfrac{1}{3}} \text{ (per cent.)}$$

Example ii. *In how many* **Years** *will* \$660 *amount to* \$748 at $5\frac{1}{3}$ per cent. (per annum, Simple Interest)?

[N.B.—It is *Interest* and not *Amount* which varies as the time.]

The required number of years is the number of times the total Interest contains the Interest for 1 year.

Here, the total Interest is \$748 − \$660 = \$88,

the Interest on \$660 for 1 year at $5\frac{1}{3}$ per cent. is $\frac{66}{10} \times \frac{16}{3}$; hence,

$$\text{the required number of years is } \frac{88}{\frac{66}{10} \times \frac{16}{3}}, \text{ that is } \frac{3 \times 10 \times 88}{66 \times 16}$$

$$\text{that is } \frac{3 \times 10 \times 22 \times 4}{3 \times 22 \times 4 \times 4}, \text{ that is } \frac{10}{4}, \text{ that is } 2\tfrac{1}{2} \text{ years.}$$

Example iii. *What* **Principal** *will amount to* \$748 *in* $2\frac{1}{2}$ years *at* $5\frac{1}{3}$ per cent. (per annum, Simple Interest)?

Here \$100 Principal gives $\$(100 + 2\frac{1}{2} \times 5\frac{1}{3})$ or $\$113\frac{1}{3}$ Amount in the given time at the given rate; hence

the Principal which amounts to $\$113\frac{1}{3}$ is \$100,

[Thus the Principal which amounts to $\$113\frac{1}{3}$ is found by multiplying $\$113\frac{1}{3}$ by the diminishing factor $\frac{100}{113\frac{1}{3}}$], therefore

the Principal which amounts to \$748 is $\frac{100}{113\frac{1}{3}}$ of \$748,

$$\text{that is } \frac{3 \times 100}{340} \text{ of \$748,}$$

$$\text{that is } \frac{3 \times 10 \times 10 \times 34 \times 22}{10 \times 34}$$

$$\text{that is } \underline{\$660.}$$

EXAMPLES. CXXVII.

1. At what rate per cent. will the Simple Interest on \$400 amount to \$80 in 5 years?

2. At what rate per cent. will \$350 produce \$28 Simple Interest in 2 years?

3. At what rate per cent. will the Interest on $2630 amount to $460.25 in 3½ years?

4. At what rate per cent. will the *Amount* on $425 be $573.75 in 5 years at Simple Interest?

5. At what rate per cent. will $3000 amount to $3438.75 in 3¼ years at Simple Interest?

6. At what rate per cent. will $1700 amount to $2273.75 in 7½ years at Simple Interest?

7. In how many years will the Simple Interest on $400 amount to $80 at 5 per cent.?

8. In how many years will the amount at Simple Interest on $550 be $660 at 4 per cent.?

9. In what time will the Simple Interest on $3000 amount to $438.75 at 4½ per cent.?

10. In how many years will $1846 amount to $2445.95 at 2½ per cent. Simple Interest?

11. In what time will the Simple Interest on $3560 amount to $333.75 at 2½ per cent. per annum?

12. The Simple Interest on $2300 amounts to $431.25 at 2½ per cent. For how many years is it calculated?

13. What Principal amounts to $406 in 4 years at 4 per cent. Simple Interest?

14. What Principal will produce $533 in 4 years at 3¼ per cent.?

15. The Simple Interest on a certain sum for 7½ years at 2½ per cent. is $431.25; what is the sum?

16. The Amount on a certain sum after 2½ years at 5 per cent. Simple Interest is $1075.50; what is the sum?

17. What Principal is required to produce $450.15 Simple Interest in 3 years at 5 per cent.?

18. What Principal will amount to $24307.55 in 3½ years at 4¼ per cent. Simple Interest?

Example iv. *What* Principal *will amount to* $2163.20 *in* 2 years *at* 4 per cent. per annum **Compound** Interest?

In this case $100 Principal gives in one year $104 Amount.

Therefore the Amount at the end of any one year is converted into the Principal at the beginning of the year by the diminishing factor $\frac{100}{104}$; hence

the required Principal $= \frac{100}{104} \times$ (Amount at end of 1st year)
$= \frac{100}{104} \times (\frac{100}{104} \times$ Amount at end of 2nd year)
$= \frac{100}{104} \times \frac{100}{104} \times \$2163.20$ or $\underline{\$2000}$.

MISCELLANEOUS EXAMPLES. CXXVIII.

1. Given that the Amount at Simple Interest is $2445.95, the rate 2½, and time 13 years, find the Principal.

2. Find the rate when $18760 amounts to $23332.75 in 6½ years at Simple Interest.

3. In how many years will the Simple Interest on $17400 amount to $4893.75 at 3¾ per cent.?

4. Given that the Interest is $10221.25 for 9¼ years at 4¼ per cent. Simple Interest, find the Principal.

5. How long will it take $1046.40 to produce $480.69 Simple Interest, at 5¼ per cent. per annum?

6. At what rate per cent. Simple Interest will a sum of money be doubled in 10 years?

7. What Principal amounts to $441 in 2 years at 5 per cent. Compound Interest?

8. What Principal will produce $816 Compound Interest in 2 years at 4 per cent.?

9. At what rate per cent. per annum will the Simple Interest on 2938·25 francs amount to 56·35 francs in 200 days?

10. Find the Compound Interest on $473 for ¾ year at 5 per cent., the Interest being added to the Principal every quarter.

11. On what day will the Interest on $7300 borrowed on March 18th amount to $54 at 3¾ per cent.?

12. What sum amounts to $1000 in 21 years at 3 per cent. Simple Interest?

13. Find the Simple Interest from June 1st to Sept. 1st on $1806.55 at 5 per cent. per annum.

14. What Principal will produce $9158.52 (Interest) in 3 years at 4 per cent. per annum Simple Interest?

15. Find the difference between the Simple and the Compound Interest on $8750 for 3 years at 4 per cent.

16. Prove that the Amount at Compound Interest for 3 years at 3 per cent. is 1·092727 times the Principal.

17. Prove that the difference between the Simple and the Compound Interest for 3 years at 5 per cent. is ·007625 times the Principal.

18. Prove that the difference between the Simple and the Compound Interest for 3 years at 4 per cent. is ·004864 times the Principal.

19. On what day will the Simple Interest on $485 lent on April 1st amount to $6.60 at 7½ per cent.?

20. Find to the nearest cent the Interest per day on $10000 at 4 per cent. Simple Interest.

21. Find the difference between the Simple and Compound Interest on \$274 for 2 years, 73 days, at $2\frac{1}{2}$ per cent. per annum.

22. On what sum does the difference between the Simple and Compound Interest for 2 years at 4 per cent. amount to \$5?

23. Find the Principal which will produce \$1262 (Interest) in 3 years at 5 per cent. Compound Interest.

24. What Principal will produce \$1000 Interest in 3 years at 4 per cent. Compound Interest?

187. When a reduction in an account is allowed for immediate payment, the seller is said to allow 'discount.'

Example. I buy goods to the amount of \$210, *and am allowed discount* 5 per cent. *for cash; what do I pay?*

There is here no mention of *time*, the question can therefore only mean that 5 per cent. is deducted from the amount of my account (that is \$10.50 is deducted), so that I pay \$199.50 cash instead of \$210.

The reduction here of \$10.50 is not (strictly speaking) of the nature of **Interest** for money, because time is an essential part of questions on interest.

EXAMPLES. CXXIX.

1. What reduction shall I get from the price of a book, of which the published price is \$10, when the bookseller deducts 10 per cent. for cash?

2. I am allowed a discount of 15 per cent. for cash. Find the cash price of articles marked respectively (i) \$5, (ii) \$3.50, (iii) \$22.50, (iv) \$7.66.

3. An English bookseller allows a discount of 2d. in the shilling on the published price of books; find the cash price of books published at (i) 7s. 6d., (ii) 4s. 6d., (iii) 6s., (iv) 12s. 6d.

4. The cash price of a book published at \$12.50 is \$10.00, what percentage is the reduction on the published price?

188. In commercial or bank discount, the reduction is of the nature of **Interest.**

Discount is the *deduction* made from a sum of money falling due at a future time, in consideration of immediate payment.

Discount usually arises as follows. Suppose a merchant in the course of business to want ready money, which he knows he will be able to pay back at the end of three months. He obtains it by **getting a bill discounted**; that is, he signs a document (a **bill**) by which he undertakes to pay a certain sum, say $5050, to the holder of the bill in three months, and sells this bill in the money market.

Suppose the 'Bill Broker' who buys (**discounts**) the bill wishes to put out his money at 4 per cent. per annum; in this case he ought to pay for the bill just so much Principal as will Amount to $5050 in 3 months, at 4 per cent.; he ought to pay in fact $5000. In such a case $5000 is the **present value**, $50 is the (*true*) **discount**.

[In **practice**, however, when a Broker or Banker discounts a bill for $5050, for 3 months at 4 per cent., he deducts $\frac{1}{4}$ of 4 per cent. of $5050, that is $50.50 as discount, thereby actually getting more than 4 per cent. for his money. The discount deducted by a Banker is called **Commercial** or **Bankers'** Discount.

In Arithmetic, 'Discount' is always understood to mean *true* or *theoretical* discount and not *commercial* discount.]

A **Bill** for a certain sum at a certain time is a promise to pay that sum after the end of that interval.

The **Present Value** of a Bill is that sum of money which with its interest for the given time will just amount to the sum due.

Hence the Discount on the Sum due is the same as Interest on the Present Value for the same time, at the same rate.

Commercial or Bankers' Discount is greater than True Discount. The difference between them is equal to the Interest on the Discount.

For, since Amount = Present Value + (True) Discount;
therefore, Interest on Amt. = Int. on Pres. Val. + Int. on Discount.
That is, Bankers' Disc. = True Disc. + Int. on True Discount.

Example i. *Find the* Present Value *of a Bill for* $907.20 *due* 2 years *hence at* 4 per cent.

In 2 years at 4 per cent. $100 amounts to $(100 + 2 × 4), or $108; hence, the Present Value of $108 is $100.

[Thus the Present Value of $108 is found by multiplying $108 by the diminishing factor $\frac{100}{108}$.]

Therefore the Present Value of $907.20 is $\frac{100}{108}$ of $907.20,

that is $\$\frac{7560}{9}$, or $\underline{\$840.}$

Example ii. *Find the* Discount *on a Bill for* $2850.75 *due in* $1\frac{1}{2}$ years *at* $3\frac{1}{3}$ per cent.

Here, $100 amounts to $\$(100 + 1\frac{1}{2} \times 3\frac{1}{3})$, that is, to $105;
hence, the Discount on $105 is $5 [and the Discount on $1 is $\$\frac{5}{105}$].

Therefore the Discount on $2850.75 is $\frac{5}{105}$ of $2850.75,

that is, $\frac{1}{21}$ of $\$2850.75 = \frac{1}{7}$ of $\$950.25 = \$135.75$.

EXAMPLES. CXXX.

Find the Present Value of

1. $260 due 1 year hence at 4 per cent. per annum.
2. $3030 due 3 months hence at 4 per cent. per annum.
3. $4050 due 3 months hence at 5 per cent. per annum.
4. $1386 due $\frac{1}{5}$ of a year hence at 4 per cent. per annum.
5. $1446 due 1 month hence at 5 per cent. per annum.
6. $1465 due 2 years hence at 4 per cent. per annum.

Find the Discount on

7. $3908.10 for $2\frac{1}{2}$ years at 8 per cent. per annum.
8. $2604.50 for $1\frac{1}{2}$ years at 9 per cent. per annum.
9. $857.15 for 7 months at $3\frac{1}{2}$ per cent. per annum.
10. $5555.55 for 2 years at $2\frac{1}{2}$ per cent. per annum.
11. $100 due 3 years hence at 5 per cent.
12. $100 due 2 years hence at 6 per cent.

Example. Find to the nearest cent the Discount *on* $1234.30 *due* $1\frac{1}{4}$ years *hence at* $4\frac{1}{2}$ per cent.

Here, $100 amounts to $\$(100 + 1\frac{1}{4} \times 4\frac{1}{2})$, that is, to $\$105\frac{5}{8}$ in the given time at the given rate.

Hence, the Discount on $\$105\frac{5}{8}$ is $\$5\frac{5}{8}$.

Therefore the Discount on $1234.30 is $\dfrac{5\frac{5}{8}}{105\frac{5}{8}}$ of $1234.30;

that is, $\frac{45}{845}$ of $1234.30, or $\frac{9}{169}$ of $1234.30;
that is, $65.732, or $\underline{\$65.73.}$

EXAMPLES. CXXXI.

Find to the nearest cent the Present Value and the Discount on

1. $1000 due 3 years hence at $3\frac{1}{3}$ per cent. per annum.
2. $5000 due 4 years hence at 5 per cent. per annum.
3. $8750 due $1\frac{1}{2}$ years hence at $6\frac{2}{3}$ per cent. per annum.
4. $4721 due $1\frac{1}{3}$ years hence at 4 per cent. per annum.
5. $8120 due $\frac{3}{4}$ year hence at 4 per cent. per annum.
6. $14275 due $1\frac{1}{3}$ years hence at $3\frac{3}{4}$ per cent. per annum.

189. The purchaser of a bill may **sell** it again at any time before it is due; when he does so, he is said to **get it discounted.**

There is a *custom* in business, which has the force of law, by which 3 days (called **days of grace**) beyond the day on which the bill becomes nominally due are always allowed before the amount of the bill must be paid.

In working an example in which the exact number of days is given the 3 days of grace should be added.

Example i. *Find the* Present Value *of a Bill for* $7051.75 *drawn on* March 21st *at* 7 months, *discounted on* August 12th *at* $2\frac{1}{2}$ per cent. per annum.

In this case a merchant on March 21st accepted (*i.e.* signed) a bill, by which he undertook to pay $7051.75 at the end of 7 months. He sells it, and it may have had many owners; its history, however, does not concern the question; what we have to find is, *the value of the bill on* August 12th, *interest on money being reckoned at* $2\frac{1}{2}$ per cent. per annum.

The bill having been drawn on March 21st at 7 months is nominally due on Oct. 21st, but, adding the 3 days of grace, is not actually due until Oct. 24th. From Aug. 12th to Oct. 24th [including one of those days, Art. 182] there are 73 days; that is, $\frac{1}{5}$ of a year. The Interest on $100 for $\frac{1}{5}$ of a year at $2\frac{1}{2}$ per cent. is $\$\frac{1}{2}$.

Hence the Present Value of $\$100\frac{1}{2}$ is $100,

$\left[\text{therefore the Present Value of \$1 is } \$\frac{100}{100\frac{1}{2}},\right]$

and the Present Value of $7051.75 is $\frac{200}{201}$ of $7051.75,

[that is, $1 - \frac{1}{201}$ of $7051.75]; that is, $\underline{\$7016.67}$.

Example ii. *The bill of Example* i. *is discounted again on* August 22nd *at* $2\frac{1}{2}$ per cent.; *find the* Discount.

Here, the number of days from Aug. 22nd to Oct. 21st + 3 is 63;
the int. on $100 for 63 days at $2\frac{1}{2}$ per cent. is $\frac{63}{365}$ of $\$\frac{5}{2}$, or $\$\frac{63}{146}$.
Hence, the discount on $\$100\frac{63}{146}$ is $\$\frac{63}{146}$,
[therefore the discount on $1 is $\$\frac{63}{14663}$,]
and the discount on $7051.75 is $\frac{63}{14663}$ of $7051.75,
that is, $\underline{\$30.30}$ nearly.

EXAMPLES. CXXXII.

Find the Discount on each of the following bills to the nearest cent.

1. For $1692.42 drawn on March 27th at 8 months, discounted Sept. 18th at $2\frac{1}{2}$ per cent. per annum.

2. For $810.30 drawn Jan. 20th at 7 months, discounted March 30th at 5 per cent. per annum.

3. For $901.68 drawn Dec. 4th at 5 months, discounted Feb. 23rd at 10 per cent. per annum.

4. For $58593.15 drawn Sept. 10th at 8 months, discounted Oct. 6th at 7½ per cent. per annum.

5. For $9156.70 drawn June 20th, 1885, at 9 months, and discounted Aug. 16th at 5 per cent. per annum.

6. For $4992 drawn Feb. 10th, 1885, for 6 months, discounted 14th at 5 per cent. per annum.

Find the Present Value of each of the following bills:—

7. For $33813 drawn Dec. 31st, 1883, at 4 months, discounted Feb. 20th at 10 per cent. per annum.

8. For $14724.75 drawn May 10th, 1884, at 12 months, discounted April 3rd, 1885, at 9⅛ per cent. per annum.

9. For $3350.25 drawn June 1st, 1884, at 12 months, discounted March 23rd, 1885, at 6¼ per cent. per annum.

10. For $368825 drawn May 8th at 7 months, discounted Sept. 17th at 4½ per cent. per annum.

11. For $3528 drawn Sept. 28th, 1884, at 11 months, discounted Nov. 6th, 1884, at 5 per cent. per annum.

12. The same bill as in Question 11 discounted again on April 7th, 1885, at 5 per cent. per annum.

13. The bill in Question 11 was discounted again on May 1st at 4½ per cent. per annum; find the present value (to the nearest cent).

14. For $1000 drawn Jan. 11th at 5 months, discounted March 1st at 6 per cent. (to the nearest cent).

15. For $875 drawn April 1st at 6 months, discounted May 27th at 6 per cent. (to the nearest cent).

MISCELLANEOUS EXAMPLES. CXXXIII.

1. Find the difference between the Interest and the Discount on $100 for 1 year at 4 per cent.

2. The difference between the Simple Interest and the Discount on a sum of money for 6 months at 4 per cent. is $10.50; what is the sum?

3. Find the difference between the Interest and Discount on $7299 due ½ year hence at 5 per cent.

4. The difference between the Simple Interest and the Discount on a bill due 9 months hence at 4 per cent. per annum is $15. What is the amount of the bill?

5. Find the Present Value of a bill for $83371.12 due 3 years hence at 6 per cent. per annum Compound Interest.

6. Find the Discount on a bill for $46824 due 3 years hence at 4 per cent. Compound Interest.

7. Find to the nearest cent the difference between the Compound Interest on $10,000 for 3 years at 4 per cent. and the Discount on $10,000 due 3 years hence at 4 per cent. per annum Compound Interest.

8. Find the difference between the amount at Compound Interest on $100 for 2 years at 4 per cent. and the Present Value of $100 due 2 years hence at 4 per cent. per annum Compound Interest.

9. $100 is invested at the beginning of each of 5 years at 5 per cent. Compound Interest; find what the total will amount to at the end of the fifth year.

10. Find the Present Value of a Fellowship of $1200 per annum for 6 years at the date of the first payment, reckoning Simple Interest at 5 per cent.

11. If the Present Value of $3171.70 due $2\frac{1}{2}$ years hence is $2758, what is the Present Value of $8314.50 due in 5 years?

12. Show that a bill broker who deducts as Discount 5 per cent. of the amount of a bill due in 1 year gets $5\frac{5}{19}$ per cent. per annum for his money.

13. Show that a broker who deducts as Discount 4 per cent. of the amount of a bill due in 10 months gets 5 per cent. per annum for his money.

14. Show that the difference between the Interest and the true Discount on a given sum at a given rate for a given time is equal to the Interest on the Discount.

15. A legacy remained unclaimed for 4 years; at the end of that time it amounted to $29172.15 at 5 per cent. per annum Compound Interest; what was the original legacy?

16. What advantage does a man gain, who owing $1000 which ought to be paid at once, is allowed to cancel his debt by four equal quarterly payments of $250 extending over the next twelve months; money being worth 5 per cent. per annum Simple Interest?

190. It must be noticed that *True* Discount **is never used** in business. The following example will illustrate the usual practice.

Example. A bill for $1000 *at* 90 *days' sight has been presented for acceptance on* February 10, 1888, *and on* March 10 *is discounted at* 5 per cent. per annum; *how much will the discounter pay for the bill?*

Explanation. A merchant *A* in New York orders goods to the amount of $1000 from a merchant *B* in London, and authorises *B* '*to draw upon him at* 90 *days' sight.*' *B* accordingly draws up a 'bill' for $1000 payable in 90 days after *A* has seen and accepted it. *B* now sells this bill for cash to *C*, a banker in London, at a rate of exchange to be agreed upon between them.*

C forthwith despatches the bill by the first and fastest mail steamer to his agent in America (sending also two duplicates, called the Second and Third of Exchange, by other steamers in case of accident). On its arrival in America, *C*'s agents present the bill to *A* for acceptance. The bill is then said to be *sighted*. (That is, *C*'s agents send the bill to *A*'s house of business and call for it again the next day, when) if the business is all in due form, the bill will be returned to *C*'s agent *accepted* [that is, *A* signs it and writes on it the date at which it *matures* (*i.e.* becomes due) and the place at which he is prepared to *meet* (*i.e.* pay) it (generally at *A*'s bankers' or his own place of business)]. This bill having been presented to *A* for acceptance on February 10, 1888, must accordingly be met in (90 + 3) days. [*Three days of grace* are allowed all over the world.]

This bill may now be used in business and passed from hand to hand. Suppose, for instance, the holder of it wishes to obtain cash for it on March 10; he goes to a bill broker who *discounts* it (*i.e.* pays him cash, making, however, a reduction from the $1000; which reduction is called Discount). In the above example the discounter discounts at 5 per cent. per annum. That is to say, the number of days between March 10 and the end of the 93 days is ascertained to be (including February 29) 64 days, and the **Simple Interest** for 64 days on $1000 at 5 per cent. per annum is ascertained from a book of tables; this amount is deducted from the $1000, and the discounter pays the holder of the bill the remainder.

Process. From February 10, 1888, we have in . . . .

Feb., 19 days.
Mar., 31 days.
April, 30 days.
May, 10 days.
3 days of grace.
—
93 total.

Hence the bill is due on May 13.

From March 10 to May 13, including one of them, the number of days is 64.

We have therefore to find the **Simple Interest** on $1000 for 64 days at 5 per cent.

---

* It will be seen that *B* has now received his money for the goods, and has nothing more to do with the transaction. The **Bill of Lading**, which is the receipt for the goods from the captain of the ship which carries the goods, as well as the policy of insurance of the goods, is often attached to the bill of exchange as a security to the holder. In this case the bill is called a *Documentary* bill to distinguish it from a *Clean* bill which has no document attached. The captain of the ship will not *deliver* the goods to any one who does not hold the bill of lading. In the case of a clean bill *B* could send the bill of lading direct to *A*. In the case of a documentary bill *A* could not get the goods until he had purchased the bill, *i.e.* paid for the goods.

Thus the commercial discount is* $\frac{64}{365} \times \frac{5}{100}$ of \$1000 = \$$\frac{640}{73}$ = \$8.77 nearly.

Therefore the price the discounter would pay for the bill is \$1000 − \$8.77, that is \$991.23.

EXAMPLES. CXXXIV.

1. Find the discount at 4 per cent. on a bill for \$3700 due on May 15th 1888 which is discounted on January 30th 1888.

2. Find the discount at 4 per cent. on a bill for \$5470 due in 100 days from November 20th 1888 which is discounted on January 3d 1889.

3. A bill for \$1350 at 30 days' sight has been presented for acceptance on January 11th 1887 and on January 20th is discounted at 4 per cent. per annum ; how much cash is paid for the bill ?

4. A bill for \$2000 at 50 days' sight has been presented for acceptance on May 11th 1888 and was retired† under discount on June 20th at 5 per cent. per annum ; how much was the discount ?

5. A bill for \$1500 at 100 days' sight matures on November 20th 1888 ; it is retired under discount at 4 per cent. per annum on August 30th 1888 ; what will the trader pay for the bill ?

6. A bill for \$2700 at 80 days' sight is accepted on August 30th 1888 ; it is discounted at 4½ per cent. per annum on October 20th 1888 ; how much discount is deducted ?

FOREIGN EXCHANGE.

191. Commercial transactions between different countries are understood to be carried on through the medium of **gold**.

[The medium of commerce in the case of India, Russia, Japan, Mexico, and Peru, is not gold, but *silver*.]

There are three ways in which the payment of money due by a merchant *A* in Paris to a merchant *B* in London might be made ; (i) he might send gold coins (**specie**) ; (ii) he might send gold in bars (**bullion**) ; (iii) he might send a **bill of exchange.** This last is the method adopted in practice.

When a merchant *A* in London sends goods to a merchant *B* in Paris, *A draws a bill* upon *B* for the value of the goods, say £100.

---

* It might be said that the fraction should be $\frac{64}{366}$ths of a year, but the tables would give $\frac{64}{365}$, as there are no special tables for leap year.

† A bill is said to be *retired* when the acceptor pays it under discount, and payment is acknowledged on the back ; the whole affair then comes to an end.

This bill *B* accepts either personally or through his agent in London. The accepting a bill consists in *B* (or his agent) signing the paper on which the bill is drawn, and thereby undertaking to pay in Paris the amount of the bill to whoever shall bring the bill up (*present it*) for payment. [See pages 202, 203.]

Now *A* wants the money in London; but there are other merchants who want to send money to Paris; therefore *A* sells this bill to another merchant *C* (or to a banker who deals in these bills, and the banker sells it to *C*); *C* sends it to his creditor *D* in Paris who presents it to *B*, and receives from *B* money due to him from *C*.

In this way goods are exchanged and trade is carried on without the necessity of sending *gold* backwards and forwards.

But it must be noticed that *A* receives English money (sovereigns) for the bill, and *D* receives French money (25 f.) for the same bill.

Now it is known that the gold in 1000 sovereigns is worth exactly the same as that in 25170 gold francs. Hence £1 is worth 25.17 francs. And this value 25.17 f. is called the par of exchange of the sovereign.

It may, however, happen that there is a great demand in London for bills on Paris; in that case a merchant may have to pay more than £1 for a bill for 25.17 francs payable in Paris; in fact, as in all other things, the price of bills of exchange varies with the supply and demand.

The price paid in one country for bills of exchange on another is called the *Course of Exchange*.

The following appears in the *New York Tribune* of May 13, 1890:— French francs at 5.18 3-4 and 5.16 1-4; reichmarks at 95 1-8 and 95 5-8, and guilders at 40 1-8 and 40 3-8.

In the quotations on Paris, the franc is the unit, and we are given the number of francs and centimes to the dollar; so that from this a merchant having to make payments in Paris would know that in order to buy bills from the bankers on Paris, he would have had to pay on May 13th, 1890, $1 for each $5.16\frac{1}{4}$ francs payable in Paris.

'Reichmarks at $95\frac{5}{8}$.' The quotation gives the number of cents in 4 reichmarks, so that for 4 marks, $95\frac{5}{8}$ cents would have to be paid.

'Guilders at $40\frac{3}{8}$.' Quotations for Amsterdam give the number of cents to the guilder.

The two different quotations give the rates for bills payable at sight and after 60 days, the cost of the latter being *lower* (compare § 188). Thus the course of exchange being as above, bills on Paris payable in 60 days are paid for at the rate of $1 for $5.18\frac{3}{4}$ francs, bills on Hamburg payable in 60 days are paid for at the rate of $95\frac{1}{8}$ cents for 4 marks, and bills on Amsterdam at the rate of $40\frac{1}{8}$ cents to the guilder.

Example i. *A merchant wishes to purchase a bill on Paris for* 7740 francs; *how much will he have to pay when the course of exchange on Paris is* 5.16?

For a bill for 5.16 fr. he pays $1.

Therefore for a bill for 7740 fr. he pays $\frac{7740}{5.16}$ times $1,

that is, $\frac{774000}{516}$ times $1, or $1500.

Example ii. *Find the cost in New York of a bill for* 7632 marks, *to be paid in Berlin, the course of exchange being* 95.

A bill for 4 marks costs 95 cents.

Therefore a bill for 7632 marks costs $\frac{7632}{4}$ times 95 cents,

that is, 1908 times 95 cents, or $1812.60.

Example iii. *What is the face of a bill on London for which* $6500 *was paid in New York, at* 4.86 (dollars *for* £1)?

[The *face* of a bill is the amount stated on it.]

Here 4.86 dollars will buy a bill for £1.

Therefore 6500 dollars will buy a bill for $\frac{6500}{4.86}$ times £1,

that is, £1337. 8*s*. 11½*d*.

192. When the standard coins of two countries are gold, the rate of exchange will not vary very greatly from year to year.

In no case would the difference between the exchange value and the real value of a gold 25 franc piece (*e.g.*) exceed the cost of transporting the bullion from the one country to the other.

When the standard coin of the one country is *silver*, and that of the other country is *gold*, then the rate of exchange will not only depend on the balance of trade, but on the relative value of silver and gold.

The relative value of silver compared with that of gold has been (1888) steadily growing less for some years.

One effect of this is, that while the purchasing power of the rupee *in India* has not necessarily changed, its power of purchasing bills for £ *s*. *d*. in London has steadily diminished; so that a lac of rupees *when sent over to England* is much less valuable than it used to be.

Foreign Money.

		Approx. Val. in Cents.
Austria	100 kreuzers = 1 *florin*	40.2
Belgium	100 centimes = 1 *franc*	19.3
British Colonies	£ *s*. *d*.	
Canada	100 cents = 1 *dollar*	100

		Approx. Val. in Cents.
Denmark	100 öre = 1 *krone*	26.8
England, etc.	*Pound Sterling*	$4.865
France	100 centimes = 1 *franc*	19.3
Germany	100 pfennig = 1 *mark*	23.8
Greece	100 Lepta = 1 *drachma* (franc)	19.3
Holland	100 cents = 1 *florin*	40.2
India	192 pies = 16 annas = 1 *rupee*	34.4*
	100,000 rupees = 1 lac	
Italy	100 centesimi = 1 *lira* (franc)	19.3
Japan	100 sen = 1 *yen*	99.7*
Mexico	*dollar*	90.9*
Norway	100 öre = 1 *krone*	26.8
Portugal	1000 reis = 10 teston = 1 *milrea*	108
Russia	100 kopecks = 1 *rouble*	66.9*
Spain	100 centimos = 1 *peseta* (franc)	19.0
Sweden	100 öre = 1 *krone*	26.8
Turkey	40 paras = 1 *piastre*	4.4
	100 piastres = 1 pound Turkish (1£T)	

* In these countries the standard coins are *silver*.

EXAMPLES. CXXXV.

1. What is the cost in Paris of a bill for £126 at 25.20 fr. (per £1)?

2. What is the cost in London of a bill on New York for $7100 at $4.875 for £1?

3. What is the cost in New York of a bill for £147 at $4.875 to the £1?

4. The course of Exchange on Brussels being 5.17, how much will a merchant in New York have to pay for goods in Brussels which cost 1142.57 francs?

5. A merchant in London orders goods in New York to the amount of $3000; how much will the bills necessary for payment cost him at $4.80 to the £1?

6. A merchant in New York orders goods in Geneva to the value of 2080 francs, find the cost of the necessary bills at 5.16 fr. to the dollar.

7. A merchant in London paid £430 for a bill on Vienna; what was (the number of florins on) the face of the bill if the course of Exchange was 12.90 (florins to the £1)?

8. What is the face of a bill on Paris for which I pay $126 in New York, the course of Exchange being 5.17?

9. What is the face of a bill on London for which I pay $975 in New York, the course of Exchange being 4.875?

10. What is the face of a bill on New York for which I pay £121. 12*s.* in London, the course of Exchange being $4.864?

11. What is the face of a bill on Dublin which cost $810 in Chicago, at $4.86?

12. Find the face of a bill on Vienna costing £323 in London, the course of Exchange being 12.92.

CHAPTER XIII

PROPORTION

193. *DEF.* Four quantities are said to be **in proportion** when the first quantity is the same fraction of the second which the third is of the fourth.

As in Art. 119 we may state the same thing in several different ways.

Four quantities are said to be in proportion when

(i) The first quantity divided by the second = the third divided by the fourth.

(ii) The ratio of the first quantity to the second = the ratio of the third quantity to the fourth.

And so on.

Example i. 3 ft., 10 ft., 75 c., and \$2.50 are in proportion; for

$$\frac{3 \text{ ft.}}{10 \text{ ft.}} = \frac{3}{10}, \text{ and } \frac{75 \text{ c.}}{\$2.50} = \frac{3 \times 25}{10 \times 25} = \frac{3}{10}.$$

Example ii. \$15, \$25, 5 cwt., and 8¼ cwt. are *not* in proportion; for

$$\frac{\$15}{\$25} = \frac{3}{5}, \text{ and } \frac{5 \text{ cwt.}}{8\frac{1}{4} \text{ cwt.}} = \frac{20}{33}.$$

Example iii. *Find a* fourth (*quantity*) proportional *to* 3 lbs., 5 lbs., *and* \$12.

$$\frac{\text{the fourth proportional}}{\$12} = \frac{5 \text{ lbs.}}{3 \text{ lbs.}}.$$

Thus the required fourth proportional = $\frac{5}{3}$ of \$12, that is, \$20.

194. *DEF.* Four quantities are said to be in **inverse proportion** when the fraction which the first quantity is of the second is the inverse of the fraction which the third is of the fourth.

Example. 4 men, 7 men, \$3.50, and \$2 are in inverse proportion.

EXAMPLES. CXXXVI.

Are the following in proportion ?—

1. 7, 5, 21, 15. **2.** 8, 9, 40, 44. **3.** 3 yds. 1 ft., 1 ft. 2 in., 15, 7.

Find a fourth (*quantity or number*) proportional to

4. 6, 27, and 9. **5.** 8, 14, and 20. **6.** 4 yds., 1 ft. 6 in., and $2.

7. 15 acres, 32 acres, and 20 men.

8. 8 horses, 11 horses, and 15 acres.

9. 65 miles, 14 miles, and 13 hours. **10.** $840, $480, and 21 lbs.

Find a fourth quantity in inverse proportion with

11. 7 men, 1 man, 3 days. **12.** 5 knots, 1 knot, 75 days.

13. 5 horses, 7 horses, 7 days. **14.** 4 miles, 7 miles, 14 hrs.

15. 35 miles, 49 miles, 7 hrs. **16.** 2500 men, 2200 men, 11 months.

195. When one quantity is a certain fixed fraction of another quantity, any two pairs of values of these quantities are in proportion.

Example. The circumference *of a circle is a certain fraction of its* diameter ; *the* circumference *of a circle* 1 ft. *in* diameter *is* 3·1416 ft.; *find the* circumference *of a circle* 5 yds. *in* diameter.

Then, 1 ft., 3·1416 ft., 5 yards and the required circumference are in proportion ; hence, $\frac{\text{req. circumference}}{5 \text{ yds.}} = \frac{3{\cdot}1416 \text{ ft.}}{1 \text{ ft.}}$;

or, req. circumference $= 3{\cdot}1416 \times 5$ yds. $= \underline{15{\cdot}708 \text{ yds.}}$

EXAMPLES. CXXXVII.

1. Two quantities are in the proportion of 8 to 9 ; the first is $2 ; what is the other ?

2. The weights of equal volumes of iron and water are 7·112 : 1 ; a cub. ft. of water weighs 1000 oz.; what does a cub. ft. of iron weigh ?

3. Find the circumference of a circle 3 ft. in diameter. [Art. 152.]

4. Find the radius of a circle 12 ft. in circumference.

5. The weights of equal volumes of hydrogen and oxygen are as ·0693 : 1·107 ; how many cubic feet of hydrogen will weigh as much as 4·2 cub. ft. of oxygen ?

6. The area of a circle is proportional to the square of its radius ; if $5880 is given for a circular field whose radius is 700 ft., what should be given for a circular field whose radius is 125 ft. ?

7. If a circular plate of gold whose radius is 3 inches cost $1125, what would be the radius of a plate of the same gold of the same thickness worth $405 ?

8. The value of a diamond per carat is proportional to its weight; if a diamond weighing 4 carats is worth $60, what is the value of a diamond which weighs 12 carats?

196. When two quantities *vary as* one another [Art. 171], any two pairs of values of those quantities are *in proportion.* [For Examples, see Ex. CXII.]

Example. If 13 cwt. *cost* $42.25, *what will* 50 cwt. *cost?*

Since the price varies as the no. of cwt., the required cost is $\frac{50}{13}$ of $42.25; for, $\frac{\text{the required cost}}{\$42.25}=\frac{50 \text{ cwt.}}{13 \text{ cwt.}}$.

197. Also when two quantities *vary inversely as* one another [Art. 173], any two pairs of those quantities are *in inverse proportion.* [Ex. CXIII.]

Example. If 12 men *mow a field in* 22 days, *how long will* 33 men *take to reap the same field?*

Since the no. of men varies inversely as the no. of days, the required time is $\frac{12}{33}$ of 22 days; for, $\frac{\text{the required time}}{22 \text{ days}}=\frac{12 \text{ men}}{33 \text{ men}}$.

PROPORTIONAL PARTS.

198. A quantity is said to be divided into parts proportional to given numbers, when the first part divided by the first number = the second part divided by the second number, and so on.

Example i. *Divide* $20 *into two* parts proportional *to* 7 *and* 3.

We must find a unit such that (7 times the unit + 3 times the unit) make up $20; then, 7 units and 3 units are the required parts.

For the result of dividing 7 units by 7 = the result of dividing 3 units by 3.

In this case (7 + 3) times the required unit = $20, therefore the unit is $2 and the parts are 7 times $2 and 3 times $2.

Thus the required parts are $14 and $6.

Example ii. *Divide* $156 *into* parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, *and* $\frac{1}{4}$.

Now, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}=\frac{6}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$ respectively;
and parts which are equal when divided by $\frac{6}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$ respectively, must also be equal when divided by 6, 4, and 3 respectively; and conversely.

So that we may take the numbers 6, 4, and 3 instead of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Hence, we find a unit such that (6 + 4 + 3) units = \$156.

Thus the unit is $\frac{1}{13}$ of \$156 or \$12; and the parts are 6 times \$12, 4 times \$12, and 3 times \$12; that is, \$72, \$48, and \$36 respectively.

Example iii. *A prize was divided between* A, B, *and* C *in the* proportion *of* $\frac{3}{4}$, $\frac{14}{15}$, and $\frac{5}{8}$; A *received* \$900; *what was the prize?*

Here, $\frac{3}{4}$, $\frac{14}{15}$, and $\frac{5}{8} = \frac{90}{120}$, $\frac{112}{120}$, and $\frac{75}{120}$ respectively; hence we use the numbers 90, 112, and 75 instead of the fractions $\frac{3}{4}$, $\frac{14}{15}$, and $\frac{5}{8}$.

Hence we must find a unit such that 90 times the unit = \$900.

This unit is \$10; therefore the required amount is (90 + 112 + 75) times \$10; that is \$2770.

Example iv. *A man leaves one-third of his property to his widow and one-half to his son, who gets* \$286 *more than either of his four sisters, between whom the residue is divided equally; what is the* whole sum *left?*

Taking the whole sum left as the unit, the four sisters get $(1 - \frac{1}{3} - \frac{1}{2})$ or, $\frac{1}{6}$ of the unit between them; each sister gets $\frac{1}{4}$ of $\frac{1}{6}$, or, $\frac{1}{24}$ of the unit.

The difference between $\frac{1}{2}$ and $\frac{1}{24}$ of the unit is $\frac{11}{24}$ of the unit;

the difference between the son's and the daughter's share is \$286.

Hence $\frac{11}{24}$ of the unit is \$286.

So that the unit is $\frac{24}{11}$ of \$286; that is, \$24 × 26, or, \$624.

Example v. *How many* English florins, half-crowns, *and* shillings *of which the* **numbers** *are proportional to* $1\frac{1}{2}$, $3\frac{3}{10}$, and $2\frac{1}{2}$ are *together worth* £55? [A florin is 2 shillings, a crown is 5 shillings.]

Instead of $1\frac{1}{2}$, $3\frac{3}{10}$, $2\frac{1}{2}$, or $\frac{15}{10}$, $\frac{33}{10}$, $\frac{25}{10}$, we use 15, 33, 25; hence the **values** of the three groups of coins (florins, half-crowns, shillings), with *sixpence* as unit, are as $4 \times 15 : 5 \times 33 : 2 \times 25$; that is, as $12 : 33 : 10$.

Hence, the groups of coins are worth $\frac{12}{55}$ of £55, $\frac{33}{55}$ of £55, and $\frac{10}{55}$ of £55, or £12, £33, and £10 respectively.

Therefore there are 120 florins, 264 half-crowns, 200 shillings.

EXAMPLES. CXXXVIII.

Divide each of the following:—

1. 143 in the proportion of 2, 5, 4.
2. \$169 in the proportion of 1, 2, 3, 3, 4.
3. \$299 in the proportion of 1, 2, 3, 4.
4. \$253 in the proportion of 6, 7, 10.
5. \$1728 in the proportion of 73 and 71.
6. \$720 in the proportion of 4, 5, and 6.
7. 19 tons 10 cwt. in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

8. \$24.50 in the proportion of $\frac{1}{4}$, $\frac{2}{5}$, $\frac{1}{6}$.

9. \$1064 into parts proportional to 2, $2\frac{1}{4}$, $2\frac{2}{5}$.

10. 180 lbs. into parts proportional to 3·3, ·7, ·5.

11. \$43 into parts proportional to 10, $2\frac{1}{2}$, 1, $\frac{1}{2}$, $\frac{1}{3}$.

12. \$6336 into parts proportional to $\frac{1}{2}$, $\frac{3}{5}$, and ·7.

13. Gunpowder is composed of nitre, charcoal, and sulphur in the proportion of 33, 7, and 5. How many lbs. of sulphur are there in 180 lbs. of powder?

14. How many pounds of the above gunpowder can be made with 30 lbs. of sulphur, and how much nitre and charcoal is required?

15. How many pounds of the above gunpowder can be made with 112 lbs. of charcoal, and how much sulphur is required?

16. Divide \$96.50 into two parts, such that one of them is three-sevenths of the other.

17. Divide 27 cwt. 37 lbs. into two parts, one of which is two-fifths of the other.

18. Divide £13. 19*s.* 6*d.* into an equal number of half-guineas, crowns, half-crowns, florins, shillings, and sixpences. [One guinea = 21 shillings.]

19. Divide \$172 among *A*, *B*, and *C*, so that for every dollar given to *A*, *B* gets \$10 and *C* \$10.50.

20. Divide \$195 between *A*, *B*, and *C*, so that *A* gets twice as much as *B*, and *B* four times as much as *C*.

21. Divide \$195 between *A*, *B*, and *C*, so that *A* gets three times as much as *B*, and *B* three times as much as *C*.

22. Divide \$195 between *A*, *B*, and *C*, so that *A* gets four-sevenths of *B*'s share, and *C* gets one-half of *A*'s share.

23. Divide 1435 tons 6 cwt. 88 lbs. among 6 men, 5 women, and 6 children, so that each woman gets twice and each man three times the share of one child.

24. Divide \$11661.15 among 2 men, 3 women, and 5 children, so that each woman gets twice as much as a child, and each man as much as a woman and child together.

25. A certain sum is divided among *A*, *B*, and *C*, so that *A* gets three times as much as *B*, and *B* three times as much as *C*; *A* receives \$120 more than *B*; what is the sum divided?

26. A certain sum is divided between *A*, *B*, and *C*, so that *A* gets twice as much as *B*, and *B* five-halves of *C*'s share, which is \$857.46; what is the sum?

27. Divide \$432.90 amongst 5 men, so that their shares are in the proportion of the first five prime numbers.

28. Divide \$12.80 among 6 persons, so that the share of each (except the first) may be equal to the sum of the shares of all who come before.

29. Find the area of a field whose perimeter is a quarter of a mile, and whose length is eight-thirds of its breadth.

30. The sides of a triangle are proportional to 3, 4, and 5; the smallest side is 2 yds.; find the perimeter of the triangle.

31. The sides of a triangle are proportional to 1, $\frac{2}{3}$, and $\frac{3}{4}$, and the largest side is 1 yard; find the other two sides.

32. *A* and *B* are partners; the former furnished $840, and the latter $1080 capital; what should be their shares of $432 profit?

33. *A* and *B* enter into partnership; *A* furnishes $1500, and *B* $1875 capital; what should be *A*'s share in a loss of $270?

34. The gains of a partnership of *A*, *B*, and *C* for a year are $20515.02; *A* furnishes $8325, *B* $6660, and *C* $5550 capital; what profit does *A* receive?

35. *A* and *B* enter into partnership; *A* furnishes $2000 for 8 months, *B* $3000 for 4 months; what should *A*'s share be in $1750 profit, reckoning only simple interest on the capital?

36. *A*, *B*, and *C* form a partnership; *A* putting in $4500 for 9 months, *B* $6300 for 1 year, and *C* $10800 for 5 months; divide between them a loss of $6300.

37. A business was commenced on Jan. 1st, when *A* put in $1000; on Mar. 1st *B* put in $1200; July 1st *C* put in $1500; and Sept. 1st *D* put in $2000; divide a profit of $2050 at the end of the year, reckoning only simple interest on the capital.

38. *A*, *B*, and *C* hire a meadow for a year for $80; *A* puts in 80, *B* 120, and *C* 200 sheep respectively; what share of the rent should each pay?

39. *A* and *B* pay $49 for the rent of a field, *A* puts in 20 horses for 12 weeks, and *B* 25 horses for 10 weeks; divide the rent fairly between them.

40. *A*, *B*, *C*, and *D* engage in business with a joint capital of $22500; at the end of a certain time *A* receives $2000, *B* $2800.75, *C* $1685.25, and *D* $1014; how much capital did *D* put in?

41. *A*, *B*, and *C* purchase a farm for $12000, of which *C* pays $5000; they sell it so as to gain a certain sum, of which *A* takes $712.7 and *B* $1425.4; what is *C*'s share of the profit? (To the nearest cent.)

42. *A*, *B*, and *C* do a piece of work for $529, *C* does $\frac{1}{2}$ of the work, and the work done by *A* is to that done by *B* as 10 is to 13; what should *A* be paid?

43. *A* and *B* undertake a contract, *A* furnishes 5 men for 20 days, and in addition 6 men for 15 days; *B* furnishes 10 men for 12 days, and also 9 men for 20 days; *A* receives $1662.50; what should *B* receive?

44. Three men contribute $4600 between them, *A* gives $400 more than *B*, and *B* gives $300 more than *C*; divide a profit of $345 between them.

45. A steeple is 268 ft. high, and the height of the church is 12 ft. higher than ¾ of the height of that part of the steeple which is above the church ; what is the height of the church ?

46. A man left his property to *A*, *B*, and *C* ; to *A* he left $180 less than one-third of his property, to *B* one-fourth, to *C* the rest, which was $590 less than what *A* and *B* received ; what was the property ?

47. 310 coins consist of sovereigns, half-sovereigns, and florins ; the values of the sovereigns, half-sovereigns, and florins being as 5 : 3 : 2 ; how many florins are there.

48. £38. 4*s*. is made up of half-sovereigns, half-crowns, and threepenny-pieces ; there are twice as many half-crowns as half-sovereigns, and the half-sovereigns to the threepenny-pieces are as 3 is to 11 ; how many coins are there ?

49. A farm of 365 acres is divided, so that a man receives 3½ a., a woman 1½ a., and a child 1 a. ; there are 5 men to every 6 women ; and 2 children to each man ; how many men are there ?

50. If 3 men do as much work as 5 women, and 4 women as much as 5 children, divide a day's wage of $26.70 among 9 men, 11 women, and 12 children.

51. Divide $2450 among 2 men, 8 women, and 10 children for work done, on the supposition that 1 man does as much work as 3 women or 5 children.

52. *A*, *B*, and *C* are partners in a business, and their shares are in the proportion of ½, ⅓, and ¼ ; *A* withdraws half his capital at the end of 4 months, and after 8 months a profit of $1056 is divided ; what is *A*'s share ?

53. The areas of circles are proportional to squares on their radii ; divide a circle 1 ft. in diameter into three equal parts by concentric circles.

54. A circular plate of gold, 10 in. in diameter and 1 in. thick, is melted and formed into two other circular plates, each 1 in. thick, whose diameters are in the ratio of 3 to 4 ; find the diameters.

55. A circular plate of gold is treated as in Ex. 54, except that the new plates are ½ in. thick ; find their radii.

199. The following examples in '**Alligation**' may be solved by the method of proportional parts.

Example. In what proportion must a man mix teas at 48c. per lb. *and at* 68c. per lb. *to sell the mixture without profit at* 60c. per lb. ?

By selling at 60c. per lb.

$$\left.\begin{array}{l}\text{on 1 lb. of tea at 48c. he would gain 12c.}\\ \text{on 1 lb. of tea at 68c. he would lose 8c.}\end{array}\right\}\ \text{L.C.M. of 12 and 8 is 24 ;}$$

on 2 lbs. of tea at 48c. per lb. he would gain 2 × 12c., *i.e.* 24c. ;

on 3 lbs. of tea at 68c. per lb. he would lose 3 × 8c., *i.e.* 24c.

Thus, if with 2 lbs. at 48c. he mixes 3 lbs. at 68c., he will neither gain nor lose ; the proportion is, 2 parts to 3.

EXAMPLES. CXXXIX.

1. In what proportion should oats at 75c. per bushel be mixed with peas at $2 per bushel, that the mixture may be worth $1.25 per bushel?

2. In what proportion must teas at 56c. and at 88c. per lb. be mixed that the mixture may be worth 64c. per lb.?

3. Form a mixture worth 72c. per lb., using teas worth 68c. and 82c. respectively.

4. A grocer has 4 kinds of sugar worth $6.25, $12.50, $15.50, and $17.50 per cwt.; in what proportions may he mix them to obtain a compound worth $14.50 per cwt., using equal parts of the first three kinds?

5. A merchant has oats at $3, barley at $5, and wheat at $10 per sack; he forms two mixtures each worth $7 per sack; in the first are equal quantities of oats and barley, and in the second equal quantities of barley and wheat; find the proportions.

6. Coffee at $18.75, $22, and $27.50 per cwt., are mixed so as to form a mixture worth $24 per cwt.; show that this can be done in an unlimited number of ways; and find the proportion when equal quantities of the last two kinds are taken.

7. A dealer buys milk at 6½c. per quart, and after diluting it with water obtains as profit three-fifths of his outlay by retailing the mixture at 8c. per quart; what proportion of water does he add to the milk?

CHAPTER XIV

PROFIT AND LOSS

200. When a profit or loss is said to be so much per cent. it is always understood that the percentage is reckoned on the **cost price.**

It will be found convenient in some examples to imagine a unit such that the cost price is 100 units.

Example i. *Goods which cost* \$20 *are sold for* \$22 ; *find the profit* per cent.

Take a unit such that the cost price is 100 units.

Here, the cost price is \$20, the profit is \$2 ; hence the profit is $\frac{2}{20}$ of the cost price, that is, $\frac{2}{20}$ of 100 units, or 10 units.

Therefore the profit is at the rate of 10 per cent.

Example ii. *An article is sold for* \$1.15 *at a profit of* 15 per cent. ; *for what must it be sold so as to gain* 12 per cent. ?

Let the unit be such that the cost price is 100 units ; then the given selling price is 115 units and the required price is 112 units.

Thus the required price is $\frac{112}{115}$ of the given price ;

that is, $\frac{112}{115}$ of \$1.15 or \$1.12.

Example iii. *Having two* houses *of equal value, by selling one of them at* \$1275 *I find I lose* 15 per cent. ; *for what sum must I sell the remaining house so as to gain* 8 per cent. *on the whole transaction ?*

Let the unit be such that the cost price of each house is 100 units ; then, the first house was sold for 85 units.

The two houses together cost 200 units ; so that to gain 8 per cent. on the cost, the two together must be sold for 216 units.

Therefore the second house must be sold for 216 – 85, or, 131 units.

Hence, the selling prices of the first and of the second house are in the ratio of 85 to 131. Therefore the second house must be sold for $\frac{131}{85}$ of \$1275 or \$1965.

Example iv. *The rents on a certain estate are* 15 per cent. *higher now than they were* 10 years *ago ; the landlord allows* 20 per cent. *of* *the* total rental *for repairs, and pays his agent* 4 per cent. *on the remainder for collection; after paying* a tax of 1 cent in 30 cents *on his* gross income *his* net income *is* $6670; *what was the rental of the estate* 10 *years ago?*

The required *answer* is $\$6670 \times \frac{30}{29} \times \frac{100}{96} \times \frac{100}{80} \times \frac{100}{115}$;
that is $\$29 \times 23 \times 10 \times \frac{30}{29} \times \frac{25}{24} \times \frac{5}{4} \times \frac{20}{23}$ or $\underline{\$7812.50}$.

[The *mental* process is as follows: 30 of gross income gives 29 of net income; hence, gross income $= \frac{30}{29} \times$ net income; (whence the first multiplier); 100 of net rental gives 96 of gross income; 100 of gross rental gives 80 of net rental; and 100 of gross rental 10 years ago, gives 115 gross rental now; whence the other multipliers.]

EXAMPLES. CXL.

1. A knife which costs 90c. is sold for $1.08; find the profit per cent.

2. A book costs $2.94 and is sold for $3.60; what is the profit per cent.?

3. Goods which cost $35 are sold for $42; find the profit per cent.

4. Goods are sold for $189 at a profit of 5 per cent. on the cost price; find the cost price.

5. Goods are purchased at $125 per ton and sold at 5 cents per lb.; find the profit per cent.

6. Goods sold at 2 cents per lb. realise a profit of 12 per cent.; find the cost price per long ton.

7. Goods bought for $120 are sold at a profit of 10 per cent.; what did they fetch?

8. At what price must goods which were bought for $150 be sold if a loss of 18 per cent. is to be realised?

9. By selling goods for $336 I realise a profit of 12 per cent. on the cost price; what did they cost?

10. If I sell goods which cost me $33 at a profit of 27½ per cent., what do I gain on the transaction?

11. If I sell goods at $47.50, losing 5 per cent., how much do I lose?

12. If I sell so as to make a profit of 23 per cent., how much do I gain on goods which cost me $3.75?

13. I sell goods for $230 at a profit of 15 per cent.; what per cent. should I have gained if I had sold them for $210?

14. By selling sugar at 9 cents per lb. a dealer loses 10 per cent.; at what price per cwt. would he have gained 10 per cent.?

15. A watch is sold for $76 at a loss of 5 per cent.; what would have been gained or lost per cent. if it had been sold for $95?

16. By selling goods for $364 a man loses 9 per cent.; find the selling price at which he would have gained 7 per cent.

17. A dealer sold 250 sheep for $2143.75, gaining thereby 16⅔ per cent.; what was the cost price of a sheep?

18. By retailing tea at 5 cents per oz. a grocer gained 33⅓ per cent.; what profit per cent. does he gain by selling at 66 cents a lb.?

19. By selling goods for $8179.50 a dealer lost 9 per cent.; at what price would he have gained 10½ per cent.?

20. A trader gains 10 per cent. by selling wine at $11.88 per doz.; what will he gain per cent. if he raises the price to $13.68?

21. The cost price of a book is $5.40; the expense of sale is 8 per cent. upon this, and the net profit 25 per cent. on the sum of these two; find the selling price.

22. By selling a horse for $212.50 a person lost 7½ per cent.; find the gain or loss per cent. by selling it for $241.25.

23. By selling a horse for $315 I lose 16 per cent.; at what price must I sell to gain 20 per cent.?

24. In an examination *A* beat *B* by 12½ per cent. of the full marks 1200; if *B*'s marks were 590 what were *A*'s?

25. The population of a town at one census was 45720 and at the next it was 51020; find the increase per thousand.

26. The annual mortality of a certain town is 28 per 1000 of the estimated population at the beginning of the year; in 1 year there are 4900 deaths; what is the population of that town at the beginning of the year?

27. The annual birth-rate of a town is 33 per 1000 and the death-rate 2 per cent.; the population increased in a certain year by 520; find (i) the population at the beginning of the year, (ii) the number of births in the year, (iii) the number of deaths.

28. A merchant sells to a retailer at 60 per cent. profit, but the latter failing he only receives $1 in $8; find the merchant's gain or loss per cent.

29. By selling an article for $5.50 I gained ⅜ of my outlay; what should I have gained per cent. if I had sold it for $6.50?

30. A man sells an article at 5 per cent. profit; if he had bought it at 5 per cent. less than he did and sold it for 25 cents less he would have gained 10 per cent.; find the cost price.

31. By selling tea at 72 cents. per lb. I gain ⅛th of the cost price; what per cent. do I gain if I raise the price to 80 cents per lb.?

32. By selling needles at 90 cents per thousand ⅓ of the *selling* price is gained; find the selling price so as to gain 60 per cent. on the prime cost.

33. A man bought a horse and sold it at a loss of 10 per cent.; if he had received $45 more he would have gained 12½ per cent. ; what did the horse cost ?

34. A man buys a quantity of wine at $4 a gallon ; 20 per cent. of it is wasted ; at what price per gallon must he sell the remainder so as to gain 20 per cent. on his outlay ?

35. A tradesman proposes to retail his goods at 10 per cent. profit, but adulterates them by adding ¼ of their weight of an inferior article which costs him ⅘ths of the price of the better ; what profit per cent. does he make ? also in what proportion must he mix the two kinds so as to gain 20 per cent. ?

36. Three sacks contain equal quantities of wheat, barley, and oats, and a mixture is formed by taking 50 per cent. of the first sack, 30 per cent. of the second, and 70 per cent. of the third ; what percentage of the whole quantity is taken ?

37. I buy coffee at $40.80 per 112 lbs. and chicory at $12.00 per 112 lbs. ; at what price per lb. must I sell a mixture of 5 parts chicory to 7 of coffee so as to gain 16⅔ per cent. on my outlay ?

38. I mix 3 cwt. of tea which cost $72 a cwt. with 1 cwt. which costs $84 ; at what price per lb. shall I gain 4 per cent. ?

39. Of the gross receipts of a railway company 41 per cent. is taken for working expenses, 56 per cent. to give the shareholders a dividend of 3½ per cent. on their shares, and the remainder $15000 to the reserve fund ; find the amount of the shares.

40. How must I mix two sorts of tea which cost 72 cents and 84 cents per lb. respectively, to sell at 88 cents per lb. and gain 10 per cent. ?

41. A grocer must sell his stock of tea at 80 cents per lb. to gain 33⅓ per cent. ; by mixing it with an inferior tea in the proportion of 4 to 1 he gains 33⅓ per cent. by selling at $11.28 for 15 lbs. ; find the cost of the inferior tea.

42. I purchase 2276 lbs. of coffee at 42 cents per lb. and mix it with chicory at 9 cents. per lb. in the proportion of 3 parts weight of the former, and 2 of the latter ; at what price per lb. must I sell the mixture to gain 25 per cent. ?

43. A tradesman buys articles at $1.92 per gross (twelve dozen), he divides them into five equal lots which he sells at 48 cents a score (20), 36 cents a dozen, 30 cents a dozen, 4 cents each, and $3.60 a hundred respectively ; find the profit per cent.

44. A boy buys a number of oranges at 3 for 4 cents and a third of the number at 1 cent each ; at what rate must he sell them to gain 20 per cent. on his outlay ? Supposing his total profit to be $1.28, how many did he buy ?

45. I buy a number of apples at 1 cent each, and half the same number at 4 for 6 cents ; at what price must I sell to gain 20 per cent. on the cost ? if my total profit is $1.12, how many did I buy ?

CHAPTER XV

AVERAGE VALUE

201. *DEF.* The **average** value of a number of things of the same kind is their sum divided by the number of them.

Hence, the sum of a number of things is equal to their average value multiplied by the number of them.

Example i. *What is the* average age *of three men who are* 70, 76, *and* 79 years old *respectively?*

The sum of their ages is $(70+76+79)$, *i.e.* 225 years; and this divided by the number of them, *i.e.* by 3, gives the average value, which is 75 *years.*

Example ii. *The* average age *of a class of* 20 boys *is* 14·95 years; *the* average age *of the class is raised to* 15 years *by the coming of a new boy; how old is this new boy?*

The sum of the ages of all the boys is the average age multiplied by the number of the boys; hence, the sum of the ages of the 20 boys is 20 times 14·95 years; the sum of the ages of the 21 boys is 21 times 15 years. The age of the new boy is therefore $(15\times21-14{\cdot}95\times20)$ years, that is, 16 *years.*

EXAMPLES. CXLI.

1. What is the average age of five men who are 89, 83, 80, 79, and 74 years old respectively?

2. The population of three towns is 19375, 24125, and 15000 respectively; what is the average population of the towns?

3. The weights of a boat's crew are respectively 9 st. 5 lbs., 10 st., 10 st. 5 lbs., 11 st. 1 lb., 12 st., 11 st. 6 lbs., 11 st. 4 lbs., and 10 st. 7 lbs.; what is the average weight of the crew? [*N.B.*—1 stone = 14 lbs.]

4. The temperatures observed at different hours in a certain day were 40°, 42°, 45°, 50°, 55°, 58°, 57°, 54°, 52°, 51°, 49°, and 47°; find the mean value of these observations. [*N.B.—Mean value* is often used for *average value.*]

5. A train travels 1 ft. in the first second of its motion, 2 ft. in the next second, 3 ft. in the next, 4 ft. in the next, 5 ft. in the next, 6 ft. in the next; find the average distance which it travels per second.

6. A train travels 3 miles in the first quarter-of-an-hour of its motion, 6 miles in the next quarter-of-an-hour, 9 miles in the next, 10 in the next, 12 in the next, 13 in the next, 12 in the next, 7 in the next; find the average distance travelled in a quarter-of-an-hour. [Compare Art. 180.]

7. A train travels 1 mile in the first 5 min., 2 miles in the next 5 min., 3 miles in the next, 3½ in the next, 3¾ in the next, 4 in the next, 4¼ in the next, 4½ in the next, 1 mile in the next; find its average speed per hour.

8. The average weight of the crew in Question **3** is diminished by 2 lbs. by including the weight of the coxswain; find the weight of the coxswain.

9. The average weight of the 8 oarsmen in a boat is increased by 2 lbs. when one of the crew who weighs 12 stone is replaced by a fresh man; what is the weight of the new man?

10. A merchant buys 50 doz. of wine at $10.50 per doz., 40 doz. at $5.25 per doz., and 36 doz. at $9⅓ per doz.; what is the average cost per doz.?

11. A landowner lets 100 acres of land at $5 per acre, 150 acres at $6.25 per acre, and 200 acres at $7.50 per acre; what average rent does he get per acre?

12. In a factory a certain number of men receive $10 per week, 4 times as many receive $9 per week, and 10 times as many receive $5 per week; what is the average weekly wage per man?

13. The average age of the boys in a school of 200 boys is 14·75 years; what will be the average age if 10 new boys come whose average age is 12·3 years?

14. The average age of the boys in a school of 750 boys is 15·4 years; 50 boys leave, thereby diminishing the average of the school to 15·3; find the average age of those who leave.

15. The average age of the boys in a school of 120 boys is 16·5 years, the average age of the 12 masters is 30 years; what is the average age of the boys and masters together?

16. The merchant in Question **10** sells the 50 doz. of wine at a profit of 10 per cent., the 40 doz. at a profit of 20 per cent., and the 36 doz. at a profit of 25 per cent.; what is the average selling price per dozen?

17. The average temperature for Monday, Tuesday, and Wednesday was 53°; the average for Tuesday, Wednesday, and Thursday was 56°, that for Thursday being 60°; what was the temperature on Monday?

CHAPTER XVI

STOCKS

202. **Stock** is the name given to money lent to some Railway or other Company, or to our own or to some Foreign Government.

When the money is lent to a Foreign Government the rate of interest is fixed, but is different under different circumstances.

Illustration.—Suppose a Company formed for the purpose of constructing a new Railway. Those who are starting the Company, the **Promoters**, agree that the **Capital** shall be so much, say $5,000,000 divided into 50,000 **shares**, each worth $100. This capital is offered for *subscription*; that is, any one may join the Company and become a **shareholder** by buying a number of shares. Now suppose the Railway made, and in working order; if the line pays well, the profit to be divided among the shareholders will be perhaps $6 for every share; in this case the Company is said to pay 6 *per cent.*: should the line not pay well, it may possibly pay only 2 or 1 or even ½ per cent.

Any shareholder in this Company may at any time sell his shares; when he does so, he will be able to get more for a share if the line pays $6 per share than if it pays only $2 per share; hence the *value* of a share depends on very many circumstances and is constantly liable to change.

Example. I have $11700 Stock *in a Railway Company's Ordinary Stock which pays* 5 per cent. *and is at* 129; *what is my income and what is the value of the Stock?*

Here $11700 *Stock* is the name for 117 shares whose original value was $100. '5 per cent.' means $5 for each $100 share.

Thus my income is 117 times $5, that is, $\underline{\$585.}$

'At 129,' means that if I sell my shares I shall get $129 for each of them; so that my 117 shares are worth 117 times $129, that is, $\underline{\$15093 \text{ cash.}}$

Again, suppose the Company described above wishes to expend more capital on its Railway. It is not usual to issue more shares, like those issued at first; the Capital originally subscribed is called **Ordinary Stock** and is not increased; instead of increasing the Ordinary Stock, the Company borrows money at a fixed rate per cent., and agrees that the interest on this money shall be paid before any interest on the original shares is paid. Money so borrowed is called **Preference Stock.** The value of Preference Stock does not in general alter so much as that of Ordinary Stock, because the rate of interest is fixed; but it *does* alter, according to the general rate of interest which money will fetch; and sometimes also according to the ability of the Company to pay the interest.

EXAMPLES. CXLII.

Find the cash value of, and the income derived from, each of the six following:—

1. $1000 stock in the 3 per cents. at 98.
2. $3000 stock in the 4 per cents. at 112.
3. $1375 stock in the 5 per cents. at 135.
4. $4350 Canada 5 per cents. at 108.
5. $4760 Railway 4 per cent. Debentures at 120.
6. $17200 Consols (3 per cent.) at 99.

203. The most convenient method of working questions on Stocks is by using $100 *stock* as a *unit*, and giving it a name, say, a **share.***

Example i. *I have* $13500 4 per cent. Preference Stock *of a Railway Company at* 105; *find the alteration in my income if I transfer my money into the* Ordinary Stock *at* 62½ *paying* 2 per cent.

---

* It must be noticed (i) that a stockholder or a shareholder has no right to demand the repayment of his original loan; he has only the right to his *dividend* or annual interest; and this is what he sells when he is said to sell his stock or his shares.

(ii) That when the capital of a company or a loan is made up of shares, each of a certain value, they are often called $100 BONDS, or $50 BONDS, or $20 BONDS, according to their nominal value; and a bondholder cannot buy or sell a *part* of a bond; while when the capital or loan is called *Stock* it may be bought or sold in any quantity. For example, I may buy $422.50 *stock*, but I can only buy so many $20 *bonds*.

(iii) In what follows a *share* is always used as a *name* for $100 *stock*, or for *bonds whose total nominal value is* $100.

That 'I have \$13500 4 per cent. Preference Stock,' means that I have 135 '*shares*,' for each of which I receive \$4 a year interest.

My income is therefore 135 times \$4, that is, \$540 a year.

I sell these 135 shares for \$105 each, and receive \$14175 cash.

With this *cash* I buy shares which cost $62\frac{1}{2}$ each. I therefore buy

$$\frac{14175}{62\frac{1}{2}}=\frac{28350}{125}=226\cdot 8 \text{ shares, that is, \$22680 Ordinary Stock.}$$

For each of these $226\frac{4}{5}$ shares I receive \$2 a year.

My income is now $226\frac{4}{5}\times \$2=\$453.60$.

Thus my income is diminished by \$86.40 a year.

Example ii. *What is the price of a* 4 per cent. stock *when a person can buy* $\$13833\frac{1}{3}$ stock *for* \$12450?

Here $\$13833\frac{1}{3}=138\frac{1}{3}$ of \$100.

So that $\$13833\frac{1}{3}$ *stock* is $138\frac{1}{3}$ '*shares.*'

Now $138\frac{1}{3}$ shares cost \$12450 *cash.*

Therefore 1 share costs $\$\frac{12450}{138\frac{1}{3}}$, that is, \$90; in other words,

the price of the stock is '90 per cent,' or, '10 *below par.*' *

Example iii. *How much money must be invested in the* $3\frac{1}{2}$ per cents. *at* $99\frac{1}{2}$ *to produce an annual income of* \$392?

To produce $\$3\frac{1}{2}$ income it costs $\$99\frac{1}{2}$ *cash.*

Therefore to produce \$392 income it costs $\frac{392}{3\frac{1}{2}}\times \$99\frac{1}{2}$ *cash*;

that is, $\frac{\$392\times 2\times 198}{7\times 2}$, or, \$11144 *cash.*

When considering the meaning of a question on Stocks the student must carefully distinguish between *dollars stock* and *dollars cash.*

We may notice (i) That money which is to be *invested* must be *cash.* (ii) That money which is to be *sold out,* that is, sold out of a Company, must be *stock.*

* NOTE.—When the market value of \$100 stock is \$100 cash, the stock is said to be **at par**; when \$100 is at \$98, it is said to be *at a discount of* 2 *per cent.*, or 2 *below par;* when it is at \$102, it is said to be *at a premium of* 2 *per cent.*, or 2 *above par.*

EXAMPLES. CXLIII.

1. How much of the Russian 5 per cent. stock at $89\frac{1}{2}$ can I buy for \$44750, and what will be the income derived therefrom?

2. How much stock in the 4 per cent. Railway Debentures at 105½ can I buy for $73850 ?

3. How much money should I obtain by selling $228375 4 per cent. New South Wales Stock at 6¾ above par ?

4. What income should I derive from the purchase of $22750 4 per cent. Railway Debentures when they are at 2½ per cent. discount, and what will they cost me ?

5. What income is derived from investing $18500 in the 3¼ per cents. at 7½ below par, and how much stock is purchased ?

6. What income shall I derive from investing $17000 in the 4½ per cents. when they are 2 per cent. above par ?

7. How much must I invest in the 3½ per cents. at 99¾ to realise an income of $4200 a year ?

8. How much must be invested in the 4¼ per cents. at 119 to purchase an annual income of $500 a year ?

9. How much 4½ per cent. Debenture Stock at 114 must be bought to provide an annual income of $900 ? What will it cost ?

10. What must be invested in 4¾ per cent. stock at 95 to bring in an income of $2190 ?

204. The buying and selling of stock is usually done through an agent, called in this case a **Broker.**

A Broker charges so much (generally 12½c. or 25c.) for selling or buying $100 stock. When he charges 12½c. (=$⅛) he is said to charge ⅛ per cent., when 25c. (=$¼) he is said to charge ¼ per cent. The brokerage is of course *added* to the cost of a share ; and *subtracted* from the money obtained from the sale of a share.

Example i. *I transfer* $4500 stock *from the* 3 per cents. *at* $97⅝ *to Railway* 4 per cent. *debentures at* 89⅞ ; *find the change in my income* (brokerage being ⅛).

My income before the transfer was 45 × $3 = $135 ; I sell 45 shares for $(97⅝ − ⅛) each ; I receive therefore $45 × 97½ cash.

With this cash I buy shares which cost me $(89⅞ + ⅛), or $90 each ; so that I buy $\frac{45 \times 97\frac{1}{2}}{90}$ shares, that is, 48¾ shares.

Thus I buy $4875 debenture stock ; my new income is 48¾ × $4, or $195 yearly ; the change in my income is $60 a year, increase.

Example ii. *What percentage is obtained for money invested in the Funds* (3 per cent.) *when they are* 4¾ *below par ?*

When the Funds are 4¾ below par, each $100 stock costs $95¼ ; hence, for $95¼ *cash* I receive $3 interest. [p. 190, Ex. IV.]

And 3 on 95¼ is at the rate of $\frac{3}{95\frac{1}{4}}$ of 100 per cent.

That is, the interest on *money* is at the rate of $\frac{1200}{381}$ per cent.; that is, $\frac{400}{127}$, or $3\frac{19}{127}$ per cent., or 3·15 (nearly) per cent.

Example iii. *Which is the better investment, the* 3 per cents. *at* 89, *or the* 4 per cents. *at* 110?

Suppose we invest \$89 × 110 *cash* in each stock.

In the first case we shall buy 110 shares, the income being 110 × \$3, or \$330.

In the second case we shall buy 89 shares, the income being 89 × \$4, or \$356.

So that the 4 per cents. at 110 yield the best income.

EXAMPLES. CXLIV.

Find the change of income in the following cases:—

1. \$9500 transferred from the 3 per cents. at 99 to the Egyptian 7 per cents. at $47\frac{3}{8}$ (brokerage $\frac{1}{8}$).

2. \$5875 in the Funds (3 per cent.) at $100\frac{5}{8}$ transferred to a 6 per cent. stock at $140\frac{7}{8}$ (brokerage $\frac{1}{8}$).

3. \$71400 Ordinary Stock paying $5\frac{7}{8}$ per cent. at $137\frac{1}{8}$ transferred to Ordinary Stock paying $9\frac{1}{4}$ per cent. at $218\frac{3}{4}$ (brokerage $\frac{1}{4}$).

4. \$12000 in the Egyptian 7 per cent. Loan at $87\frac{5}{8}$ transferred to the Funds (3 per cent.) at $99\frac{7}{8}$ (brokerage $\frac{1}{8}$).

5. \$2550 in Ordinary Stock at $63\frac{1}{2}$ paying $1\frac{3}{8}$ per cent. transferred to 4 per cent. Debentures at $100\frac{7}{8}$ (brokerage $\frac{1}{8}$).

What percentage is paid in each of the seven following cases?—

6. Railway Stock at 136 paying $5\frac{7}{8}$ per cent.

7. Consols (3 per cent.) at $97\frac{1}{2}$.

8. Turkish 7 per cents. at 82.

9. Canada 5 per cents. at 105.

10. N. S. Wales Inscribed 4 per cent. stock at 109.

11. Metropolitan Board of Works $3\frac{1}{2}$ per cents. at $106\frac{1}{2}$.

12. Ordinary Stock paying $7\frac{1}{2}$ per cent. at 157.

13. Which is the better investment, 4 per cents. at 110 or $3\frac{1}{2}$ per cents. at par?

14. Which is the better investment, a stock paying $3\frac{1}{2}$ per cent. at 88 or one paying 4·4 per cent. at $96\frac{2}{3}$?

15. Which is the better investment, 3 per cent. stock at 81 or 5 per cent. stock at 27 above par?

16. Which is the better investment, 3 per cent. stock at 87 or 6 per cent. stock at 168?

17. Which would yield the greater return for an investment, Midland stock at 138¼ or Great Northern stock at 125½, the former paying 5⅝ per cent., the latter 5⅛ per cent. ?

18. Which is the more profitable stock to invest in, 3 per cents. at 83½ or 3½ per cents. at 3 per cent. discount ?

19. I invest $21390 in the 5 per cents. at 114⅞, and afterwards sell at 135⅛ and invest in the 4 per cents. at 92⅞ ; what is the change in my income (brokerage ⅛) ?

20. A man derives an income of $777 from an investment in the 3½ % stock at 67½ ; how much stock does he own and how much is it worth ? (*N.B.* % stands for '*per cent.*')

21. I invest $27225 in the 3 per cents. at 90⅝, and when they have risen to 91⅛ I sell out and invest in the 3½ per cents. at 97⅜ ; what is the change in my income (brokerage ⅛) ?

22. A person invests $2645 in railway shares at 114¾, the annual dividend being $5 on each share ; afterwards he sells out at 125¼ and invests in 3 per cent. Consols at 93⅝ (brokerage on railway stock ¼, on Consols ⅛). Find the change in income.

23. A person has $5000 stock in the 3 per cents. which he sells and reinvests in the 3½ per cents. at 87⅜ (brokerage ⅛) and increases his income by $5 ; find the price of the 3 per cents.

24. If I sell out $5610 3 % stock at 97⅝ and invest the proceeds half in 3½ % stock and half in 4 % stock, both at 116¾, find the change in my income (brokerage ⅛).

25. What amount of stock in the 3½ per cents. will produce the same income as $2492 in the 3 per cents. ?

26. By selling $1000 3 per cents. at 95 and reinvesting it I increase my income by $10 a year. If the dividend on the new shares is 8 per cent., what is the price of them ?

27. A man invested the same sum in two different stocks, a 3 per cent. stock at 87½ and a 3½ per cent. stock at 93¼, and his income from one is $212 more than from the other ; what sum has been invested in each ?

28. What sum must be invested in the 3 per cents. at 90 to amount in 23½ years at simple interest to $3210 cash ; the price of stock remaining unchanged ?

29. How many years sooner would the required amount in Question 28 be realised if the price of stock rose to 96 ?

30. A man has $4100 which he invests in 3 per cent. stock at 87 and 5 per cent. stock at 104 ; what sums must he invest in the respective stocks to make 3½ per cent. on the whole ?

31. A man has $3000 which he invests in 3 per cent. stock at 96 and 6 per cent. stock at 168 ; what must be the amount invested in each that he may make 3½ per cent. on the whole ?

32. How much 3 per cent. stock has been sold out at 88, if when the proceeds are invested in $100 shares at 24, paying 4 per cent., the owner's income is reduced by $10 a year ?

33. How much 3 per cent. stock has been sold out at 89 if the owner's income is increased by $12 a year by investing the proceeds in 4 per cent. stock at 92?

34. A man derives an income of $760 from money invested in the 3 per cent. Consols; he sells out when they are at 93 and buys $3\frac{1}{2}$ per cents. at 95; how much is his income increased by the transfer?

35. $7800 New Zealand 5 per cents. are sold at 104, and the proceeds invested in railway shares paying 6 per cent.; what is the price of the latter if the income derived is increased by $26?

36. A person sells some $25 shares in 3 per cents. at 24, and invests his money in 5 per cent. debenture at $$106\frac{3}{8}$; find how much per cent. his income is increased.

37. When the 3 per cents. are at $89\frac{1}{2}$, at what price would the $3\frac{1}{2}$ per cents. pay the same rate of interest?

38. The difference between the incomes derived from investing a certain sum in 4 per cent. stock at $112\frac{1}{2}$ and in $2\frac{1}{2}$ per cents. at 75 is $84; find the sum invested.

39. A man sells 46 $100 shares (4 per cents.) at $18\frac{3}{8}$ and invests the proceeds in $3\frac{1}{2}$ per cents., thereby increasing his annual income by $40; what was the price of the latter stock?

40. 3 per cent. stock bought at $90\frac{1}{8}$ is held for 6 months, at the end of which time the interest is paid; it is then sold at the same price at which it was bought, find the rate per cent. per annum of interest obtained for the money used (brokerage $\frac{1}{8}$).

41. I invest a certain sum in the 4 per cents. at $102\frac{1}{4}$ and after receiving a half-year's dividend I sell out at $105\frac{3}{8}$, gaining thereby $1036 in all; what sum did I invest (brokerage $\frac{1}{4}$)?

42. *A* and *B* invest each the same sum of money in 3 per cent. stock at 91 and $3\frac{1}{4}$ per cent. stock at $97\frac{1}{2}$, *A* purchasing an equal amount of each kind of stock, *B* dividing his money equally between the two stocks; which receives the larger interest? and if their incomes differ by 10 cents, how much money did they invest?

43. A man invests $8190 in the 3 per cents. at $90\frac{3}{4}$ and sells out part of the stock when they have risen to $93\frac{3}{4}$, and the remainder when they have fallen to $85\frac{1}{4}$, he lost $30 by the transaction; how much did he sell out at first? He invests the proceeds in $4\frac{1}{2}$ per cents. so as to cause an increase in his income of $50; at what *price* did he buy the $4\frac{1}{2}$ per cents. (brokerage $\frac{1}{4}$)?

CHAPTER XVII

SQUARE ROOT

205. *DEF.* The **square** of a number is the product of *two* factors each equal to the number.

The square of 2 is 2 × 2, that is, 4 ; the *square* of 7 is 7 × 7, that is, 49.

The square of a number is indicated by a small ^{2} placed thus, $2^2 = 2 \times 2$; $3^2 = 3 \times 3$.

A small figure used in this way is called an **index.**

The **cube** of a number is the product of *three* factors each equal to the number.

The cube of 5 is $5 \times 5 \times 5$, that is, 125.

The cube is denoted by the index ^{3}, thus

$$3^3 = 3 \times 3 \times 3.$$

The fourth, fifth, etc. **power** of a number is the product of four, five, etc. factors each equal to the number.

The square is the *second* power ; the cube is the *third* power ; the number itself is the *first* power of the number.

$$3^4 = 3 \times 3 \times 3 \times 3 = 81 \; ; \; 3^1 = 3.$$

The powers of 10 are 10^1, 10^2, 10^3, 10^4, etc. ;
that is 10, 100, 1000, 10000, etc. [see Art. 46.*]

---

[*] It will be noticed that when we multiply any power of 10 by another power of 10, we *add* the indices ; thus $10^5 \times 10^3 = 10^8$.

Let us suppose this rule of addition to be always true ; then

206. It is always possible, by multiplication, to find any required power of a number.

Example. $5^3 = 5 \times 5 \times 5 = 125$.

EXAMPLES. CXLV.

Find the value of the following powers :—

1. 3^2. **2.** 3^4. **3.** 3^5. **4.** 2^7. **5.** 5^4.
6. 7^3. **7.** 8^3. **8.** $(21)^2$. **9.** $(35)^2$. **10.** $(21)^3$.

Prove the following statements :—

11. $36^2 = 2^4 \times 3^4$. **12.** $25^2 = 5^4$. **13.** $8^3 = 2^9$.
14. $27^2 = 9^3 = 3^6$. **15.** $8^4 = 4^6 = 16^3 = 2^{12}$. **16.** $72^2 = 2^6 \times 3^4 = 6^4 \times 2^4$.

207. A number is called the *square* root of its square.

2 is the *square root of* 4 ; 12 is the *square root of* 144.

The *square root of* a number is indicated by the symbol $\surd$.

The '*square root of*' 4 is written $\sqrt{4}$.

208. A number is called the *cube root* of its cube.

2 is the cube root of 8 ; 12 is the cube root of 1728.

The '*cube root of* 8' is written $\sqrt[3]{8}$.

A number is called the *fourth root* of its fourth power.

2 is the fourth root of 16 ; 3 is the fourth root of 81.

The '*fourth root of* 16' is written thus $\sqrt[4]{(16)}$.

Similar definitions apply to *fifth root*, *sixth root*, etc.

EXAMPLES. CXLVI.

Prove the following statements :—

1. $\sqrt[3]{(64)} = 4$. **2.** $\sqrt[5]{(32)} = 2$. **3.** $(\sqrt[2]{4})^3 = 8$.
4. $\sqrt[3]{(27)} = \sqrt{9}$ **5.** $\{\sqrt[6]{(64)}\}^2 = \sqrt{16}$. **6.** $\{\sqrt[5]{(243)}\}^2 = \sqrt{(81)}$.
7. $\left(\dfrac{2\frac{1}{4} \div \frac{4}{9}}{1\frac{1}{80}}\right)^2 \times \sqrt{\left(\dfrac{\cdot\dot{3} \text{ of } 1\frac{1}{15}}{\cdot 5 \times \cdot 1} + \dfrac{\cdot 3\dot{5}}{\cdot 0\dot{8}}\right)} = \frac{2}{3} \text{ of } 5^3$.

$10^5 \times 10^0 = 10^5$, or the *factor* 10^0 has no effect ; but the factor 1 has no effect ; so that $10^0 = 1$.

Again, $10^5 \times 10^{-1} = 10^4$ and $10^5 \times \frac{1}{10} = 10^4$; therefore $10^{-1} = \frac{1}{10}$.

Similarly $10^{-2} = \frac{1}{100}$, and so on.

This is the reason why 1 is called the *zero* power of ten ; $\frac{1}{10}$ the *first negative* power of ten ; and so on. [Art. 46.]

209. The square of a number has either twice as many digits as the number, or one less than twice as many.

For a number less than 1000 and not less than 100 has for its square a number less than 1000000 and not less than 10000; for example, a number of 3 digits has for its square a number of 6 or 5 digits.

210. In the following list we notice

number	1	2	3	4	5	6	7	8	9
square	1	4	9	16	25	36	49	64	81
cube	1	8	27	64	125	216	343	512	729

(i) that there is no square number which ends in 2, or in 3, or in 7, or in 8;

(ii) that when a number ends in 1, 2, 3, 4, 5, 6, 7, 8, or 9
its cube ends in 1, 8, 7, 4, 5, 6, 3, 2, or 9.

211. To find the square root of a given square number we proceed as follows:

I. When the given square number is less than 100, the square root is given by the above table.

II. When the square number is greater than 100 we first consider the **form** of a square number.

Example i.
$$\begin{aligned} 73^2 &= (70+3) \text{ times } (70+3) \\ &= 70 \text{ times } (70+3) + 3 \text{ times } (70+3) \\ &= (70)^2 + 70 \times 3 + 3 \times 70 + 3^2 \\ &= (70)^2 + \text{twice } 70 \times 3 + 3^2. \end{aligned}$$

Example ii. $(850)^2 = (800)^2 + \text{twice } 800 \times 50 + (50)^2.$

The method of 'square root' is based upon the *form* of the above result; which may be stated thus, *The square of the sum of two numbers = the square of the first + twice the product of the first and second + the square of the second.*

This is the algebraic formula $(a+b)^2 = a^2 + 2ab + b^2$.

Just as in long division we subtract products of the divisor, gradually diminishing the remainder; so in square root we subtract the squares of numbers, gradually diminishing the remainder.

Example i. *Find the square root of* 5329.

78)53'29(70 + 3
49 00
twice 70 + 3)4 29
4 29
that is
7)53'29(73
49
143)4 29
4 29

There are *two* digits in the answer [Art. 205]; hence we mark off the digits in pairs from the right.

Now $(70)^2 = 4900$ and $(80)^2 = 6400$; hence the required number is between 70 and 80.

We have to subtract $(70 + \text{some digit})^2$. That is

$(70)^2 +$ *twice* $70 \times$ *this digit* $+$ (*this digit*)^2.

We first subtract $(70)^2$; the remainder is 429.

We now have to *guess* the other digit; we shall have to subtract *twice* $70 \times$ *the digit* $+$ (*the digit*)^2, which is $(140 +$ *the digit*$) \times$ *the digit.*

Thus 140 serves as a guide, and the fact that 429 is a little more than 3 times 140, suggests that the required digit is 3.

On trial, we find that when $(70 + 3)^2$ is subtracted from 5329 there is no remainder; hence $(73)^2 = 5329$.

Example ii. *Find the square root of* 734449.

800)73'44'49(800 + 50
64 00 00
twice 800 + 50)9 44 49
8 25 00
1 19 49

There are *three* digits in the answer [Art. 209]; hence we mark off the digits in pairs.

Now $(800)^2 = 640000$ and $(900)^2 = 810000$, hence the required number is less than 800 and greater than 900.

We begin by subtracting 800^2; then, 50 times (twice 800 + 50); these together are $(800 + 50)^2$. The remainder is 11949; hence,

the required number is 850 + *some digit*; we have subtracted $(850)^2$; we now have to subtract (*twice* 850 + *some digit*) × *the digit.*

1700 + 7)11949(850 + 7
11949

Twice 850, that is, 1700, serves as a trial divisor, and suggests 7 as the required digit.

Thus, $(857)^2$ subtracted from 734449 leaves no remainder.

The whole process may be arranged thus:—

8)73'44'49(857
64
165)9 44
8 25
1707)1 19 49
1 19 49

or using the abbreviated method of subtraction [Art. 36].

8)73'44'49(857
165)9 44
1707)1 19 49

Thus, the square root of 734449 is 857.

NOTE.—The RULE for forming the successive '*divisors*' is, *Multiply the answer as far as it has been obtained by* 2, *and after this place the new digit of the answer.*

EXAMPLES. CXLVII.

Find the square root of each of the following :—

1. 729. **2.** 1024. **3.** 1764. **4.** $104\frac{1}{25}$.
5. $62\frac{67}{121}$. **6.** 3969. **7.** 8649. **8.** 4761.
9. 23409. **10.** 133956. **11.** 114921. **12.** 94249.
13. 383161. **14.** 646416. **15.** 844561. **16.** 942841.
17. 12809241. **18.** 21224449. **19.** 2819041. **20.** 82264900.
21. 322669·4416. **22.** 6407·522209. **23.** 50136170·8761.

Find the number having four significant figures whose square is nearest to each of the following :—

24. 5. **25.** 10. **26.** 7. **27.** ·1. **28.** $\cdot\dot{3}$.
29. $\frac{2}{3}$. **30.** $\frac{2}{7}$. **31.** $\frac{22}{7}$. **32.** $\cdot 1\dot{5}$. **33.** $\cdot 3\dot{7}\dot{5}$.
34. 3·1416. **35.** $\cdot\dot{3}4\dot{5}$. **36.** $4\cdot\dot{0}\dot{1}$. **37.** 1000. **38.** $33\cdot\dot{3}$.

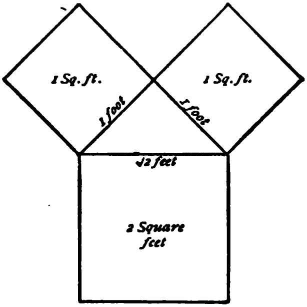
On Incommensurable Numbers.

212. The quantity indicated by $\sqrt{2}$ *does* exist.

By Euclid I. 47, the surface of the square described on the side opposite the right angle of a right-angled triangle is equal to the sum of the surfaces of the two squares described on the other two sides.

Consider a right-angled triangle having each of two sides 1 ft. long. Describe on each of its three sides a square.

The surface of the largest square = the sum of the surfaces of the two smaller squares.



Thus we have a square whose area = 2 square feet.

The number of feet in each of the equal sides of this square must be such that the square of the number is 2.

So that the length of each side of this square is $\sqrt{2}$ feet.

213. A line can be drawn such that the measure of its length is the square root of any given number.

For if we can draw a line the measure of whose length is the square root of any number say $\sqrt{3456}$, we can draw a line the measure of whose length is $\sqrt{3457}$ as follows:—

Take the line of length $\sqrt{3456}$ units, draw at its extremity a line of length 1 unit at right angles to it; then the line completing the right-angled triangle is $\sqrt{3456+1}$ units long.

214. The square root of any given number can be expressed as a decimal, correct to any required number of significant figures.

Example. Find $\sqrt{2}$ correct to 7 significant figures.[*]

We perform the process of subtracting from 2·00,000,000,000,000 the square number nearest to and less than it; thus

$$\begin{array}{r|l}
1 & \text{2·00'00'00'00'00'00'00(1·4142135} \\
24 & 1\,00 \\
281 & 400 \\
2824 & 11900 \\
28282 & 60400 \\
282841 & 383600 \\
2828423 & 10075900 \\
28284265 & 159063100 \\
 & 17641775
\end{array}$$

Hence the difference between $(1{\cdot}4142135)^2$ and 2 is ·00000017641775;
the difference between $(1{\cdot}14142136)^2$ and 2 is ·00000010642469.

Thus $\sqrt{2} = \underline{1{\cdot}414214}$ for all practical purposes [Art. 150]. *Answer.*

---

[*] It will be noticed that when five digits 14142 of the above result have been obtained, the next four digits depend only on the remainder 3836 and the divisor 28282 and are not affected by the additional digits to the right of these.

Hence when five digits in a square root have been obtained four more may be found by contracted division, thus

$$\begin{array}{r|ll}
1 & 2\cdot & (1{\cdot}4142 \\
24 & 100 & \phantom{(1{\cdot}4142}1356 \\
281 & 400 & \\
2824 & 11900 & \\
2\cancel{8}\cancel{2}\cancel{8}\cancel{2} & 60400 & \\
 & 3836 & \\
 & 1008 & \\
 & 160 & \\
 & 19 & \text{Sq. rt.} = 1{\cdot}41421356.
\end{array}$$

EXAMPLES. CXLVIII.

1. Prove that $\sqrt{3}$ lies between 1·7320508 and 1·7320509.

Find each of the following correct to 5 significant figures:

2. $\sqrt{10}$. **3.** $\sqrt{1+\sqrt{2}}$. **4.** $\sqrt{7}-\sqrt{5}$. **5.** $\sqrt{200}$.

6. $\sqrt{\frac{3}{7}}$. **7.** $\sqrt{\frac{5}{9}}$. **8.** $\sqrt{20}$. **9.** $\sqrt{4840}$.

10. $\sqrt{3{\cdot}14159}$. **11.** $\sqrt{2{\cdot}718282}$. **12.** $\sqrt{{\cdot}4342945}$. **13.** $\sqrt{(2\sqrt{2})}$.

14. Find the square number having 7 significant figures nearest to and greater than 1·196033.

Example. Find the length of the side and of the diagonal of a square field containing 10 acres.

10 acres = 48400 sq. yds. The square root of 48400 = 220.

Hence the length of each side is 220 yds.

By (Euclid I. 47) the square on the diagonal = the sum of the squares on two of the sides.

Hence a square one of whose sides is the diagonal contains 2 × 48400 sq. yds.

$$\text{The square root of } 2 \times 48400 = 220 \times \text{the square root of } 2$$
$$= 220 \times 1{\cdot}4142\ldots$$

Hence the length of the diagonal is $\underline{311{\cdot}12}$ + yds.*

EXAMPLES. CXLIX.

1. Find the length of the side of a square field containing 1000 acres.

2. A rectangular field of 10 acres is twice as long as it is broad; find its length to the nearest tenth of a yard.

3. The length of a rectangular field of 100 acres is to its breadth as 3 is to 2; find the length to the nearest yard.

4. The length of a rectangular field of 37·25 acres is to its breadth as 4 is to 3; find the length to the tenth of a foot.

5. The areas of circles are as the squares on their radii; the radius of one circle being 100 ft., find to the 10th of an inch the radius of another circle having twice the area of the first.

6. Find the radius of a circle enclosing an acre. [See Art. 154.]

7. A square tank 3 ft. deep is to hold 1000 gallons; find its other dimensions.

8. A round stone pillar 10 ft. high weighs 20 tons; find its diameter, assuming that stone weighs 195·32 lbs. per cub. ft.

---

* Suppose a square to be described, one of whose sides is 1 ft.; and suppose we attempt to measure the diagonal; the example of Art. 210 tells us that the length of the diagonal is $\sqrt{2}$ ft., *i.e.* 1·4142135 + feet. We cannot express this length with 1 ft. as unit as an exact decimal; at the same time it should be noticed that it is quite impossible to measure a line to the degree of accuracy indicated by this expression. [Art. 150.]

215. There is always some remainder in the process of finding $\sqrt{2}$; hence it is not possible to find a decimal number whose square is equal to 2 exactly.

A number which cannot be exactly expressed as a decimal number is said to be **incommensurable** with decimal numbers; and is called *an incommensurable number.*

Two quantities, whose ratio is an incommensurable number, are said to be incommensurable.

The length of the diagonal and of a side of a square are incommensurable.

CUBE ROOT.

216. The cube of a number contains either thrice as many digits, or one less or two less than thrice as many digits as the number.

For the cube of a number less than 1000 and not less than 100 must be less than 1000000000 and not less than 1000000; so that it must contain either 9 or 8 or 7 digits.

217. To find the cube root of a given cube number we proceed as follows:—

I. When the cube number is less than 1000 its cube root is given by the Table on page 231.

II. When the cube number is greater than 1000, we first consider the **form** of the cube of a number expressed by two digits.

Example i.

$$\begin{aligned}(76)^3 &= (70+6) \text{ times } (70+6)^2 \\ &= (70+6) \text{ times } \{(70)^2 + \text{twice } 70 \times 6 + 6^2\} \\ &= (70)^3 + \text{thrice } (70)^2 \times 6 + \text{thrice } 70 \times 6^2 + 6^3.\end{aligned}$$

Example ii.

$$(850)^3 = (800)^3 + \text{thrice } (800)^2 \times 50 + \text{thrice } 800 \times (50)^2 + (50)^3.$$

The method of 'cube root' is based upon the form of the above result, which may be stated thus:—

The cube of the sum of two numbers = the cube of the first number + thrice the product of the square of the first by the second + thrice the product of the first by the square of the second + the cube of the third number.

This is the algebraic formula $(a+b)^3=a^3+3a^2b+3ab^2+b^3$.

Example i. *Find the* cube root *of* 438796.

$$\begin{array}{rrl}
7^2= & 49) & 438'976(70+6\\
 & & \underline{343}\\
\text{thrice }(70)^2= & 14700 & \multicolumn{1}{|l}{95\,976}\\
\text{thrice }70\times 6= & 1260 & \multicolumn{1}{|l}{}\\
6^2= & \underline{36} & \multicolumn{1}{|l}{}\\
 & 15996 & \multicolumn{1}{|l}{\underline{95\,976}}
\end{array}$$

The digits are marked off in threes [Art. 216].

We observe that

$$(70)^3=343000,$$

and that $(80)^3=512000$; so that the required cube is $(70+\text{some digit})^3$. Accordingly we commence the subtraction of $(70+\text{some digit})^3$ by subtracting $(70)^3$.

To complete the subtraction of $(70+\text{some digit})^3$ we must also subtract $\{\text{thrice }(70)^2\times\text{this digit}+\text{thrice }70\times(\text{this digit})^2+(\text{this digit})^3\}$, that is, $\{\text{thrice }(70)^2+\text{thrice }70\times\text{this digit}+(\text{this digit})^2\}\times$ the digit. Hence our new 'divisor' is the above bracket.

Accordingly we find that *three times* $(70)^2=14700$; and we *judge from the size of the remainder* that the digit required is 6: on trial it is found to be so.

Example ii. *Find the* cube root *of* 233744896.

We first mark off the digits in *threes*, thus 233'744'896, and subtract the largest possible cube number as in Example (i) from 233'744; thus

$$\begin{array}{rrl}
6^2= & 36) & 233'744(61\\
 & & \underline{216}\\
\text{thrice }(60)^2= & 10800 & \multicolumn{1}{|l}{17\,744}\\
\text{thrice }60\times 1= & 180 & \multicolumn{1}{|l}{}\\
1^2= & \underline{1} & \multicolumn{1}{|l}{}\\
 & 10981 & \multicolumn{1}{|l}{\underline{10\,981}}\\
 & & 6\,763
\end{array}$$

We have really subtracted $(610)^3$, that is, $(61)^3\times 1000$ from 233744896. The remainder is 6763896.

We must now *complete* the subtraction of $(610+\text{some digit})^3$ from 233744896; thus

$$\begin{array}{rr|l}
\text{thrice }(610)^2= & 1116300 & 6763896(610+6\\
\text{thrice }610\times 6= & 10980 & \\
6^2= & \underline{36} & \\
 & 1127316 & \underline{6763896}
\end{array}$$

thus $(610+6)^3$ subtracted from 233744896 leaves *no remainder.*

The whole process may be arranged as follows :—

$$\begin{array}{rr|l}
6^2 = & 36 &)233'744'896(\underline{616.}\quad \textit{Answer.} \\
 & & \underline{216\quad\ } \\
\text{thrice } (60)^2 = & 10800 & \ 17744 \\
\text{thrice } 60 \times 1 = & 180 & \\
1^2 = & \underline{1} & \\
 & 10981 & \ \underline{10981\qquad} \\
\text{thrice } (610)^2 = & 1116300 & \ 6763896 \\
\text{thrice } 610 \times 6 = & 10980 & \\
6^2 = & \underline{36} & \\
 & 1127316 & \ \underline{6763896}
\end{array}$$

The *Rule* for forming the successive 'divisors' is:—Take (*thrice the square of the answer as far as it has been obtained*) × 100 + *thrice the product of* 10 *times the answer by the new digit* + *the* (*new digit*)^2.

EXAMPLES. CL.

Find the cube root of each of the following :—

1. 1728. **2.** 29791. **3.** 85184. **4.** 531441.
5. $25239\frac{19}{27}$. **6.** $82312\frac{7}{8}$. **7.** 5000211. **8.** 5268024.
9. 38272753 **10.** 31855013. **11.** 15069223. **12.** 28372625.
13. 45·499293. **14.** ·559476224. **15.** 353393·243.
16. 1647·212741. **17.** 21975528·401. **18.** 1249939619·648.
19. 151334·226289. **20.** 2222·447625. **21.** $49\cdot\dot{2}9\dot{6}$.
22. ·00017646408152900. **23.** $\sqrt[3]{2}$ to 4 sign. figs.
24. Prove that the cube root of 3 lies between 1·442 and 1·443.

Find the cube roots of the following to 3 significant figures :—

25. 2. **26.** ·2. **27.** ·02. **28.** 10. **29.** 100.

30. Find the length of the edge of the cube which contains 1481544 cubic inches.

31. The contents of two cubes are respectively 5359·375 and 5·359375 cubic inches ; find the difference between the lengths of their edges in inches.

32. A cube contains 11 cubic feet 675 cubic inches ; find the length (1) of its edge, (2) of its diagonal.

33. Find the edge to the nearest inch of a cube which contains 10 cubic yds. [see Ex. 28].

34. The volume of a sphere varies as the cube of its diameter ; find the diameter of a sphere which will contain twice as much as a sphere 10 in. in diameter.

CHAPTER XVIII

SCALES OF NUMERATION AND NOTATION

218. A **scale** of numeration, with a given number as **radix**, is a method for the systematic arrangement of numbers in groups, in which each group consists of some power of the radix.

Example i. The method of arranging numbers with radix ten is explained in Art. 7.

Example ii. Let a number of counters be taken, and let any number, say 7, be chosen as radix.

Let all the counters be arranged in *groups* of 7 counters each, until the remaining number of counters is less than 7.

Let all the groups of 7 be arranged in *sets* of 7 groups, each set containing 7^2 counters, until the remaining number of groups is less than 7.

Let all the sets of 7^2 be arranged in *heaps* of 7 sets, each heap containing 7^3 counters, until the remaining number of sets is less than 7.

And so on, as in Art. 7.

By the above process the number (of the counters) will have been enumerated in the scale of 7.

219. The scale of ten is called the *decimal* scale (or the *denary*), the scales of *twelve*, *eleven*, *nine*, *eight*, *seven*, etc., are called respectively the *duodecimal* (or *duodenary*), the *undenary*, the *nonary*, the *octenary*, the *septenary*, etc., scales.

220. The notation of Art. 12 may be used for any scale provided the radix of the scale is clearly indicated.

Example. Suppose that in Example ii. Art. 218, the successive *remainders* are 3, 5, 0, 4, and that finally there are 2 heaps of 7^4 counters, the number would be

$$\left.\begin{array}{c|c|c|c|c} (\text{seven})^4 & (\text{seven})^3 & (\text{seven})^2 & \text{seven} & \text{one} \\ 2 & 4 & 0 & 5 & 3 \end{array}\right\}\text{or}\quad \overbrace{24053}^{\text{scale of } 7}$$

221. In what follows the radix will be indicated by a figure placed above the number.

Thus, $\overbrace{24053}^{7}$ indicates $2\times7^4+4\times7^3+5\times7+3$.

When no radix is indicated, *ten* is to be understood.

N.B. **t** and **e** are used as *digits* for *ten* and *eleven.*

Example i. *Express* 45678 *in the scale of* 7.

$$\left.\begin{array}{r l} \text{Process, } 7)\overbrace{45678}^{\text{(ten)}} & \\ 7)\overline{6525} & \text{times } 7+3 \\ 7)\overline{932} & \text{times } 7^2+1\times7 \\ 7)\overline{133} & \text{times } 7^3+1\times7^2 \\ 7)\overline{19} & \text{times } 7^4+0\times7^3 \\ \overline{2} & \text{times } 7^5+5\times7^4 \end{array}\right\rangle \begin{array}{l} \text{Thus, } 45678 \\ =2\times7^5+5\times7^4+1\times7^2+1\times7+3 \\ =\underline{\overbrace{250113}^{7}}. \end{array}$$

Example ii. *Express* $\overbrace{45678}^{\text{eleven}}$ *in the scale of* ten.

$$\left.\begin{array}{r l} \mathbf{t})\overbrace{45678}^{\text{eleven}} & \\ \mathbf{t})\overline{4\mathbf{t}63} & \text{times ten}+0 \\ \mathbf{t})\overline{550} & \text{times } (\mathbf{t}\times\mathbf{t})+3\times\mathbf{t} \\ \mathbf{t})\overline{60} & \text{times } (\mathbf{t}\times\mathbf{t}\times\mathbf{t})+0 \\ \overline{6} & \text{times } \mathbf{t}^4+6\times\mathbf{t}^3 \end{array}\right\}$$

The work is similar to the above, except that we have to remember that *the moving a digit one place to the left* increases its value *eleven*-fold [see Art. 47]; thus in the scale of eleven $45\div\mathbf{t} =(4\times e+5)\div\mathbf{t}=4$ and 9 over.

Hence, $\overbrace{45678}^{\text{eleven}}=6\times\mathbf{t}^4+6\times\mathbf{t}^3+3\times\mathbf{t}=\underline{66030}.$

EXAMPLES. CLI.

Express the following *decimal* numbers in the scale (i) of 7, (ii) of 8, (iii) of 2, (iv) of 3, (v) of twelve. [Also arrange by means of counters.]

1. 1000. 2. 456. 3. 679. 4. 700. 5. 789.

Express the following numbers in the scales (i) of ten, (ii) of 5, (iii) of 2, (iv) of twelve.

6. $\overbrace{564355}^{7}$. **7.** $\overbrace{116100}^{8}$. **8.** $\overbrace{11010010}^{2}$. **9.** $\overbrace{84000}^{\text{twelve}}$. **10.** $\overbrace{8t5212}^{\text{eleven}}$.

222. In Arts. 43, 44, the methods of Arts. 7 and 12 are employed to express any *part* of a unit. In a similar way we may express any part of a unit by a **radix-fraction.** The **point** used in a similar manner to the decimal point, is called the **radix-point.**

Example. Express $\frac{3}{4}$ of a unit *in the scale of* 7.

We first find how many *sevenths of a unit* this part ($\frac{3}{4}$) of the unit contains.

$\frac{3}{4}$ *of a unit* contains $\frac{3}{4} \times 7$ of a seventh; that is, $\frac{21}{4}$ of a seventh; that is (5-sevenths + $\frac{1}{4}$ of a seventh) of the unit.

Hence, the *first* digit to the right of the *radix-point* is 5 and the remainder is $\frac{1}{4}$ of a seventh (compare Art. 43).

We next find how many ($\frac{1}{7}$ of $\frac{1}{7}$) of the unit this remaining part, $\frac{1}{4}$ of a seventh, contains.

$\frac{1}{4}$ of a seventh contains $\frac{1}{4} \times 7$ of ($\frac{1}{7}$ of $\frac{1}{7}$) of the unit,

that is, [1 of ($\frac{1}{7} \times \frac{1}{7}$) + $\frac{3}{4}$ of ($\frac{1}{7}$ of $\frac{1}{7}$)] of the unit.

Hence, the *second* digit to the right is 1 and the remainder is $\frac{3}{4}$ of ($\frac{1}{7}$ of $\frac{1}{7}$) of the unit;

$\frac{3}{4}$ of ($\frac{1}{7}$ of $\frac{1}{7}$) = $\frac{3}{4} \times 7$ of ($\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7}$) = ($5 + \frac{1}{4}$) of ($\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7}$) of the unit.

Hence, the third digit is 5. Proceeding in this way we see that

	one	$\frac{1}{7}$th	$\frac{1}{7} \times \frac{1}{7}$th	$\frac{1}{7} \times \frac{1}{7} \times \frac{1}{7}$th	$(\frac{1}{7})^4$th	$(\frac{1}{7})^5$th	$(\frac{1}{7})^6$th	
$\frac{3}{4}$ =	0·	5	1	5	1	5	1	etc. = $\overbrace{\cdot\dot{5}\dot{1}}^{7}$

223. The process may be abbreviated thus :—

$$\begin{array}{l} \overbrace{4)3\cdot0000}^{7} \text{ etc.} \\ \quad\ \underline{\cdot5151} \text{ etc.} \end{array} \left.\vphantom{\begin{array}{l}a\\a\\a\\a\end{array}}\right\}$$

3·0000 etc. is in the scale of 7; we say mentally 30 divided by 4 is 3 × 7 divided by 4, put down 5 and carry 1; 10 divided by 4 is 1 × 7 divided by 4, put down 1 and carry 3; and so on.

224. A radix-fraction is transformed from one scale to another as follows :—

Example i. *Express* ·04973 *in the scale of* 7.

$$\begin{array}{rl}
\overbrace{\cdot 04973}^{\text{(ten)}} & \text{of a unit} \\
\underline{7} & \\
0\cdot 34811 & \text{of } \tfrac{1}{7} \text{ of a unit} \\
\underline{7} & \\
2\cdot 43677 & \text{of } (\tfrac{1}{7}\times\tfrac{1}{7}) \text{ of a unit} \\
\underline{7} & \\
3\cdot 05739 & \text{of } (\tfrac{1}{7}\times\tfrac{1}{7}\times\tfrac{1}{7}) \text{ of a unit} \\
\underline{7} & \\
0\cdot 40173 & \text{of } (\tfrac{1}{7})^4 \text{ of a unit} \\
\underline{7} & \\
2\cdot 81211 & \text{of } (\tfrac{1}{7})^5 \text{ of a unit} \\
 & \text{and so on.}
\end{array}$$

We reduce the number to sevenths, and then to $(\frac{1}{7}\times\frac{1}{7})$ths, and so on; compare Example (i), Art. 123.

Hence $\cdot 04973 = \{0\times\frac{1}{7} + 2\times(\frac{1}{7})^2 + 3\times(\frac{1}{7})^3 + 0\times(\frac{1}{7})^4 + 2\times(\frac{1}{7})^5 + \text{etc.}\}$,

that is, $\cdot 04973 = \underline{\overbrace{\cdot 02302}^{7}}\ldots$

Example ii. *Express* $\overbrace{\cdot 41036}^{7}$ *in the scale of* twelve.

$$\begin{array}{r}
\overbrace{\cdot 41036}^{} \\
\underline{\omega} \\
7\cdot 05642 \\
\underline{\omega} \\
1\cdot 31223 \\
\underline{\omega} \\
5\cdot 32011 \\
\underline{\omega} \\
5\cdot 43165 \\
\text{and so} \\
\text{on.}
\end{array}$$

ω stands for twelve; the process is the same as the above, except that the multiplication is in the scale of 7.

Mentally thus: twelve times $6 = 72 =$ ten sevens $+ 2$; put down 2, carry ten. Twelve times $3 +$ ten $= 46 =$ 6 sevens $+ 4$; put down 4, carry 6. Twelve times $0 + 6 = 6$; put down 6. Twelve times $1 = 12 =$ seven $+ 5$; put down 5, carry 1. Twelve times $4 + 1 = 49 = 7$ sevens; put down 0, carry 7. And so on.

Therefore, $\overbrace{\cdot 41036}^{7} = \underline{\overbrace{\cdot 7155}^{\text{twelve}}}\ldots$

225. Hence, to reduce a fraction to a radix-fraction with a given number as radix, we multiply the fraction repeatedly by the given radix and take the whole number in each result as a digit in the answer.

Example. $\frac{1}{4}$ expressed in the scale of 7 must be *a recurring* radix-fraction; for the multiplication of $\frac{1}{4}$ by any power of 7 will not produce a whole number. We may notice that in this case the number of possible 'remainders' is not greater than 3, so that there cannot be *more* than 3 digits in the *period*.

EXAMPLES. CLII.

Express as radix-fractions in the scale (i) of ten, (ii) of 7, (iii) of twelve, (iv) of 3,

1. $\frac{1}{3}$. **2.** $\frac{1}{5}$. **3.** $\frac{5}{6}$. **4.** $\frac{3}{7}$. **5.** $\frac{11}{36}$.

Express as radix-fractions in the scale (i) of 6, (ii) of 8, (iii) of eleven, (iv) of 2,

6. $\frac{2}{3}$. **7.** $\frac{4}{5}$. **8.** $\frac{51}{64}$. **9.** $\frac{35}{36}$. **10.** $\frac{5}{11}$.

Express as radix-fractions in the scale (i) of 7, (ii) of 9, (iii) of twelve.

11. ·25. **12.** 5·375. **13.** 2·123. **14.** 4·08725. **15.** $\cdot\dot{3}$.

Express as decimals, and also in the scales of 2 and of 7,

16. $\overbrace{\cdot 201}^{3}$. **17.** $\overbrace{\cdot 431}^{5}$. **18.** $\overbrace{\cdot 101101}^{2}$. **19.** $\overbrace{\cdot 1946}^{\text{twelve}}$.

20. Prove that $\overbrace{\cdot 12\dot{3}\dot{4}}^{7} = \dfrac{\overbrace{1234-12}}{6600}$.

21. Prove that $\overbrace{\cdot 34\dot{7}0\dot{9}}^{\text{eleven}} = \dfrac{\overbrace{34709-34}^{\text{eleven}}}{ttt00}$.

Express as (decimal) vulgar fractions—

22. $\overbrace{401\cdot 36\dot{0}\dot{2}}^{7}$. **23.** $\overbrace{571\cdot 04\dot{7}}^{8}$. **24.** $\overbrace{4\cdot 50\dot{1}0\dot{9}}^{\text{twelve}}$.

Prove the following:—

25. $\overbrace{\cdot 034\dot{6}}^{7} = \overbrace{\cdot 035}^{7}$. **26.** $\overbrace{4\cdot 70\dot{t}}^{\text{eleven}} = \overbrace{4\cdot 71}^{\text{eleven}}$.

In what scales (from 2 to twelve) can the following fractions be expressed in (i) a recurring, (ii) a terminating form?

27. $\frac{1}{3}$. **28.** $\frac{4}{5}$. **29.** $\frac{5}{6}$. **30.** $\frac{5}{36}$. **31.** $\frac{7}{72}$.

32. $\frac{3}{22}$. **33.** $\frac{5}{42}$. **34.** $\frac{7}{64}$. **35.** $\frac{7}{81}$. **36.** $\frac{11}{60}$.

226. The *decimal* system of numeration was in use among the Egyptians, Greeks, and Romans long before the *notation*, which is based on the idea of *locality*, was invented. Thus, the notation of Art. 12 is comparatively a modern invention; it is an improvement of that used by the later Greeks, and was invented by the Hindoos.*

---

* See *Principles of Arithmetic*, by Homersham Cox, Cambridge, 1885.

227. The **Roman notation** (derived from the Egyptian Hieroglyphics) is obsolete for the purposes of *calculation*, but is still often used in other ways.

For example, in the present work, Roman numerals are used to number the sets of Examples.

The principal **symbols** of the Roman Notation are

I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1000.

When a line is drawn over any one of these symbols its value is increased a thousandfold; thus $\overline{\text{X}} = 10000$, $\overline{\text{M}} = 1000000$.

The chief *rules* are, (i) each symbol when it is on the right of one of higher value adds its own value to the number indicated;

(ii) each symbol when it is on the left of one of higher value subtracts its own value from the number indicated.

Examples. 1886 = MDCCCLXXXVI.
3044599 = $\overline{\text{MMMXLIV}}\text{DXCIX}$.

EXAMPLES FOR EXERCISE CLIII.

1. State in words the value of the figures 1000300500.
2. Find the cost of 759 articles costing \$5.77 each.
3. Two sorts of tea cost respectively \$22.25 and \$24.75 per cwt., find the difference in price per lb.
4. Find the L.C.M. of 16, 18, 24, 30, 15, 12.
5. Multiply 17 yds. 1 ft. $11\frac{5}{8}$ in. by 84.
6. From $15\frac{1}{22}$ take $11\frac{101}{168}$.
7. Express 3 tons 2 cwts. 87 lbs. 8 oz. as the fraction of $5\frac{1}{2}$ tons.
8. Reduce 6 yds. $4\frac{1}{2}$ in. to the decimal of 200 yds.
9. Find the value of 1·125 of 2 tons 13 cwts. 50 lbs.
10. Find the area of the walls of a room 16 ft. 8 in. long, 14 ft. broad, 12 ft. high; express the answer in yards.

---

11. Reduce $\frac{13875}{92015}$ to its lowest terms.
12. Find the value of $6\frac{7}{8}+11\frac{7}{12}+4\frac{5}{24}+\frac{7}{32}$.
13. Simplify $\dfrac{7\frac{7}{12}\times\frac{2}{13}}{1\frac{1}{2}-\frac{1}{6}}$.
14. Find the value of $\frac{1}{12}$ of \$6 + $\frac{3}{4}$ of \$12.60 − $\frac{5}{9}$ of 99c.
15. What fraction multiplied by $16\frac{4}{5}$ will produce $3\frac{1}{12}$?
16. Reduce $\frac{7}{50}$ to a decimal; and ·00028 to a vulgar fraction.
17. Multiply 3·125 by 5·12.
18. Divide ·0117 by 4·16.
19. Find the value of 1·31875 of 10 tons.
20. A man's income is \$2964.50, and his daily expenditure is \$6.30: how much did he save in 1885, when he had to pay a tax of \$1 in \$30?

---

21. Multiply 354612 by 8602.

22. Reduce 800000 inches to miles.

23. If 178 cwts. 87 lbs. 8 oz. be divided into 53 equal parts, find the weight of one such part.

24. Reduce to lowest terms $\frac{900}{1260}$, $\frac{327}{1458}$.

25. Add together $6\frac{3}{7}$, $4\frac{3}{14}$, $8\frac{5}{6}$, $2\frac{4}{21}$.

26. Simplify $\dfrac{\frac{4}{9}-\frac{2}{7}}{\frac{4}{9}+\frac{2}{7}}$.

27. Multiply 3 days 15 hrs. 47 min. 30 secs. by $2\frac{3}{7}$.

28. Multiply ·006134 by 80·032, and divide the result by ·0032.

29. Find the value of ·07 of 2 tons 10 cwts.

30. If a person travelling 12 hours a day can perform a journey in 24 days, in how many days could he perform the same journey, supposing he travelled 16 hours a day?

---

31. How many quarter inches are there in 1262 yds. $5\frac{1}{4}$ in.?

32. Reduce 78160 shillings to half-crowns.

33. Find the L.C.M. of 2, 3, 4, 5, 6, 7, 8, 12, 44, 132.

34. Multiply $2\frac{1}{4}$ by $5\frac{1}{3}$; and divide $3\frac{1}{6}$ by 4.

35. Simplify $4\frac{1}{4}$ of $1\frac{1}{68}$ of $\frac{2}{23}+3\frac{1}{2}+2\frac{1}{4}$ of $1\frac{1}{3}$.

36. Reduce 15 yds. 1 ft. $5\frac{1}{2}$ in. to a fraction of 32 yds. 3 in.

37. Explain the meaning of the symbols 500, ·5, ·0005.

38. Find the sum and the difference of ·79864 and ·801.

39. Reduce $\frac{21}{15000}$ of a year to the decimal of a day.

40. A field is 121 yards long, 86 yards broad: find its value at $400 an acre.

---

41. Reduce 50 tons 4 cwts. 54 lbs. 2 oz. to oz.

42. Find the area of a room 12 yds. 2 ft. 3 in. long by 8 yds. 1 ft. 4 in. broad.

43. Reduce to lowest terms $\frac{45045}{76230}$ and $\frac{1261}{2134}$.

44. Subtract $31\frac{3}{13}$ from $89\frac{4}{11}$.

45. Express 2 cwts. 89 lbs. as a fraction of 17 cwts. 34 lbs.

46. Divide 1 day 19 hrs. $37\frac{6}{7}$ min. by 21.

47. Divide 1·724 by ·032, and 7 by 87·5.

48. Reduce 3 tons 8 cwt. 25 lbs. to the decimal of 6 tons.

49. If 7 men take 4 days to mow a field of 5 a. 1 rood, how long would 8 men take to do the same?

50. In a mile race *A* gives *B* 100 yards start, and beats him by 20 yards. If *B* can run the mile in 5 min. 8 secs., how long did *A* take?

51. Divide 300 tons of coal equally among 168 people.

52. Find the L.C.M. of 12, 21, 33, 28, 14, 11.

53. Add together $2\frac{2}{3}$, $13\frac{7}{12}$, $4\frac{4}{21}$, $21\frac{13}{42}$.

54. Simplify $\dfrac{3\frac{2}{3} \text{ of } 1\frac{7}{22}}{\frac{2}{5} \text{ of } 6\frac{2}{3}}$.

55. Multiply 89 yds. 0 ft. $7\frac{2}{3}$ in. by $12\frac{3}{7}$.

56. A man gave $\frac{3}{7}$ of his money to a beggar and had 88c. left, how much had he at first?

57. Reduce ·072 to a vulgar fraction in lowest terms; $\frac{137}{125}$ to a decimal fraction.

58. Find the value of 7·4453125 of 8 days.

59. If 7 men mow $5\frac{1}{4}$ acres in a certain time, how much will 16 men mow?

60. How many half-crowns are equal to a thousand half-guineas?

---

61. Reduce 7 tons 13 cwts. 62 lbs. 8 oz. to ounces.

62. How many half-guineas are there in £3003?

63. Find the area of a field 102 yds. 2 ft. 6 in. long and 21 yds. 1 ft. 3 in. wide.

64. Reduce to lowest terms $\frac{825}{1155}$ and $\frac{559}{731}$.

65. Add together $1\frac{13}{15}$, $2\frac{11}{20}$, $5\frac{7}{24}$, $\frac{5}{48}$.

66. Express 14 yds. 2 ft. $3\frac{1}{4}$ in. as a fraction of 8 yds. 2 ft. $\frac{1}{2}$ in.

67. Find the value of $\frac{2}{3}$ of \$2.52 + $\frac{7}{8}$ of \$6.48 − $\frac{6}{11}$ of 88c.

68. Multiply 32·52 by 31·5; and divide 20349·48 by ·276.

69. Reduce 5 yds. 2 ft. $7\frac{1}{2}$ in. to the decimal of 66 yds. 2 ft.

70. If a man earns \$3.36 a week, how much a year has he to live upon after paying \$1.80 a month for rent? (Reckoning 12 months or 52 weeks to the year.)

---

71. Find the cost of 367 things at \$23.34 each.

72. Divide 61 days 16 hours by 4 hours.

73. Find the L.C.M. of 18, 27, 36, 40, 45, 50.

74. Simplify $\frac{1}{16}$ of $4\frac{1}{5}$ of $\frac{4}{33}$ of $4\frac{6}{7}$ of $3\frac{4}{17}$.

75. Subtract $2\frac{4}{5}$ from $4\frac{3}{7}$.

76. Divide 11 tons 12 cwts. 50 lbs. by $\frac{9}{16}$.

77. A man went $\frac{3}{7}$ of a journey by train, $\frac{5}{8}$ of the remainder by coach, and walked the last 3 miles: how long was the journey?

78. Reduce $\frac{7}{80}$ to decimals; and ·8725 to a vulgar fraction.

79. Find the value of 3·125 of 1 week 4 days 16 hours.

80. If the carriage of 13 cwts. 68 lbs. 12 oz. costs £3. 19*s.* 1*d.*, what will the carriage of 3 cwts. 37 lbs. 8 oz. cost for the same distance?

81. Reduce 6 tons 4 cwts. 51 lbs. to pounds.

82. A certain sum of money was divided among 162 people, and they each received $11.19 : what was the sum ?

83. Find the L.C.M. of 16, 18, 24, 30, 15, 12.

84. Multiply 17 tons 98 lbs. $9\frac{1}{7}$ oz. by 84.

85. From $15\frac{1}{22}$ take $11\frac{101}{165}$.

86. Express 3 tons 2 cwts. 87 lbs. 8 oz. as the fraction of $3\frac{1}{2}$ tons.

87. Reduce $\frac{1}{6250}$ to a decimal.

88. Divide ·002 by ·0125.

89. Reduce 2 tons 12 cwts. 25 lbs. to the decimal of 5 tons.

90. A man spends $132 every 30 days, and his yearly income is $2000 : how much does he save in a year ?

---

91. Reduce 176285 inches to yds. ft. in.

92. A room has each of its sides 10 yds. 2 ft. long: what is its area ?

93. Reduce $\frac{35786}{125309}$ to its lowest terms.

94. Find the value of $6\frac{7}{8}+1\frac{7}{12}+4\frac{5}{24}-\frac{7}{32}$.

95. Simplify $\dfrac{1\frac{1}{12}\times\frac{11}{15}}{1\frac{1}{2}-1\frac{1}{6}}$.

96. Find the value of $\frac{11}{12}$ of $300 + $\frac{3}{4}$ of $189 − $\frac{5}{8}$ of $20.25.

97. What fraction multiplied by $13\frac{4}{9}$ will produce $4\frac{7}{12}$?

98. Multiply 40·12 by 42·01.

99. Find the value of 7·4453125 of 10 tons.

100. If 5 yards of cloth cost $5.25, what is the cost of 365 yards ?

---

101. Write in words the number 2071003601.

102. Divide 3455 yds. 1 ft. 5 in. by 91. Can this be done in more than one way ?

103. Find the area of a room 18 ft. 9 in. by 16 ft. 8 in. Find the length of an equally large room whose breadth is 12 ft. 6 in.

104. Reduce $\frac{41265}{60024}$ to its lowest terms.

105. Arrange $\frac{3}{5}$, $\frac{3}{7}$, $\frac{13}{30}$, $\frac{27}{70}$ in order of magnitude.

106. Divide $2\frac{3}{4}$ of $1\frac{1}{11}+\frac{8}{17}$ of $2\frac{5}{8}$ by $2\frac{3}{4}$ of $1\frac{1}{11}-\frac{8}{17}$ of $2\frac{5}{8}$.

107. Find the value of $1\frac{1}{6}$ of $6 + $\frac{5}{38}$ of $468.35 − $12\frac{1}{2}$ of 75c.

108. Reduce ·0288 to a vulgar fraction in lowest terms; and $\frac{101}{1250}$ to a decimal fraction.

109. Find the value of 2·890625 of £6.

110. In the number 1023·045 how many times greater value has the 2 than the 4 ? how many times less the 5 than the 1 ? and what is the difference in value between the two 0's ?

111. Reduce 31 lbs. 9 oz. 15 dwts. 21 grs. to grains; and ten thousand and fifty-seven ounces to cwts.

112. If 5 lbs. 7 oz. of tea cost $3.54, what will 29 lbs. cost?

113. Find the L.C.M. of 12, 15, 20, 24, 30, and 40.

114. Divide 163 yds. 2 ft. $8\frac{4}{7}$ in. by 28.

115. Find the value of $2\frac{1}{5}+3\frac{2}{5}+7\frac{2}{3}+8+1\frac{1}{15}$.

116. Reduce 7 cwts. 84 lbs. 6 oz. to the fraction of $2\frac{1}{2}$ tons.

117. A man left his son one-third of his property, each of his three daughters one-seventh, and the remaining $2500 he left in legacies and charity: what was his whole property worth?

118. Divide 312·8 by ·64 and ·3375 by 13·5.

119. Reduce 14 yds. 1 ft. $1\frac{1}{2}$ in. to the decimal fraction of 100 yds.

120. Divide $350 among 26 men so as to give one of them $25 more than each of the others.

---

121. Reduce 14 lbs. 5 oz. 17 dwts. to grains.

122. Multiply 33 tons 15 cwts. 68 lbs. 12 oz. by 15.

123. Reduce to its lowest terms $\frac{5643}{8217}$.

124. Simplify $3\frac{1}{7}$ of $1\frac{3}{11}-\frac{4}{5}$ of $2\frac{2}{5}$.

125. Simplify $\dfrac{\frac{3}{5}+5\frac{5}{6}\text{ of }\frac{2}{7}}{\frac{3}{5}\text{ of }5\frac{5}{6}+\frac{2}{7}}$.

126. Reduce 2 fur. 97 yds. 1 ft. to the fraction of 1 m. 439 yds.

127. Add together 5·314, 20·1, ·336; subtract the result from 30.

128. Multiply 7·28 by ·105; and divide 12 by 4·8.

129. Reduce 4 hrs. 48 min. to the decimal of 6 hours.

130. One man takes 120 steps a minute, each 30 inches long; another walks $3\frac{1}{4}$ miles an hour: if they start together, how soon will one of them be 100 yards ahead of the other?

---

131. Divide 46 cwts. 92 lbs. by 23.

132. A room is 19 ft. 6 in. long and 6 yds. 0 ft. 8 in. wide: find the area of its floor.

133. Find the L.C.M. of 16, 18, 24, 28, 42.

134. Divide 17 tons 6 cwts. 87 lbs. 8 oz. by $3\frac{1}{7}$.

135. Simplify $12\frac{3}{8}-4\frac{7}{12}+2\frac{1}{5}-\frac{11}{24}$.

136. Find the value of $\frac{3}{4}$ of $7.20 + $\frac{5}{11}$ of $3.96 + $\frac{1}{6}$ of 66c.

137. A boy spent $\frac{3}{7}$ of his money on pictures and $\frac{5}{9}$ on eatables, and had 24c. left: what had he at first?

138. Reduce $\frac{5}{8}$ to a decimal; and 4·0128 to a vulgar fraction in its lowest terms.

139. Find the value of 12·75 of 21 yds. 1 ft. 5 in.

140. If 50 yards of silk cost $43.75, how many yards may be bought for $17.50?

141. Subtract five thousand and four from one hundred and twenty thousand ; and give the answer in words.

142. Multiply 5 weeks 2 days 23 hours by 205.

143. How many francs worth 19c. each should be given in exchange for $18.43 ?

144. Reduce to its lowest terms $\frac{285714}{714285}$.

145. Subtract $3\frac{3}{7}$ from $5\frac{2}{13}$.

146. Find the value of 3 tons 14 cwts. 70 lbs. divided by $2\frac{1}{3}$.

147. In a division sum the dividend is 136954 ; the quotient is 3604 : find the remainder and the divisor.

148. Add together ·0012, 5·721, 13·04, and 12.

149. Multiply 8·16 by 4·6875 : (i) by decimals ; (ii) by vulgar fractions, reducing the result to a decimal.

150. Find the value of 3·678125 of 12 tons.

---

151. Reduce 120573 lbs. to tons.

152. Find the area of the floor of a room 17 yds. 2 ft. long and 3 yds. 1 ft. wide.

153. Find the L.C.M. of 14, 21, 24, 30, 42.

154. Add together $\frac{7}{11}$, $\frac{2}{33}$, $3\frac{4}{15}$, $4\frac{4}{55}$.

155. Simplify $2\frac{1}{3}$ of $3\frac{3}{5}$ of $\frac{5}{14}+3\frac{1}{7}$ of $\frac{3}{5}\times 3\frac{1}{2}+10\frac{2}{7}$.

156. Reduce 3 cwts. 56 lbs. 4 oz. to a fraction of 3 tons.

157. Reduce ·018125 to a vulgar fraction in lowest terms.

158. Subtract 8·0808 from 80·808.

159. Divide ·066625 by 5·33.

160. If $15.86 be charged for digging 61 poles of ground, what will be charged for digging 101 poles ?

---

161. Multiply 10 sq. yds. 4 sq. ft. 76 sq. in. by 132.

162. Reduce to its lowest terms $\frac{527781}{846093}$.

163. Simplify $\frac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}-2\frac{1}{7}}$.

164. Multiply 78·67 by ·2013.

165. Find the value of 7·125 of $17.50.

166. What will it cost to pave a court 146 ft. 9 in. long and 88 ft. 9 in. broad at $22\frac{1}{2}$c. per square yard ?

167. Find the Compound Interest on $15,000 for 3 years at $4\frac{1}{2}$ per cent.

168. A battalion of 1921 men is to be raised from four towns in proportion to their populations. Find the number of men raised from each if their inhabitants number 4150, 12450, 24900, and 29050 respectively.

169. Divide 423840 tons 6 cwts. 62 lbs. 8 oz. by 651.

170. Simplify $3\frac{1}{21} + 2\frac{3}{7} - 3\frac{3}{4} + 6\frac{1}{2} + 10\frac{2}{3}$.

171. Divide 92 yds. 1 ft. $8\frac{1}{3}$ in. by $2\frac{1}{3}$.

172. Divide 3444·72101 by 7·01.

173. A man left his eldest son one-fourth of his property; to each of his two other sons one-sixth; and to each of his three daughters one-ninth: the remaining $2400 he left in legacies. What was the whole amount of his property?

174. Find the amount of $1427 in 6 years at $3\frac{1}{2}$ per cent. Simple Interest.

175. If 3 cwts. cost $26.25, what is the cost of 7 cwts. $62\frac{1}{2}$ lbs.? (to the nearest cent).

176. If 2 horses can be kept 5 months for $128.30, how much will it cost to keep 3 horses for 11 months?

---

177. Reduce 56721394 ounces to tons.

178. Simplify $4\frac{5}{12} + 5\frac{7}{9} - 6\frac{3}{4} + 1\frac{7}{8}$.

179. Reduce $\frac{5}{11}$ of 8 cwts. 25 lbs. to the fraction of 5 cwts. $62\frac{1}{2}$ lbs.

180. Divide 3·333651 by 61·7; and $\cdot 0\dot{6}5\dot{4}$ by 1·2.

181. Find the value of 1·875 of $76 – $3\frac{1}{7}$ of $36·75.

182. If a debtor can pay only $168 out of a bill of $560, how much does he pay in $240?

183. Find the Compound Interest upon $5500 at 4 per cent. for $2\frac{1}{2}$ years.

184. Find the area of a square room whose side is $19\frac{1}{2}$ ft. long.

---

185. Find the value of $11\frac{2}{3} \times 5\frac{1}{7} \times 4\frac{1}{2}$ of $\frac{1}{12}$.

186. Reduce to a simple fraction $\frac{7\frac{2}{5}}{2\frac{2}{5}} - \frac{2\frac{1}{3} + \frac{1}{33}}{1\frac{3}{11}}$ of $1\frac{5}{16}$.

187. Multiply ·0391 by 4·005; and ·00002 by ·0000431.

188. Add up $\cdot 0\dot{3}$, $5\cdot 1\dot{7}\dot{2}$, $6\cdot \dot{9}4\dot{3}$, and $11\cdot 7\dot{8}42\dot{1}$, making the answer correct to 7 places of decimals.

189. Divide 168 tons 18 cwts. 37 lbs. 8 oz. by 546.

190. Find the amount at Simple Interest upon $6376 for $4\frac{1}{6}$ years at 3 per cent.

191. If the carriage of 130 lbs. for 90 miles cost $2.10, how much can be carried 240 miles for $5.60?

192. A father left his eldest son one-fourth of his property, and each of his three other sons one-fifth; the remaining $6000 he left in legacies: what was his whole property worth?

193. Reduce 123456 square feet to acres.

194. Simplify $(2\frac{1}{3}+1\frac{7}{8})\div(2\frac{1}{3}-1\frac{7}{8})$.

195. What part of $84 is $69.10 ? Express the result as a decimal.

196. Multiply 6·012 by ·0612. Divide ·0842 by ·00125.

197. Find the value of ·00625 of 7 tons 4 cwts.

198. If a printer sets up 8500 letters a day, and is paid 11c. for every 1000, how long will he take to earn $16.83 ?

199. If 142 lbs. of salt cost $17.75, what will be the cost of 30 lbs. ?

200. Find the Simple Interest on $333750 for 5 years at $3\frac{1}{2}$ per cent.

201. Simplify $2\frac{1}{2}+3\frac{1}{3}-4\frac{1}{4}+5+\frac{1}{5}$ of $6\frac{1}{6}$.

202. Divide 4 weeks 4 days 16 hours by $5\frac{2}{3}$.

203. Reduce $\frac{1}{7}$ to a decimal, and test your result by bringing it to a vulgar fraction.

204. Find the value of 1·5625 of 7 cwts.

205. If the rent of land in London be $112.75 per square yard, what is the rent per acre ?

206. If when labour is paid at the rate of 18 cents an hour the cost of building a house be $1866.24, what would be the cost if labour were paid at 16 cents an hour ? Also find how long a gang of 32 men working 9 hours a day would take in building it.

207. Find the Compound Interest on $7500 for 3 years at 4 per cent.

208. If on an average *A* make 5 runs to *B*'s 2, and *B* make 2 runs to *C*'s 3 during the season, find how many runs they make respectively of a total of 750 runs.

209. A room is 20 ft. 3 in. long and 16 ft. 8 in. wide, find the area of its floor.

210. Add together $3\frac{1}{8}$, $2\frac{3}{16}$, $1\frac{4}{5}$, $\frac{19}{20}$, and $4\frac{7}{10}$.

211. Simplify $\frac{1}{3}$ of $2\frac{1}{7}+1\frac{1}{5}$ of $\frac{2}{3}$ of $2\frac{3}{8}-\dfrac{2\frac{1}{7}}{1\frac{1}{2}}$.

212. Bring 15·375 and $2\cdot38\dot{6}$ to vulgar fractions.

213. Divide 75·62 by ·064 ; and $\cdot9\dot{2}$ by $\cdot1\dot{6}\dot{7}$.

214. If 6 men can dig a trench 34 yards long in 10 days, how many yards can 20 men dig in 15 days ?

215. Find the Compound Interest upon $3750 at 3 per cent. for 3 years.

216. Divide $1347.50 among *A*, *B*, and *C*, so that *A* shall have five times as much as *B*, and *B* eight times as much as *C*.

217. Reduce 56789 ounces to cwts.

218. Divide 15 tons 8 cwts. 80 lbs. by $5\frac{1}{4}$.

219. Simplify $\dfrac{3\frac{1}{3}+2\frac{1}{5}\text{ of }3\frac{1}{3}}{2\frac{1}{3}\text{ of }1\frac{4}{5}-2\frac{1}{3}\text{ of }\frac{2}{5}}$.

220. Divide 9·001728 by 14·4, and divide 1 by 500.

221. Find the area of a table 9 ft. 9 in. long, 2 ft. 7 in. broad.

222. The highest score in an innings was $\frac{2}{9}$ of the total, and the next highest was $\frac{3}{7}$ of the remainder. These two scores differed by 8 runs. What was the total?

223. If the distances of Venus and the Earth from the Sun be in the proportion of 7 to 10, and the Earth's distance from the Sun be 91,500,000 miles, find that of Venus.

224. Find the Compound Interest on $20000 for 3 years at 6 per cent.

---

225. Add together $1\frac{5}{9}$, $2\frac{2}{15}$, $3\frac{14}{25}$, $\frac{7}{45}$.

226. Reduce $5\frac{2}{3}$ of 4 yds. 1 ft. to the fraction of 2 yds. 2 ft. 6 in.

227. Express $\frac{2}{15}$ as a decimal, also ·36 and $3\cdot6\dot{3}\dot{6}$ as vulgar fractions.

228. Find the difference between $\frac{5}{18}$ of $4.50, and ·0625 of $21, and express $1.02 as a decimal of $32.64.

229. Find the rent of 15 a. 138 p. at $16.24 per acre.

230. If a man go from London to Edinburgh (400 miles) in 16 days, walking 8 hours a day, how many hours a day must he walk to perform a distance of 1125 miles in 60 days?

231. Find the Simple Interest on $7541.25 for 5 years at 4 per cent.

232. Divide $1277.50 among three persons, in the proportion of 12, 15, and 23.

---

233. Add together $3\frac{1}{2}$, $4\frac{5}{12}$, $7\frac{1}{3}$, and $\frac{9}{16}$, and subtract $14\frac{41}{48}$ from the result.

234. Reduce $\frac{5}{8}$ of 7 yds. 1 ft. 6 in. to the fraction of 6 yds. 1 ft. $8\frac{1}{4}$ in.

235. Reduce $4\frac{5}{8}$ and $11\frac{1}{16}$ to decimals.

236. Multiply ·024 by 2·5, and divide 6 by ·125.

237. Reduce 62 lbs. 6 oz. to the decimal of 2 cwt.

238. If the carriage of 25 tons for a certain distance cost $21.75, how much will be charged for the carriage of 37 tons for the same distance?

239. Find the Simple Interest and amount on $6243 for 5 years at $3\frac{1}{2}$ per cent.

240. Find the area of a floor 33 ft. 2 in. long and 18 ft. broad.

241. Simplify $\dfrac{2\frac{1}{2}+3\frac{1}{4}-4\frac{1}{7}}{1\frac{3}{7}\text{ of }2\frac{1}{4}}$.

242. Multiply $44.25 by $2\frac{1}{3}$, and divide the result by $17\frac{1}{10}$.

243. Reduce 1·175 and $4{\cdot}1\dot{6}$ to vulgar fractions in lowest terms.

244. Find the value of 2·3125 of $8.50.

245. Divide 2960 tons 10 cwts. $12\frac{1}{2}$ lbs. by 249.

246. If 8 men can mow 56 acres of wheat in 14 days, how many acres can 9 men mow in 15 days?

247. Find the Comp. Interest on $8400 for 3 years at 5 per cent.

248. The wages of 2 men, 4 women, and 6 children amount in 6 days to $77.04; find how much the men receive per day if each person shares equally with the rest.

---

249. Reduce £1517. 13*s*. $7\frac{3}{4}$*d*. to farthings.

250. Simplify $\dfrac{3\frac{5}{6}+1\frac{1}{12}-4\frac{3}{4}}{\frac{14}{15}\text{ of }\frac{5}{28}}$.

251. Reduce $\dfrac{1\frac{1}{8}+\frac{1}{4}}{1\frac{1}{8}-\frac{1}{4}}$ to a simple decimal.

252. Add together 9·25, $11{\cdot}0\dot{3}$, and ·0016, and reduce the result to a vulgar fraction.

253. Find the value of ·375 of $21 + $\cdot\dot{5}\dot{4}$ of $8.25 + $\cdot0\dot{2}\dot{7}$ of $55 − ·775 of $5.

254. The rent of 286 a. 144 p. is $3586.25; of how many acres of the same land is the rent $3443.75?

255. If 30 tons can be carried 10 miles for $18.75, how far ought 27 tons to be carried for $18?

256. Three-fifths of the income ($600) of a certain hospital is to be divided equally among three persons, the rest in proportion to their days of residence. The first resided 351 days, the second 312 days, and the third 273 days; how much will each receive?

---

257. Multiply together $4\frac{4}{5}$, $7\frac{1}{3}$, $\frac{15}{16}$, $\frac{19}{38}$, and $1\frac{55}{57}$.

258. What fraction of 66 yds. 2 ft. is $\frac{11}{12}$ of 6 yds. 4 in.?

259. Multiply ·000116 by 1·25; and divide 738·95271 by ·07.

260. Find the value of $1\frac{1}{8}$ of a ton + $\frac{3}{4}$ of a cwt. + $\frac{11}{16}$ of 25 lbs., and reduce the result to the decimal of 10 tons.

261. Find the value of 65 things at $12.43 each.

262. Find the amount of $1575 at $5\frac{1}{4}$ per cent. Simple Interest, for $4\frac{2}{3}$ years.

263. Find the Compound Interest on $15625 for 3 years at 6 per cent.

264. Find the cost of 2 cub. yds. 8 cub. ft. 120 cub. in. at $7.50 per cub. ft.

---

265. Reduce 721035 crowns to guineas.

266. Add together $1\frac{3}{4}$, $3\frac{1}{6}$, $5\frac{3}{7}$, $2\frac{3}{20}$, $1\frac{5}{21}$, and $\frac{7}{80}$.

267. Divide 5 yds. 2 ft. $9\frac{3}{5}$ in. by $2\frac{1}{3}$.

268. Express 2 cwts. 31 lbs. 4 oz. as a decimal of 1 ton.

269. Multiply 4·125 by $8{\cdot}1\dot{4}\dot{5}$, and give the answer in decimals.

270. Find the Simple Interest on $1513 for 4 yrs. at 3 per cent.

271. A man leaving his property by will to his three sons, left a third of it and $1000 to the eldest; a half of the remainder and $1000 to the second; and the rest to the third, who found that his share amounted to $2500; what was the total value of the property?

272. Four men are employed to reap a field, and, after working 5 days, they have cut 10 acres; 2 more men are then put on, who work at the same rate, and the whole is finished in 3 more days. How many acres are there in the field?

273. Find the time of walking 18 miles 4 furlongs 219 yards at the rate of 1 mile in $14\frac{3}{8}$ minutes.

274. What fraction of 30 wks. 6 days 8 hrs. is 11 wks. 4 days 3 hrs.?

275. Divide 3·2121 by 49·8.

276. Find the value of 3·825 of $37.60.

277. Express $7\frac{5}{16}$ and $7\frac{5}{18}$ as decimals, and subtract one result from the other.

278. Find the Compound Interest on $52500 for 3 years at 4 per cent.

279. If 100 men do half of a certain piece of work in $\frac{1}{5}$th of a year, how many will it take to do two-thirds of the remainder in $\frac{1}{6}$th of a year?

280. A bankrupt owes $\frac{1}{3}$ of his debts to *A*, and $\frac{1}{5}$ to *B*, and to his other creditors in all $160 less than to *A* and *B* together; what is his total debt?

281. Add together $5\frac{4}{15} + 6\frac{15}{16} + 10\frac{9}{20}$.

282. Find the time of emptying a tank of 100,000 gallons at the rate of $1\frac{3}{7}$ pints in 37 seconds.

283. Multiply 13 days 5 hrs. 43 mins. by $3\frac{3}{7}$.

284. Divide 2·45 by ·0784.

285. Find the value of 14·1875 of $16.64.

286. Find the value of 36 cwts. $37\frac{1}{2}$ lbs. at $12.40 per cwt.

287. If I can walk a certain distance in 114 days when I rest 9 hours each day, how long will it take me to walk twice as far if I walk twice as fast and rest twice as long each day?

288. After paying a tax of $1 in $40 on his income, and spending $\frac{4}{5}$ of the remainder, and lending a friend $240, a man has ·1 of his income left; what was his income?

289. Find the quotient and the remainder when 72 a. is divided by 3 a. 147 p.

290. Divide $\dfrac{4\frac{1}{2}+2\frac{2}{3}}{6\frac{2}{7}-4\frac{1}{3}\text{ of }1\frac{7}{33}}$ by $4\frac{7}{8}$.

291. Reduce $\frac{1}{125}$ and $12\frac{365}{1000}$ to decimals.

292. Reduce 12 days 23 hours 40 minutes to the decimal of 15 days 14 hours.

293. Multiply $2{\cdot}8\dot{0}\dot{3}$ by $9{\cdot}1\dot{6}$, and give the answer as a decimal.

294. Find the amount at Compound Interest of $125 × 25 × 25 for 3 years at 4 per cent.

295. Find the cubic content of a vessel 111 yds. 4 in. long, 55 yds. 2 ft. 3 in. broad, 27 yds. 2 ft. 5 in. high.

296. Coals travel by canal at the rate of 1 mile in $21\frac{3}{8}$ min., by road 3 times as fast, by rail 9 times as fast; they go $4\frac{1}{3}$ hours by canal, $16\frac{1}{4}$ by road, and $10\frac{5}{8}$ by rail; how far are they carried?

---

297. Find the G.C.M. and the L.C.M. of 3432 and 3575.

298. Reduce 280436 cubic inches to cubic yards.

299. Find the value of $\frac{2}{3}$ of $8.40 + $\frac{3}{5}$ of $8 + $\frac{3}{4}$ of $1.

300. Multiply 6·04 by ·35; and divide 131·95 by 4·375.

301. Add together £·032, 2·35*s.*, and 5·42*d.*, and subtract ·35 of 2*s.* 6*d.* from the result.

302. Find the Simple Int. upon $1875 for $12\frac{1}{2}$ years at $3\frac{3}{4}$ per cent.

303. A room is 15 ft. long, 12 ft. broad, and 9 ft. 9 in. high. Find the cost of painting the walls at 60c. per sq. yard.

304. Find the side of the largest square tile with which a court 33 yds. 1 ft. 7 in. long and 20 yds. 11 in. broad can be paved.

---

305. Multiply 3 mi. 372 yds. 2 ft. 5 in. by $7\frac{1}{5}$.

306. Subtract $1\frac{2}{3}$ of $5\frac{1}{11}$ from $25\frac{2}{3}$.

307. Reduce 12 wks. 6 days 19 hrs. to the fraction of 45 wks. 2 days $18\frac{1}{2}$ hrs., and express the fraction in its lowest terms.

308. Express the difference between ·25 and ·025 as a vulgar fraction.

309. Reduce $26\frac{5}{18}$ to a decimal; and $\cdot008\dot{3}$ to a vulgar fraction.

310. What is the length of the side of a square garden whose area is 66 poles 28 sq. yds. $4\frac{1}{2}$ sq. feet?

311. Find the Comp. Int. upon $3906.25 for 3 years at 4 per cent.

312. Two men do half a piece of work in 5 days, and then one alone finishes it in 8 days more. How long would the other man alone take to do the whole work?

313. Find the Least Common Multiple of 21, 95, 99, 133, 154.

314. Divide 642 yds. 2 ft. $5\frac{1}{2}$ in. by $2\frac{1}{15}$.

315. Simplify $\dfrac{3\frac{1}{4}-2\frac{1}{8}}{3\frac{1}{2}+3\frac{1}{6}}+7\frac{1}{4}$.

316. Reduce $\frac{6744}{128}$ and $\frac{54}{81}$ to decimals.

317. Multiply ·000431 by 87·2. Divide ·0002992 by ·0187.

318. If 336 bushels of wheat cost $336, what will 616 bushels cost when the price of wheat has risen 4 cents per bushel?

319. Find the interest, and the discount, for 8 months on $21630 at $4\frac{1}{2}$ per cent. per annum.

320. *A* starts business with a capital of $8000, and after six months admits a partner *B* with a capital of $7000. The profits at the end of the year are $943: what is the share of each?

---

321. Reduce ten million pounds to long tons.

322. Simplify $3\frac{1}{6}+\frac{2}{5}$ of $\frac{5}{12}-2\frac{1}{4}$.

323. Reduce 3 cwts. 43 lbs. 12 oz. to the fraction of 2 cwts. 62 lbs. 8 oz.

324. Express as vulgar fractions, in their lowest terms, ·001625 and $\cdot 9\dot{8}\dot{1}$.

325. If a pace is 2 ft. 7 in., reduce forty thousand paces to miles.

326. Find the square root of 117094041.

327. A lawn, which is twice as long as it is broad, contains $840\frac{1}{2}$ square yards of grass. Find its length and breadth.

328. A person transfers $16,500 stock from the 3 per cents. at 90 to the $3\frac{1}{2}$ per cents. at 99: find the change in his income.

---

329. Divide 10 cwts. 97 lbs. $9\frac{1}{7}$ oz. by 13.

330. How many cubic feet of air are there in a room 8 yards long, $5\frac{1}{2}$ yards broad, and 10 ft. 5 in. high?

331. Simplify $\frac{3}{4}$ of $2\frac{4}{7}+\frac{5}{9}$ of $\frac{3}{7}$ of $6+1\frac{3}{28}$.

332. Reduce $\frac{1}{2}$ of 7 yds. 1 ft. 7 in. to the fraction of 3 yds. 1 ft. 6 in.

333. From 3·1 subtract ·469. Multiply 23·041 by ·01036. Divide ·00001 by ·32.

334. Find the Simple Interest on $13312.50 for 6 years at $3\frac{1}{4}$ per cent.

335. An explorer accomplishes a journey of 160 miles in 3 days, travelling 11 hours a day; in how many days would he accomplish 2000 miles, travelling 15 hours a day?

336. $59118 is left to three legatees in the proportion of $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$. What does each receive?

337. Find the G.C.M. and the L.C.M. of 273 and 4131.

338. Simplify $\dfrac{3 - 4\frac{1}{7} + 2\frac{1}{3}}{3 \times 2\frac{1}{3} - 4\frac{1}{7}}$.

339. Express as vulgar fractions in lowest terms, ·00345 and $2{\cdot}07\dot{8}$.

340. Find the value of 3·4125 of $201.60.

341. Find the value of 16 tons 12 cwts. 81 lbs. 4 oz. at $171.52 per ton.

342. To what will $16000 amount at the end of three years, allowing Compound Interest at $2\frac{1}{2}$ per cent. per annum ?

343. A tradesman receives $525 for goods which cost him $480 five months ago. What rate of profit does he make per cent. per annum ?

344. If 112 lbs. of copper be drawn in 1 mile of wire, find the area of a section, copper being 8·96 times as heavy as water, and 1 cubic foot of water weighing 1000 oz. Avoirdupois.

---

345. A man walks 20 miles in 5 hrs. 20 minutes ; how many yards does he walk each hour ?

346. Multiply 240 yds. 2 ft. $7\frac{7}{12}$ in. by 81.

347. Add together $3\frac{1}{12}$, $\frac{4}{9}$ of $6\frac{3}{4}$, $\frac{19}{100}$, and $\frac{17}{360}$.

348. Reduce $2\frac{1}{10}$ of 5 wks. 4 days 2 hrs. to the fraction of 33 wks. 3 days 12 hrs.

349. Find the value of $\cdot 01\dot{6} + 4{\cdot}0808 - \cdot 0\dot{0}0\dot{8} + 50{\cdot}1 - \cdot 1966$.

350. Express as a vulgar fraction in its lowest terms $\cdot 03\dot{2}8145\dot{7}$.

351. How many yards of carpet 25 inches wide are required for a floor 22 ft. 6 in. long and 16 ft. 3 in. broad ?

352. A tradesman makes 35 per cent. profit on his outlay ; what did he pay for goods which he sells at $315 ?

---

353. Divide 5 yds. 1 ft. 10 in. by $5\frac{1}{3}$.

354. Find the difference between 4 tons 5 cwts. $28\frac{7}{16}$lbs. and 5 tons 4 cwts. $6\frac{1}{3}$ lbs.

355. Find the continued product of 3·125, ·016, ·012, and 80.

356. Simplify $\dfrac{6\frac{1}{4} \text{ of } 4\frac{4}{5}}{11\frac{1}{5} - 6\frac{7}{10}}$.

357. If 33 bus. of corn cost $46.09, what will 75 cost ?

358. Find the amount at Simple Interest upon $5074.50 for $2\frac{1}{2}$ years at 4 per cent.

359. Find the cost of 54 tons 10 cwts. 62 lbs. at $45 per ton.

360. A man has £25. 17*s.* 6*d.*, consisting of sovereigns, half-crowns, and shillings in the proportion 1, 3, 7. How many has he of each coin ?

---

361. Simplify $4\frac{4}{11}$ of $7\frac{5}{16}+1\frac{5}{8}\div2\frac{3}{4}+2\frac{1}{2}$ of 11.

362. The proprietor of $3\frac{1}{3}$ shares of a mine receives every year $205.75 as interest; what income is derived from holding $15\frac{1}{4}$ shares in the mine?

363. Find the value of ·46875 of $30.72 + $\cdot2\dot{4}\dot{5}$ of $16.50.

364. Find the value of $5\cdot0\dot{6}\dot{2}+17\cdot36\dot{5}9\dot{3}-12\cdot4\dot{3}\dot{7}$.

365. *A* walks $2\frac{1}{2}$ miles in 40 minutes, taking exactly a yard each step; in what time will *B* walk $4\frac{3}{8}$ miles when his stride is 40 inches and he takes only 21 steps while *A* takes 22?

366. Find the present worth of $370607.60 due in $4\frac{1}{2}$ years at 4 per cent.

367. By selling wine at $4.18 per gallon, a merchant loses 5 per cent. on his outlay: what price per gallon should he charge so as to gain 15 per cent.?

---

368. Reduce 188 wks. 4 days 9 hrs. to the fraction of 251 wks. 3 days 12 hours.

369. Divide 530 weeks 5 days 4 hrs. 1 min. by $5\frac{1}{20}$.

370. Simplify $\dfrac{\cdot04478256\div5\cdot48}{\cdot036\times2\cdot043}$.

371. Find the weight of 783 things at 14 cwts. 48 lbs. 12 oz. each, by Practice.

372. What decimal of 15 tons 12 cwts. 50 lbs. is 7 tons 17 cwts. 50 lbs.?

373. A certain box is 3 ft. 6 in. long, 2 ft. 3 in. wide, and 1 ft. 4 in. high, find (i) the contents of the box, (ii) the cost of covering both ends with velvet at 14c. per square foot.

374. *A* can do a certain piece of work alone in 72 days, *B* in 96, *C* in 120. *C* begins alone for the first day, then *A* and *B* join in and all three work for 12 days, after which *A* leaves off: in how many more days will *B* and *C* finish it?

---

375. Add together $2\frac{2}{5}$, $5\frac{7}{10}$, $12\frac{9}{20}$, $9\frac{12}{25}$, $\frac{97}{100}$.

376. Find the value of 3 tons 1 cwt. 93 lbs. 12 oz. at $149.32 a cwt.

377. Multiply $7\cdot1\dot{0}3\dot{7}$ by ·027. Divide ·01 by 6·25.

378. Find the Comp. Int. on $6250 for 3 years at 6 per cent.

379. Find the cubic contents of a box 7 ft. 9 in. in length, 2 ft. 6 in. in breadth, and 1 ft. 4 in. in depth.

380. A farmer buys 10 oxen for $66 each, and 10 more for $72 each: what loss per cent. does he incur by selling them for $65.55 each?

381. What income is derived from holding $1000 stock at $72\frac{1}{2}$ paying $2\frac{9}{10}$ per cent.?

---

382. How many times is 4 tons 1 cwt. 52 lbs. contained in 4462 tons 7 cwts. 20 lbs.?

383. Find the value of $\frac{5}{66}$ of 5 miles + $\frac{4}{77}$ of 4 furlongs + $\frac{5}{12}$ of a foot − 159$\frac{1}{8}$ yards.

384. Express 3 tons 2 cwts. 50 lbs. as the decimal of 5 tons 17 cwts. 50 lbs.

385. If 24 men can build a wall in 10 days, in how many days will 36 men do it?

386. If $4335.75 will provide food for 41 persons for 94 days, what will be the expense of keeping 10 persons for 2 days?

387. Find the discount on $3165 for 11 months at 6 per cent.

388. What percentage is received upon money invested in stock at 72$\frac{1}{2}$ paying 2$\frac{9}{10}$ per cent.?

389. *A* runs 6$\frac{2}{3}$ yards while *B* runs 7 yards; *B* runs 16$\frac{1}{2}$ yards while *C* runs 15 yards: if *A* can run a mile in 5 minutes 15 seconds, what time will *C* take to do it?

---

390. Simplify $1\frac{6}{7}$ of $1\frac{3}{4} + \dfrac{6\frac{2}{7} - 5\frac{1}{2}}{6\frac{2}{7} + 5\frac{1}{2}} - 2\frac{2}{3}$.

391. Divide ·0000567 by ·0875.

392. Reduce 4 furlongs 3 poles 3 yds. 1 ft. 6 in. to yards.

393. Find the amount at Simple Interest on $2800, for 4$\frac{1}{2}$ years at 3$\frac{1}{3}$ per cent.

394. If I buy $\frac{3}{7}$ of an estate of 200 acres for $64164, how much should I have to give for $\frac{9}{10}$ of the same?

395. If it cost $12.60 to paper a room 9 ft. high with paper 2 ft. 3 in. wide, what would it cost if the room were two feet higher and the paper 2 ft. 6 in. wide at the same price per yard?

396. Which is the better investment, the 4$\frac{1}{4}$ per cents. at 114 or the 3$\frac{1}{2}$ per cents. at 89?

---

397. What fraction of 40 yds. is 1$\frac{3}{7}$ of 19 yds. 1 ft. 11 in.?

398. Find the difference between $6\cdot0\dot{3}\dot{9}$ and $8\cdot1\dot{2}$.

399. Find the value of 3 cwts. 62 lbs. 8 oz. at $357.68 per cwt.

400. How many yards of carpet 2 ft. 3 in. wide will cover a floor 27 ft. long and 16 ft. 6 in. wide? and what will it cost at $1.15 a yard?

401. Divide $7000 between *A*, *B*, and *C*, so that for every $1 given to *A*, *B* shall get $·3 and *C* $·45.

402. If sugar sold at 9 cents per lb. give a profit of 14 per cent., find the cost price per cwt.

403. An open cistern, measuring on the outside 20 ft. square and 9 ft. deep, has sides and floor of brick 1 ft. thick, and is full of water: find the total weight of the cistern and its contents; a cubic foot of water weighs 1000 oz., brick weighing half as much again as water.

404. Find the value of $1\frac{2}{9}$ of 38 yds. 2 ft. $4\frac{3}{7}$ in.

405. Reduce ·009024 and $\cdot 4\dot{7}\dot{8}$ to vulgar fr. in their lowest terms.

406. If a man walk 200 miles in 8 days, travelling 8 hours a day, how many hours a day must he travel to perform a journey of 1125 miles in 60 days?

407. Find the area of the walls of a room 6 yds. 2 ft. long, 5 yards wide, and 3 yards high.

408. Find the discount on $2435 due five months hence at $3\frac{1}{2}$ per cent. per annum.

409. Divide $372 among 12 men, 20 women, and 33 children, so that the share of a man, woman, and child is in the ratio of 5, 3, and 2.

410. How much 3 per cent. Stock must a person hold that he may receive from it an income of $450?

---

411. Simplify $6\frac{1}{4}$ of $1\frac{3}{5}+\frac{7}{10}+2\frac{1}{2}$ of $\frac{3\frac{2}{5}}{1\frac{2}{15}}+2\frac{1}{9}$.

412. Divide ·16133 by 16·25, and multiply $\cdot 34\dot{0}\dot{9}$ by $1\cdot\dot{5}4\dot{1}$.

413. Find the cost of papering the walls of a room 26 ft. 8 in. long, 16 ft. 4 in. wide, and 12 ft. high, with paper a yard wide, at $1.08 per dozen yards.

414. In what time will $1613.50 amount to $1774.85 at $2\frac{1}{2}$ per cent.?

415. If I sell goods at $126, having bought them for $78.75, what is my gain per cent.?

416. How much stock in the $3\frac{3}{4}$ per cents. can be purchased at $99\frac{3}{4}$, by selling out $4200 from the $3\frac{1}{2}$ per cents. at 95?

417. If a court 100 feet long and 30 feet wide have a grass-plot in its centre and a gravel path 4 ft. wide running round it, find the area of the grass-plot, and reduce it to the decimal of an acre.

---

418. Multiply $4\frac{7}{18}+\frac{3}{8}$ of $2\frac{1}{3}$ by $7\frac{1}{5}-5\frac{1}{7}$.

419. Express 32 lbs. 7 oz. as the decimal of 3 cwts.

420. If 14 men reap a field in $16\frac{1}{2}$ hours, how long would 11 men take to reap it?

421. Find the area of the four walls of a room 18 ft. 10 in. long, 15 ft. 8 in. wide, and 14 ft. high.

422. Find the tax on $9891 at $1 in $60.

423. In what time will $300 produce $54 at Simple Interest at 4 per cent.?

424. Find the income obtained from investing $25500 in stock paying $4\frac{1}{4}$ per cent., at 102.

425. The width of a river is 110 yards, and its average depth $4\frac{1}{2}$ feet: find how many cubic yards of water will flow past a given point in one minute, if the average rate of the stream be 2 miles an hour.

426. Divide $1{\cdot}28$ by ${\cdot}0625$.

427. Multiply $2{\cdot}02\dot{3}\dot{6}$ by $2\frac{3}{4}$.

428. Find the value of 1 ton 0 cwts. 87 lbs. 8 oz. at $6.72 per ton.

429. Find the Compound Interest on $718.75 for 3 years at 10 per cent.

430. Divide $176.22 among *A*, *B*, *C*, so that *B* may get twice, and *C* three times, as much as *A*.

431. A grocer buys tea at $31.25 per cwt.; what must he charge per lb. that he may gain 20 per cent.?

432. A cistern has 3 pipes, which can fill it in 2 hours, $2\frac{1}{2}$ hours, and 40 minutes respectively: how long would they take to fill it, if they were all open together?

433. Simplify $\dfrac{2\frac{1}{2}-\frac{5}{6}}{2\frac{1}{2}+\frac{5}{6}}-\frac{1}{4}$ of $29\frac{1}{3}+\dfrac{2\frac{5}{18}}{\frac{1}{3}}$.

434. Find the value of $7{\cdot}934375$ of 2 yds. 8 in.

435. If a railway train goes at the rate of $27\frac{3}{4}$ miles in 2 hours and 45 minutes, how many miles will it go in an hour?

436. The area of a rectangular plot of ground is 101 sq. yards 3 sq. ft. 90 sq. in., and the length is 36 ft. 9 in.: find the breadth.

437. *A*, *B*, and *C* enter into partnership: *A* puts in $630; *B* $450; *C* $540. How ought a profit of $162 to be divided amongst them at the end of the year?

438. Find the discount on $1504.50 due 6 months hence at 4 per cent.

439. How much money must be invested in the 3 per cents. at $73\frac{5}{8}$ to produce $390 per annum?

440. A man rows 10 miles in $2\frac{1}{2}$ hours against the stream, the rate of which is 3 miles an hour; how long will he be in rowing 5 miles with the stream?

441. Divide ${\cdot}004789875$ by $7{\cdot}5$.

442. Multiply ${\cdot}01\dot{2}\dot{6}$ by $17{\cdot}28$.

443. If 3 horses plough 4980 square yards in 5 days, how long will it take 7 horses to plough 87150 square yards?

444. Find the cost of painting both sides of a door 8 ft. 9 in. high and 4 ft. 10 in. wide, at 30c. per square foot.

445. The telegraph posts on a railway are 77 yards apart: find the smallest number of miles that correspond to an exact number of posts.

446. If, by selling an article at $4.35 a pound, I gain 16 per cent. on my outlay, what was its prime cost?

447. *A* can reap a field in 5 days, and *B* in 6 days, each working 11 hours a day; in what time could *A* and *B* together reap it working 10 hours a day?

448. Simplify $3\frac{1}{4}+2\frac{1}{3}$ of $5\frac{1}{4}$ of $1\frac{2}{49}-2\frac{1}{10}\times\frac{3}{7}$.

449. Find the value of ·01953125 of $32.

450. If 1 cwt. 12 lbs. of goods cost $14, find the value of 7 cwts. 15 lbs. of the same.

451. How many yards of carpet, 2 ft. 3 in. wide, are required for the floor of a room 18 ft. 4 in. long, 10 ft. 6 in. wide?

452. Reduce to its simplest form $3\frac{1}{125}$ of $3\frac{4}{7}\div(\frac{47}{345}$ of 9).

453. Find the present value of $2457.65, due 7 months hence, at 5 per cent.

454. If by selling an article for $1807.30 I make 6 per cent. profit, for what must I sell it to make 7 per cent. profit?

455. How many ducats are worth £980, two guineas being equal to 3 Napoleons, 15 dollars to 4 Napoleons, and 6 ducats to 7 dollars?

---

456. Divide ·000124 by 640.

457. Multiply $\cdot\dot{1}$ by $\cdot\dot{1}$, giving the answer as a decimal.

458. If 10 men earn $93.75 in 6 months, what will be the wages of 16 men for 20 months?

459. Find the Compound Interest for 2½ years on $156.25 at 4 per cent.

460. Find the rent of 72 a. 145 p. at $12.80 per acre.

461. A man sells a horse for $351 at a profit of 17 per cent.; what did the horse cost him?

462. What income would be derived from investing $5118.75 in the 3½ per cents. at 97½?

463. An express train goes eight times as fast as a man who walks 7 feet in a second; how many miles per hour does it go?

---

464. Divide 9 lbs. 8 oz. 1 dwt. 16 grs. by 2⅔.

465. What decimal of four miles is 27 yards 1 foot 6 inches?

466. If 1 metre = 1·1 yards, and 1100 yards = 1 verst, find how many versts there are in 5750 metres.

467. How many yards of carpet, 4 ft. 9 in. wide, will be required to cover the floor of a square room whose length is 14 ft. 3 in.?

468. Divide 3 furlongs into 4 parts proportional to the quantities 9½, 8⅓, 13⅙, 5⅔; the parts to be given in yards.

469. Find how much I shall get off, if I pay ready money, a bill of $2030.05 due 3¾ years hence at 2⅔ per cent. interest.

470. Find the income obtained by investing $2085 in the 3½ per cents. at $92⅔ per bond.

471. How much is gained or lost per cent. by selling a basket of apples at one cent each, if half of them were bought at 5 for 6c. and the rest at 3 for 2c.?

472. Divide 5·8 by ·003625.

473. Determine the exact value of $3{\cdot}01 - 5{\cdot}\dot{3}1\dot{4} + 2{\cdot}4$.

474. Find, by Practice, the value of 3 a. 3 r. 32 p. at $407.50 per acre.

475. Determine the amount, at Simple Interest, of $2775 after 8 years 4 months at 4½ per cent.

476. Three-sevenths of a gang of workmen receive 18c. a day each, and the rest 30c. a day; the whole sum divided per week of 6 days is $4384.80. Find how many workmen there were.

477. What is the profit per cent. to a market-woman who buys ducks at 84c. each and sells them for $11.04 a dozen?

478. If each of two pipes can fill a cistern in 7 minutes, while a third can empty it in 2¼ minutes, how long will it take to empty the cistern if all three taps are turned on when it is full?

---

479. What number added to $\frac{5}{8} + 2\frac{23}{24} + \frac{13}{56} - \frac{1}{16}$ will make the total equal to 4?

480. Divide $9{\cdot}4\dot{1}\dot{7}$ by $1{\cdot}24\dot{9}$.

481. Find, by Practice, the value of 3 tons 3 cwts. 48 lbs. of tin, at $571.34 per ton.

482. Find the Compound Int. on $32,000 for 4 years at 2½ per cent.

483. If 27 men can do some work in 14 days, working 10 hours a day, how many hours a day must 12 men work to do it in 45 days?

484. A man sold 72 yards of cloth for $50, his profit being the cost of 11·52 yards; what did he gain per cent.?

485. How much stock must be bought in the 3 per cents. at 92¼ to produce an income of $135?

---

486. Find the value of 3·953125 of 8 weeks 4 days 16 hrs.

487. If 5 tons be carried 59½ miles for $63.75, how far may 11 tons be carried for $41.25?

488. How many pieces of paper ¾ yd. wide must be bought for papering a wall 15 ft. 8 in. long, 11 ft. 3 in. high, the piece containing 12 yards?

489. A bankrupt owes 3 creditors $450, $560, $670 respectively; divide his property worth $352.80 equitably among them.

490. In what time will $9960 amount to $10105.25 at 2½ per cent.?

491. Find the price of 3 per cent. Stock if an investment of $4120.75 produces an income of $159.

492. Find the true Discount on $3723.85, due 3½ years hence, at 3 per cent. interest.

493. Find the value of $\dfrac{15\frac{3}{4} - 4\frac{3}{5} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 23\frac{1}{3} + 2\frac{19}{36}}$.

494. Find the value of ·0625 of 21 yds. 1 ft.

495. If a package weighing $7\frac{1}{2}$ cwts. be carried 125 miles for $3.50, how much will be charged for the carriage of 3 tons 15 cwts. for a distance of 200 miles ?

496. Divide $847 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

497. Find the amount of tax payable on a salary of $1260 per annum, when the tax is $1 in $48.

498. If tea be bought at 88c. per lb. and sold at 96c. per lb., what is the gain per cent. ?

499. Three men working all day can plant a field in 10 days ; but one of them having other employment can only work half-time : how long will it take them to complete the work ?

---

500. Divide ·02048 by ·00003125.

501. Divide $2\cdot3\dot{4}\dot{5}$ by $\cdot0\dot{0}\dot{3}$.

502. If a board be 9 inches broad, what must be its length if it contain 15 square feet ?

503. *A*, *B*, and *C* engage in trade, investing capital to the amount of $256, $352, $384 respectively. Their profits amount to $558 : what were their shares ?

504. The rate of interest being 7 per cent., what is the discount on a sum of $4070 due 3 months hence ?

505. How much 3 per cent. Stock at 90 must be sold to realise $3735 ?

506. *A* and *B* have equal sums of money : they buy into the 3 per cents. and $4\frac{1}{2}$ per cents. respectively, and get the same interest ; the 3 per cents. being at 85. At what are the $4\frac{1}{2}$ per cents. ?

---

507. Divide 13 weeks 5 days 15 hours 12 min. by $2\frac{3}{4}$.

508. Multiply $102\cdot84\dot{9}$ by $\frac{10}{17}$.

509. If 70 yards of matting 2 feet wide cost $31.50, what should be the price of 85 yards of the same material 6 feet wide ?

510. Find the cost of painting the wainscoting, which is $4\frac{1}{2}$ ft. high, in a room 26 ft. by 17 ft. 9 in., paint costing $1.56 per sq. yd.

511. If $\frac{5}{21}$ of a property is worth $9478·50, what is the value of the remainder ?

512. A grocer wishes to make a profit of 20 per cent. : for how much must he sell goods which cost him $12.50 ?

513. Divide $255 among 7 men, 11 women, and 17 children, so that two women may have as much as three children, and a man may have as much as a woman and a child together.

514. Multiply 11·625 by ·0032.

515. Find the value of 5061 things at $108.61 each.

516. Find the cubic contents of a room 22 ft. 6 in. long, 13 ft. 6 in. wide, and 12 ft. high.

517. Divide 220 yds. 2 ft. 9 in. into parts having the ratio 15 : 10 : 8.

518. Find the present value of $155.25 due in 1 year at $3\frac{1}{2}$ per cent.

519. A policeman goes after a thief who has 528 yards start: if the policeman goes at the rate of a mile in 7 minutes, and the thief at the rate of a mile in 10 minutes, how far will the thief have got when he is overtaken?

520. A merchant holds half a million rupees valued at 46c. each: what will be the amount of his loss if the value of silver falls from $1.15 to $$1.00\frac{1}{2}$ per oz.?

---

521. Divide 2004 weeks 6 days $20\frac{1}{2}$ hours by 673.

522. Find the area of a floor 13 yds. 2 ft. 3 in. long and 10 yds. 1 ft. 3 in. wide.

523. Reduce $11\frac{1}{4}$ of 84 lbs. 8 oz. to the fraction of 6 cwts. 72 lbs. 3 oz.

524. Find the poor rate on $732.80 at $1 in $64.

525. If 75 men can build 528 yards of wall in 28 days, working 12 hours a day, how many hours a day must 80 men work to build 704 yards in 40 days?

526. In how many years will $1800 amount to $2067.75 at $4\frac{1}{4}$ per cent.?

527. What is the difference per cent. profit between selling 8 cwts. for the cost price of 9 cwts., and selling 9 cwts. for the cost price of 10 cwts.?

---

528. If 27 cwts. 19 lbs. cost $163.14, what weight can be bought for $25.74?

529. Find the value of $7\frac{1}{2}$ of $6\frac{2}{3}$ of $1\frac{1}{3}+4\frac{5}{6}+2\frac{2}{3}$ of $7\frac{1}{4}$.

530. Reduce $\frac{17}{1600}$ to a decimal. Divide ·001 by ·0005. Multiply $\cdot 02\dot{3}$ by $\cdot 8\dot{3}$.

531. How many square yards are there in 6·66875 acres?

532. Find the Compound Interest on $3200 for 2 years at 3 per cent.

533. A man holding $8800 Turkish 6 per cents. sells out at $91\frac{3}{4}$ and invests the proceeds in 3 per cent. Consols at 88: find the diminution in his income.

534. In a half-mile race *A* gives *B* 10 yards start, and beats him by 20 yards; *B* gives *C* 30 yards start in half a mile, and is beaten by 60 yards: which runs the faster, *A* or *C*?

---

535. Divide 44 lbs. 5 oz. 17 dwts. 1 gr. by $4\frac{3}{7}$.

536. Simplify $\frac{17\frac{1}{2}}{1\frac{1}{4}} \div 12\frac{5}{6} + \frac{4}{55}$ of $9\frac{3}{8}$.

537. Find the value of $\cdot 9\dot{4}\dot{5}$ of 1 ton 14 cwts. $37\frac{1}{2}$ lbs.

538. If 4 men can reap a field of 12 acres in 5 days, how long will they take to reap 19 acres 2 roods?

539. What is the principal which, invested for 19 years at $3\frac{1}{2}$ per cent. Simple Interest, produces an interest of $133?

540. Find the cost of carpeting a room whose length is 16 ft. 6 in. and breadth 13 ft. 4 in., with carpet $\frac{2}{3}$ yard wide, and of which the price is $1.08 per yard.

541. A coach starts at 6 A.M. and travels at the rate of $9\frac{1}{2}$ miles an hour: at 7.45 A.M. a horseman starts from the same place and follows the coach; he rides at the rate of 13 miles an hour: at what o'clock will he overtake the coach?

---

542. How many pieces of matting, each 6 ft. long and 30 in. wide, will cover the floor of a hall 54 ft. long and 27 ft. 6 in. wide?

543. Reduce 17 cwts. $6\frac{1}{4}$ lbs. to the decimal of 2 tons.

544. Multiply 16·018 by 2·034 and ·001 by $\cdot\dot{0}\dot{2}$. Divide 1·0860204 by 80·09.

545. If the 2 cent loaf weigh 6 oz. when wheat costs $1.20 per bushel, what ought the 24 cent loaf to weigh when wheat is selling at $1.80 per bushel?

546. At what rate per cent. would $900 become $1068.75 in 3 years 9 months?

547. A man walks to a town at the rate of $3\frac{3}{4}$ miles per hour, rests there $\frac{1}{2}$ hour, and rides back at the rate of $7\frac{1}{4}$ miles per hour: how far has he walked, the whole time occupied having been 4 hrs. 10 mins.?

---

548. Reduce to its lowest terms $\frac{3564}{8415}$.

549. Reduce to its simplest form $\frac{5\frac{1}{2} \text{ of } \frac{13}{15}}{9\frac{29}{30} - 4\frac{17}{20}}$.

550. Divide 2 tons 16 cwts. 50 lbs. in the ratio of $3\frac{3}{10} : \frac{1}{3} : \frac{2}{15}$.

551. Explain what is meant by a decimal not being finite. Divide $\cdot 03\dot{8}$ by $\cdot 002\dot{7}$.

552. If 89 lbs. cost $8.01, find the cost of 5 tons 4 cwts. 75 lbs.

553. Divide $100 between *A*, *B*, and *C*, so that *A* may receive $7 as often as *B* receives $8, and *C* $5 as often as *B* receives $4.

554. If a terrestrial globe were made of 36 inches diameter, find the size on its surface of France, which contains 200,000 square miles, the earth's diameter being 7920 miles.

---

555. Add together $5\frac{1}{7}$, $3\frac{1}{21}$, $2\frac{13}{14}$, and $\frac{1}{2}$ of $\frac{3}{7}$.

556. Express $3\frac{3}{4}$ of 5 cwts. 75 lbs. as a fraction of 15 tons.

557. Express $\frac{5}{12}$ and $\frac{5}{13}$ as decimals, and ·01625 and $\cdot 343\dot{5}\dot{8}$ as vulgar fractions.

558. Find the value of 3·08375 of 11 yds. 4 in.

559. If 29 men earn \$643.80 in 54 days, how much will 600 men earn in 12 days?

560. A person holding \$10,000 in the 3 per cent. Consols sells out at 95 and invests his money in the Spanish 3 per cents. at \$$12\frac{3}{8}$: find the change in his income.

561. If by selling at 19c. I lose 5 per cent., at what price must I sell to gain 5 per cent.?

---

562. Reduce 9 tons 12 cwts. 50 lbs. to the fraction of 25 tons $13\frac{1}{3}$ cwts.

563. Simplify $12\frac{1}{33} - \dfrac{4\frac{2}{3} - 2\frac{4}{5}}{3\frac{2}{3} \text{ of } 1\frac{4}{5}}$.

564. Multiply $3\cdot\dot{6}\dot{3}$ by $\cdot 08\dot{2}$. Subtract $2\cdot\dot{7}1\dot{9}$ from $3\cdot 0\dot{6}$ without the use of vulgar fractions.

565. How many square feet of wall-paper will be required for a room whose dimensions are—length 23 ft. 5 in., breadth 18 ft. 7 in., height 9 ft. 6 in.?

566. Find the Discount on \$1964.15 due 5 months hence at $4\frac{1}{2}$ per cent.

567. How many days will 64 men, working 12 hours a day, take to do what 48 men, working 9 hours a day, do in 16 days?

568. 3 women can do as much work as 5 children, and a man twice as much work as a woman. If the total wages of 6 men, 10 women, 15 children be \$155, how ought they to be divided?

---

569. Find, by Practice, the weight of 768 things at 1 ton 7 cwts. $37\frac{1}{2}$ lbs. each.

570. Explain your method of placing the decimal point in the quotient when one decimal fraction is divided by another.

571. Find the value of 3·7953125 of 2 tons, and reduce 7 yds. $1\frac{1}{4}$ in. to the decimal of 20 yds.

572. A person transfers \$5000 stock from the 3 per cents. at 72 and invests the proceeds in a 4 per cent. stock at 90: what is the difference in his income?

573. Find the cost of papering a room 18 ft. 1 in. by 14 ft. 8 in. and 12 ft. 6 in. high, with paper 30 in. broad and costing 9c. per yard.

574. A room is $18\frac{1}{4}$ ft. long, $13\frac{1}{4}$ ft. wide, 10 ft. high: find the difference in the cost of papering the walls with English paper which is 21 in. wide and costs 12c. a yard and with French paper which is 18 in. wide and costs 11c. a yard, allowing 14 sq. yds. for doors, etc.

575. Divide ·282892 by ·394, and ·01 by 500.

576. What is the value of 5 cwts. 93 lbs. at $13.75 per cwt.?

577. *A* puts into a business $80 for 5 months; *B* puts in $90 for 8 months; *C* puts in $66 for one year; divide the profits, $47.80, fairly between *A*, *B*, and *C*.

578. Divide $900 among 3 persons, so that the first may have twice as much as the second, and the third four times as much as the other two together.

579. Find the Compound Interest on $8333.33 for $2\frac{1}{2}$ years at 5 per cent.

580. A goods-train leaves Bristol at 12 noon, for London, 120 miles distant, and travels at the rate of 20 miles per hour; another train leaves London for Bristol at 1.30 P.M., and travels at the rate of 25 miles per hour; when, and where, will the trains pass each other?

---

581. Find the value of 4·01875 of $652.

582. If the wages of 29 men for 54 days amount to $482.85, how many men must work 12 days to receive $2442?

583. How many casks, some of them holding $2\frac{1}{4}$ cwts., and the others each 12 cwts. $12\frac{1}{2}$ lbs., are required for 30 tons?

584. Find the Simple Interest on $4208 from Aug. 13th to Oct. 25th at $2\frac{1}{2}$ per cent. per annum.

585. Find the present value of $1105 due in 6 years, interest being at 5 per cent.

586. A mixture is made of 1 gallon of ale at 10c. a gallon, 3 at 18c., 4 at 24c., and 12 at 8c.: how much per gallon is the mixture worth?

---

587. Find the value of $5\frac{3}{4}+6\frac{3}{7}+1\frac{6}{35}-12\frac{7}{20}$.

588. Divide 13·8125 by 3·25; and 1·05 by $1\cdot1\dot{6}$.

589. A purse and the money it contains are worth $9.24, and the money is 10 times the value of the purse: how much money does it contain?

590. Find the value of $3\frac{3}{8}$ of $1·10 + 1·35 of $5.90 − 2·4375 of $4.80.

591. A box made of board an inch thick measures on the outside 20 in. long, 14 in. wide, and 8 in. deep: find the cubic contents of the interior, and the cost of painting the outside, at 18c. per square foot.

592. In what time will $220 gain $56.10, interest being at $4\frac{1}{2}$ per cent.?

593. Three men have together completed a piece of work for which they receive $19.10: *A* has worked 4 days of $10\frac{1}{2}$ hours, *B* for 11 days of 7 hours, and *C* (who does in 2 hours as much as either *A* or *B* can do in 3) for 6 days of 8 hours: how much money ought each to have?

594. Reduce $4\frac{3}{4}$ guineas to the decimal of £50.

595. Multiply 102 yds. 1 ft. $9\frac{3}{5}$ inches by $3\frac{4}{7}$.

596. Divide $\dfrac{5\frac{1}{8}+4\frac{3}{4}}{5\frac{1}{8}-4\frac{3}{4}}$ by $1\frac{4}{7}$ of $1\frac{4}{7}$.

597. If it takes 135 men to supply, in 5 days' work, a fortress with provisions for 5 months when the garrison is 650 strong, how many will be required to supply it in 3 days for 4 months, after the garrison has been reduced by 130 men?

598. A man whose nominal income is $4250, and who rents a house at $500 a year, finds his income reduced by a tax of $2\frac{1}{2}$c. in the $1 and local rates to $4006.25. What do the rates (which are levied on rental) amount to in $100?

599. Find the Present Value of $3640 due in 10 months, interest being at $4\frac{4}{5}$ per cent.

600. A man bought a certain number of Bonds of the 3 per cents. at 84, and the same number of $3\frac{1}{2}$ per cents. at 91; his income from both investments together was $260; how much money did he invest?

---

601. Simplify $5\frac{7}{8}-3\frac{1}{4}$ of $1\frac{1}{13}+\dfrac{4\frac{1}{2}}{5\frac{2}{5}}$.

602. Express $\frac{23}{320}$ and $\frac{320}{23}$ as decimals: the latter to 6 places. To how many places might it have to go before recurring?

603. Multiply $6\cdot0\dot{5}$ by $\cdot1\dot{2}\dot{7}$. (*Answer in decimals.*)

604. Calculate the cost of papering a room 20 ft. long, 17 ft. broad, and 12 ft. high, allowing 42 sq. ft. for doors and windows, the pieces of paper being 64c. per piece of 12 yards long and 21 in. broad, and the cost of putting up the paper 25c. per piece.

605. Find the Compound Interest on $1800 at 6 per cent. for 3 years. How much does it exceed the Simple Interest?

606. A man's income is diminished by $1500; but a tax that he pays of $2\frac{1}{2}$ c. per $1 being raised by one-sixth of its amount, he pays the same amount of tax as before; find his income.

---

607. What fraction of $7\frac{1}{2}$ tons is $1\frac{7}{8}$ of $5\frac{3}{4}$ cwts.?

608. State a rule for division in decimals and apply it to divide 17·35 by 62·5 and by ·0125.

609. Bring 19 tons 13 cwts. $37\frac{1}{2}$ lbs. to decimal of 50 tons; and find the value of 3·1024 of $75.

610. Find the value of 780 things at $4.05 each.

611. Find the difference between the amount of $1205 for 2 years at 5 per cent. Compound Int. and the present value of the same sum due 2 years hence at 5 per cent. Comp. Int.

612. What will be the change of income obtained by transferring $7600 Russian 5 per cent. stock at 84 to Rio Improvement 7 per cent., at $114 for each $100 share?

613. Reduce 18750 inches to fur. po. yds., &c.

614. Reduce $3\frac{1}{5}$ of 8 lbs. 7 oz. to the fraction of 1 cwt. 11 lbs. 9 oz.

615. What time should 24 men take to finish a work of which 18 men have done half in 15 days?

616. If $12 is charged for the carriage of 8 cwts. for 128 miles, how far would 4 cwts. be carried for $1.50.

617. At what rate Simple Interest would $150 amount to $250 in 8 years?

618. A man sells a horse for $203.50 at a loss of $7\frac{1}{2}$ per cent.: what did the horse cost him?

619. Calculate the cost of lining with lead the inside of a cistern (open at top) whose dimensions are, length 6 ft. 8 in., breadth 2 ft. 2 in., and height 3 ft. 2 in.; lead costing 36c. per square foot.

---

620. Simplify $3\frac{15}{16} + 2\frac{1}{4}$ of $3\frac{2}{3} - \frac{5}{9}$ of $\dfrac{1\frac{4}{5} \times 1\frac{3}{8}}{1\frac{5}{6}}$.

621. A coal merchant buys coal at $3.72 per ton, and pays $1.74 per ton for carriage by railway: at what price should it be sold to produce a profit of $33\frac{1}{3}$ per cent.?

622. Divide $1\cdot00625$ by $132\cdot5$ and $\cdot\dot{3}$ by $1\cdot\dot{9}\dot{3}$, each to five places of decimals.

623. Find the weight of 980 things at 5 tons 9 cwts. 87 lbs. 8 oz. each.

624. Required the Compound Interest on $7500 at 4 per cent. for 3 years; and also for 3 years 6 months.

625. A person owes $3689.20, but can only pay $2305.75: what dividend in the $100 can he pay?

626. Find the income obtained by investing $2717 in $3\frac{1}{2}$ per cent. stock at $96\frac{1}{4}$.

---

627. Simplify $\dfrac{\frac{3}{11}(\frac{2}{19} \text{ of } 2\frac{5}{7} + \frac{1}{3} \text{ of } 1\frac{2}{7})}{\frac{2}{13} \times 1\frac{5}{8} \times 1\frac{1}{7} - \frac{1}{11}} \div \dfrac{\frac{1}{3}}{\frac{2}{9} \text{ of } 3 - \frac{1}{17} \times 5\frac{2}{3}}$.

628. Multiply 25 yds. 2 ft. $9\frac{3}{5}$ in. by $3\frac{2}{3}$.

629. If $\frac{2}{3}$ of a field be let for $28, what will be the rent of $\frac{3}{4}$ of it?

630. Find the cost of making a road 4 miles 3 furlongs 11 yards long at $760 a mile.

631. Divide $4633.75 among 5 boys, 2 women, and 2 men, so that each man has four times as much as a woman, and each woman five times as much as a boy.

632. Find the present value of $2831.95 due 9 months hence at 6 per cent.

633. Lace, which can be purchased in Caen at 7.63 francs per metre, incurs an additional value of 65 per cent. by transit to England with the duty and the trade profit: how much will it cost in England per yard? *N.B.*—1 metre = 39·3708 inches. 55 francs = 43 shillings.

634. Divide 4·932 by 115·2 and multiply $3{\cdot}3\dot{5}\dot{7}$ by 2·5.

635. Find the value of ·05 of \$4.80 + $3{\cdot}6\dot{3}$ of \$1.20 + 2·375 of 4c.

636. Simplify $\dfrac{1\frac{1}{2} \times 2\frac{1}{4} + 2\frac{1}{3} \times 5\frac{4}{9}}{2\frac{1}{4} - 1\frac{1}{2} \text{ of } 2\frac{1}{3} + 5\frac{4}{9}}$.

637. If 7 boys catch 4 fish in 6 days, how long will it take a house of 21 boys to catch one fish at the same rate?

638. I buy teas at 72c. and 96c. per lb.; I mix them as 4 to 7; what is my gain per cent. if I sell the mixture at \$1 per lb.?

639. At what rate of interest will \$8210 amount to \$9031 in $7\frac{1}{2}$ years?

640. When and where shall I overtake a friend who starts 18 min. before me and walks at $3\frac{1}{2}$ miles an hour, if I go at $4\frac{1}{4}$ miles an hour?

---

641. Find the value of $3\frac{7}{10} + 1\frac{3}{5}$ of $6\frac{1}{4} + 2\frac{1}{5} + 3\frac{2}{5}$ of $\frac{15}{17}$ of $2\frac{1}{2}$.

642. Reduce 1243 feet to poles.

643. Find the value of $15{\cdot}2\dot{7}\dot{5} + 4{\cdot}081\dot{3} - 2{\cdot}592 + {\cdot}203\dot{5}\dot{1} - 5\frac{12}{25} + 6{\cdot}\dot{2}$.

644. If 10 men working 9 hours a day can mow 104 acres in 8 days, in how many days of $9\frac{1}{2}$ hours could 12 men mow 247 acres?

645. In what time will \$1250 put out at $4\frac{1}{2}$ per cent. Simple Interest, amount to \$1568.75.

646. If by selling a house for \$4000 I gain 25 per cent. on my outlay, what should I gain per cent. by selling it for \$3360?

647. If 3 men and 5 women can do a piece of work in 8 days which 2 men and 7 children can do in 12, how long would 12 men, 10 women, and 21 children, working together, take to do it?

---

648. Divide 716 yds. $3\frac{4}{5}$ in. by $6\frac{1}{8}$.

649. Divide ·5 by 31·25; and $3{\cdot}6\dot{1}$ by $1{\cdot}\dot{6}\dot{1}$.

650. A room is $22\frac{1}{2}$ ft. long, 15 ft. wide, and $19\frac{1}{2}$ ft. high: find the cost of papering its walls with paper a yard wide at 84c. per dozen yards.

651. Find the Present Value of \$341 due in 3 years 5 months, interest being at 4 per cent.

652. Find the present value of a bill for \$645.15 drawn on March 1 at 4 months and discounted April 22, at 10 per cent. per annum, allowing days of grace.

653. A person holding \$10000 in the 3 per cents. sells out at $93\frac{5}{8}$, and invests the proceeds in 4 per cent. Stock at $101\frac{7}{8}$: find the change in his income, allowing $\frac{1}{8}$ per cent. commission in each transaction.

654. *A* can beat *B* by five yards in a 100 yards race, and *B* can beat *C* by ten yards in a 200 yards race: by how much can *A* beat *C* in a 400 yards race?

---

655. Simplify $\frac{117}{185}$ of $1\frac{6}{91} - \frac{5\frac{2}{7}}{13\frac{2}{3}} + 2\frac{1}{7}$.

656. Reduce 17 cwts. 18 lbs. 12 oz. to the fraction of 2 tons 15 cwts.

657. Find the value of 1·55 of 24 yds. 1 in. + $4\frac{8}{45}$ of 21 yds. 9 in.

658. Divide $12{\cdot}44\dot{3}\dot{6}$ by ·0472.

659. If 1572 tiles each 10 inches square would pave a court, how many would be required if the tiles were 15 in. long, 8 in. wide?

660. What is the true discount on a bill for $8111.25 drawn on February 4th, at 10 months, and discounted July 14th at 7½ per cent. per annum? (*N.B.*—Three days of grace are allowed.)

661. What gain per cent. is made by buying pears at 3 for 4c. and selling them at 2 for 6c.?

---

662. Simplify $\frac{2\frac{1}{2}}{2\frac{1}{8}}$ of $\frac{2}{3} + \frac{1\frac{1}{4} - \frac{11}{12}}{\frac{1}{6} + \frac{11}{12}} - \frac{2\frac{11}{15}}{2\frac{3}{5}}$.

663. Simplify $11{\cdot}3\dot{0}\dot{1} + 5{\cdot}02\dot{3}4\dot{5} + 3{\cdot}50\dot{0}3\dot{7} - 5{\cdot}\dot{2} + {\cdot}000\dot{4}\dot{7}$.

664. Multiply 82·043 by ·07432; and divide ·0640475 by 3·425.

665. There are 1,836,472,000 copies of newspapers printed in America in a year. Taking a newspaper on the average at 4000 copies solid to the foot, find the number of miles in a column formed of these copies: also, taking 41 inches to be their average length, find the number of miles they would cover if placed end to end.

666. If *A* can do a piece of work in 6 days which *B* can do in 10 days, and which *A*, *B*, and *C* together can do in 2½ days; in what time can *C* do it alone?

667. A man would lose 17½ per cent. by selling a house for $5775: what price must he ask in order to gain 15 per cent.?

668. If the 3½ per cents. are at 91, find the difference in the income derived from $7800 stock and the investment of $7800.

---

669. Simplify $\frac{3\frac{1}{3} \text{ of } 4\frac{1}{3}}{(2\frac{1}{2} - \frac{1}{3}) \text{ of } (3\frac{1}{2} - \frac{1}{4})}$.

670. Find the value of $\frac{{\cdot}321 \times {\cdot}321 - {\cdot}179 \times {\cdot}179}{{\cdot}321 - {\cdot}179}$ of $5.

671. Find the square root of $27\frac{1}{25}$ and of 90018·0009.

672. In how many years will the Simple Interest on $1418 amount to $106.35 at 2½ per cent.?

673. What sum of money must be invested in 4½ per cent. Stock at 83⅞ to bring an income of $219? (Brokerage ⅛.)

674. A man walking 18 miles finds that in 1 hour 40 minutes the distance he has walked is $\frac{5}{7}$ of the remaining distance; find his speed.

675. A cubical cistern open at the top costs $75.60 to line with lead at 42c. per square foot; how many cubic feet will it contain?

---

676. Find the difference between $\frac{6}{7}$ of 15 acres 155 poles and $\frac{1}{5}$ of 31 acres 107 poles.

677. Simplify $\frac{\cdot562 \times \cdot562 - \cdot188 \times \cdot188}{\cdot562 - \cdot188}$ of \$75.

678. Find the cost of 7 cwts. 72$\frac{1}{2}$ lbs. at \$55.64 per cwt.

679. Find the present worth of \$1368.40 due 8 months hence at 5$\frac{1}{2}$ per cent.?

680. A man walking 13$\frac{1}{2}$ miles finds that in 1 hour 15 min. he has walked $\frac{4}{5}$ of the remaining distance; find his rate of walking.

681. A cubical cistern open at the top costs \$78.40 to line with lead at 32c. per sq. ft.; how many cubic feet does it contain?

682. What was the selling price of goods bought for \$1000.25 and sold at a loss of 4 per cent.?

---

683. Reduce $\frac{4489}{3125}$ to a decimal.

684. Reduce $2\cdot48\dot{0}2\dot{7}$ to a vulgar fraction in its lowest terms.

685. A litre contains 61·027 cubic inches; a gallon contains 231 cubic inches; how many gallons are there in 50 litres?

686. Add together $\frac{11}{35}$ of \$113.40, ·615 of \$24, and $3\frac{3}{11}$ of $4\frac{5}{7}$ of $1\frac{5}{9}$ of \$19.90.

687. Express as recurring decimals the square of $\cdot\dot{5}$ and the square root of $\cdot58\dot{7}$.

688. What is the cost of a piece of square matting 20 ft. 6 in. along one side, made up of strips $\frac{3}{4}$ yard wide, costing 90c. a yard?

689. On a certain line of railway the telegraph posts are placed 58 yards apart, and a passenger counts as he passes 37 in 2 minutes; at what rate is he travelling?

---

690. Simplify $\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$ of $\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{5}}$ of $\frac{\frac{1}{6}-\frac{1}{7}}{\frac{1}{6}+\frac{1}{7}}$ of 585.

691. Find the sum of $2\cdot\dot{4}$, $\cdot\dot{3}\dot{2}$, $\cdot56\dot{7}$, $7\cdot0\dot{5}\dot{6}$, $4\cdot1\dot{7}$, and $\cdot4\dot{3}0412\dot{2}$.

692. Find by Practice the value of a nugget of gold weighing 3 lbs. 11 oz. 8 dwts. 4 grs. at \$18.60 per oz.

693. Calculate the value of $\sqrt{(3+2\sqrt{2})}$ to 2 decimal places.

694. A lawn is half as long again as it is wide; the cost of levelling it at 18c. a square yard is \$846.72; find the cost of enclosing it with a fence at \$1.80 per yard.

695. Find the cubic contents of a square tank whose side is 18 feet long and whose average depth is 6 ft. 9 in.

696. What income would be derived from investing \$9350 in 5$\frac{1}{2}$ Preference Stock at 137$\frac{1}{2}$?

697. If the hands of a clock coincide every 65$\frac{1}{2}$ minutes, how much does the clock gain or lose in a day?

---

698. Simplify $5\frac{1}{3}$ of $\dfrac{1}{1\frac{1}{3}+\dfrac{1}{2\frac{1}{4}}} \div \dfrac{4\frac{1}{6}+5\frac{1}{4}}{4\frac{1}{4}+3\frac{2}{3}}$.

699. If 3 cwts. 26 lbs. of tea cost as much as 30 cwts. 97 lbs. of sugar, how much sugar would be given in exchange for 10 lbs. of tea?

700. Find the H.C.F. of 1445, 595, 408, and 4012.

701. A tradesman buys milk at 5c. a quart and sells at 6c. a quart; how much water has he put with it, if his profit is 60 %?

702. At what rate of interest will the present worth of $397.75 payable 2 years hence be $370?

703. *A* and *B* undertake to do a piece of work for $14; *A* could do it alone in 7 days, *B* in 8 days; with the assistance of a boy they do it in 3 days: how should the money be divided?

704. Which pays best, $3\frac{1}{2}$ per cents. at 96 or 4 per cents. at $109\frac{1}{2}$?

705. *A* can give *B* 10 yards and *C* 21 yards start in a race of 120 yards; *B* can give *C* $1\frac{1}{2}$ secs. start over the same distance; how long does each take to run 120 yards?

---

706. Simplify $\frac{7}{11}$ of $\dfrac{\frac{19}{81}-\frac{9}{49}-\frac{1}{27}}{\frac{1}{2}+\frac{1}{14}-\frac{5}{9}}$ of $(3\frac{2}{5}+\frac{1}{2}-2\frac{1}{10})$.

707. Divide 4203·182 by 2·3175 to 5 significant figures.

708. At Simple Interest, what sum lent at 4 per cent. produces in 3 years the same interest as $540 produces in 2 years at 5 per cent.?

709. At what rate of interest will the present worth of $402.05 payable 2 years hence be $374?

710. If 4 per cent. be lost by selling at 18c. per yard, what price will gain 12 per cent.?

711. Calculate the value of $\sqrt{(\cdot 4+2\sqrt{3})}$ to 3 significant figures.

712. A sum of $720 is due from *A* to *B* now and $1815 is due 7 months hence; show that, reckoning Simple Interest at 5 per cent. per annum, if *A* pay both debts 5 months hence neither will lose.

---

713. A clock gains ·0375 per cent.; what will it gain in a week?

714. Find the value of $\frac{2}{3}$ of 5*s*. + $\frac{3}{4}$ of 8*s*. 6*d*. + $1\frac{7}{17}$ of £2. 2*s*. 6*d*.; and reduce the result to the fraction of $\frac{1}{2}$ of £13. 18*s*. 10*d*.

715. Divide $(\cdot 00056542)^2$ by $(12\cdot 534)^2$ to 5 figures.

716. The whole outside surface of a cubic box is 2075·76 square feet; find the length of one of its edges.

717. The length of a square room is 18 ft. 9 in.; the paper for the walls costs $15.75 at 14c. per square yard; find the height.

718. The interest on $7560 amounts in 5 years to $1260 at Simple Interest; find the rate.

719. I had $5000 stock in $3\frac{1}{2}$ per cents. at 77. I sold out and bought into the $4\frac{1}{2}$ per cents. at $91\frac{2}{3}$: find how much stock I have now, and the change in my income. (*Brokerage is here neglected.*)

720. Divide $2\frac{1}{3} - 3\frac{2}{3} + 2\frac{1}{24}$ by $\frac{7}{9} - \frac{8}{81} + \frac{23}{324} + \frac{1}{3}$ of $\frac{3}{8}$.

721. Divide 6917 by 567, using factors.

722. Multiply $(23{\cdot}045)^3$ by $(13{\cdot}0134)^3$ to 4 figures.

723. Find the value of $6{\cdot}05$ of 24c. + $\cdot 54\dot{3}$ of \$17.82 − $\cdot 2\dot{7}1428\dot{5}$ of \$16.80.

724. I have $\frac{1}{8}$th of a ship, worth \$10452, which is insured for $91\frac{2}{3}$ per cent. of its value; the ship being lost, what do I lose?

725. How much must I invest in the $3\frac{1}{2}$ per cents. at 91 to have an income of \$932 after paying \$7 in \$240 tax?

726. A spirit merchant mixes 52 gallons whisky at \$3.78 per gallon with 60 gallons at \$4.06, and sells the mixture so as to gain $8\frac{1}{3}$ per cent.; at what price per gallon does he sell it?

---

727. Express $\cdot 02\dot{9}$ of \$4.50 as the decimal of \$70.

728. A cubic foot of water freezes into $1{\cdot}089$ c. ft. of ice; how much water is there in a rectangular iceberg 100 ft. by 50 ft. by 20 ft.?

729. If £3 = 20 thalers, 25 thalers = 93 francs; 27 francs = 5 scudi, 62 scudi = 135 gulden; how many gulden = £1?

730. Find the amount at Compound Interest on \$4000 in 4 years at 5 per cent.

731. Divide \$671 into parts proportional to 3, 2, $1\frac{1}{2}$, $\frac{5}{6}$.

732. Find the gain per cent. in selling at 25c. per cwt. coal bought at the rate of \$15.75 for 5 tons.

733. *A*, *B*, and *C* can do a piece of work in 2 hours; *A* and *B* can do it in 3 hours, *B* and *C* can do it in 5 hours; in what time can *A* and *C* do it?

---

734. Divide $\dfrac{3\frac{3}{5} - \frac{27}{125}}{\frac{7}{9} \text{ of } 9\frac{1}{2} \text{ of } 8\frac{1}{3}}$ by $\dfrac{2\frac{1}{2} - 1\frac{3}{5}}{3\frac{1}{2} \text{ of } 4\frac{2}{9} \text{ of } 6\frac{1}{4}}$.

735. Find the value of $\sqrt{3 + \sqrt{3}}$ to 6 figures.

736. Find the price of 3 tons 3 cwts. 85 lbs. at \$35.25 per 3 cwts.

737. *A*, *B*, and *C* can walk at the rate of 3, 4, and 5 miles an hour; they start from New York at 1, 2, and 3 o'clock respectively; when *B* catches *A* he sends him back with a message to *C*; when will *C* get the message?

738. Find the discount on \$6388.25 for 2 months at 6 per cent.

739. In a race *A* can give *B* 3 yards and *C* 5 yards in a hundred; in a mile race *B* gives *C* 32 yards start; which wins? by how much?

740. A reservoir a quarter of a mile long, 100 yards wide, and $20\frac{1}{2}$ yards deep, is emptied in 48 hours by a channel 10 feet wide and 3 ft. 5 in. deep: at what uniform rate will the water flow? (Give the result in yards per hour.)

---

741. Find the smallest multiplier which will make 114400 a perfect cube.

742. Simplify $\frac{7\frac{1}{2}+1\frac{17}{27}}{8\frac{2}{3}+3\frac{23}{33}}-\frac{2\frac{1}{5}-1\frac{3}{8}}{1\frac{2}{5}\text{ of }1\frac{3}{7}}$.

743. Find, by Practice, the value of 72 cwts. 73 lbs. 8 oz. of sugar at $6.12 per cwt.

744. An equal number of guineas, pounds, half-guineas crowns, and half-crowns amount to £398. 5*s.* How many are there of each ?

745. Find the income I derive from investing $1500 in 3 per cents. at 90, and $1470 in 4 per cents. at 112.

746. Find the sum of money which in 1 year at 3 per cent. comp. int. payable quarterly will amount to $800.

747. The price of diamonds *per carat* varies as their weight ; if a diamond of 3 carats costs $450, what does a diamond of 2 carats cost ?

---

748. Find the L.C.M. of 99, 999, and 9999.

749. Find the value of $\cdot 8\dot{3}$ of $7.20 + $\cdot 0\dot{5}$ of $12.60.

750. How long will a man be in walking from London to Eton, a distance of 21¼ miles, when he takes 110 steps of 2½ feet every minute ?

751. Find the cost of papering a room 20 ft. long. 16½ ft. broad, and 12 ft. high, the price of a piece of paper 12 yds. long and ¾ yd. broad being $1.10.

752. Find the sum of money which in 2 years at 4 per cent. comp. int. payable half-yearly will amount to $300.

753. Find the amount of a legacy which after paying 10 per cent. duty when invested in the 4 per cents. at 91 gives an income of $1616 (brokerage ⅛).

754. If 91 men 28 women and 35 boys can reap a field of corn 85 yards by 23 yards, in 4 days of 10 hours, how many women must be associated with 13 men and 5 boys to reap a field 69 yards by 17 yards in 21 days of 8 hours each ?

---

755. Simplify $\frac{5}{7}$ of $9\frac{1}{3}+\frac{2}{9}$ of $7\frac{1}{2}-\frac{5}{11}$ of $10\frac{1}{12}$.

756. Find the product of $\cdot\dot{1}4285\dot{7}$, $\cdot\dot{4}\dot{9}$ and $\cdot 34\dot{2}85714$.

757. Find the cube root of 113140331 in the scale of 7.

758. A grain of gold is beaten out into a leaf of 56 square inches ; how many gold leaves make up the thickness of a leaf of paper $\frac{1}{400}$ of an in. thick [1 cub. ft. of gold weighs 1215 lb. Troy] ?

759. If I gained 1 cent in 6 cents by selling a book for $1.12, what do I gain per cent. if I sell it at $1.02 ?

760. Determine the change of income made by transferring $1050 from the 3¾ per cents. at 105 to the 3¼ per cents. at 91.

761. In running a mile race if *A* can give *B* 22 yards, and *B* give *C* 80 yards, how much may *A* give *C* ?

762. If the price in shillings of a cwt. of goods be multiplied by 3 and divided by 7, the result is the value in farthings of 1 lb.

763. The price of gold is £3. 17*s.* 10½*d.* per oz.: what is the least number of ounces that I must take to the mint that I may receive an exact number of sovereigns in exchange?

764. Find the net income corresponding to a gross income of $4657.50, after paying taxes amounting to $1 in $15.

765. If a profit of 22½ per cent. is made by selling a book for $2.94, what would be its price if the profit were only 5 per cent.?

766. Find the present value of $450 due in 9 months, supposing the 3 per cents. to be at 99.

767. The Great Northern Express leaves London at 8.30 p.m., and reaches Edinburgh (399 miles) at 6 a.m. At what distance from London, and at what time, would it be met by a train leaving Edinburgh at 10.20 p.m., and travelling 38½ miles an hour?

768. The diagonal of a square is 1210 yards long: how many acres does the square contain?

---

769. Find the difference between ·70323 of 1 ton and 3·5646 of 1 cwt.

770. The price of 2 turkeys and 9 fowls is $12.42 and the value of a fowl to that of a turkey is as 2 to 9; find the value of a fowl.

771. An English mile is ·2136 of a German mile; how long do I take to walk a German mile at the rate of 4 English miles per hour?

772. The dimensions of a room are as 5 : 4¼ : 3$\frac{1}{7}$, find to the nearest inch the dimensions of a room containing 6000 cub. ft.

773. If 22 lbs. of tea at 48c. per lb. are mixed with 27 lbs. at 72c. and the mixture sold at 66c. per lb., what is the gain per cent.?

774. If $40.32 be spent in buying carpet ¾ yard wide at 84c. a yard for a room 27 feet long and 16 ft. 9 in. broad, how much of the floor will remain uncovered?

775. *A* does ⅔ of a piece of work in 4 hours, *B* does ¾ of what remains in 1 hour, and *C* finishes it in 20 minutes. How long would they have been doing the whole if they had worked together?

---

776. Simplify $\dfrac{2\frac{1}{3} \text{ of } 4\frac{1}{19} - (2\frac{1}{3} + 4\frac{1}{19})}{17\frac{1}{2} \text{ of } \frac{20}{57}}$.

777. If a cubic foot contain 343 units of volume, find the number of units of area in a rectangle 20 ft. by 300 ft.

778. By selling tea at $1.20 per lb. a grocer clears $\frac{3}{7}$ths of his outlay, find the profit per cent. if he sells the same at $1.12 per lb.

779. If I charge as interest 5c. per $6 per week, how much is this per cent. per annum (simple interest)?

780. Find by duodecimals the continued product of 11 ft. 7 in. 7 pts., 5 ft. 3 in. 3 pts., 2 ft. 8 in. 11 pts. (a *part* = $\frac{1}{12}$ of an inch).

781. Divide \$3382.75 among three men, so that the first shall have \$240 more than twice as much as the second, and the second four times as much as the third.

782. A sum of £251. 15*s.* 11*d.* is transmitted through Paris to New York. £1 is worth 24 fr. ·64c. at Paris, and 6 fr. 23c. are worth 2 dollars 25c. American. Find the value of the sum in dollars.

---

783. Simplify $\frac{\frac{4}{13} \text{ of } \$1.82 + \frac{3}{8} \text{ of } 35\text{c.}}{7} + 1\text{c.}$

784. Add together the circulating decimals $\cdot\dot{4}\dot{7}$, $\dot{3}8\dot{5}$, $\cdot\dot{1}3986\dot{7}$.

785. If 28 men mow 672 acres in 24 days of $8\frac{1}{2}$ hours each, how many hours a day would 120 men take to mow 1080 acres in 9 days ?

786. *A* and *B* undertake to do each $\frac{1}{2}$ of a piece of work ; *A* begins at 9, *B* at 10.30, both stop at 12, having done $\frac{1}{3}$ of the work ; they resume at 1 ; *A* finishes his share at 4 ; when will *B* finish ?

787. A money-lender charges \$12.50 for the loan of \$67.50 for three months : what rate of interest per annum does he charge ?

788. I invest \$14970 in the purchase of $3\frac{1}{2}$ per cents. at 105 and of $3\frac{1}{4}$ per cents. at 97 ; my total income being \$500, how much did I invest in each stock ?

789. The diagonal of a square field is 403 yds. 1 ft. long : how many acres does it contain ?

---

790. If 15 yards 3 qrs. 3 nails of cloth cost \$23.80, how much will be bought for \$5.60 ? (4 nails = 1 qr., 4 qrs. = 1 yd.)

791. If iron weighs 7·8 times the same volume of water, find the length of a bar of iron 2 ft. 6 in. by 1·7 ft. which weighs 10 tons (1 cub. ft. of water = 1000 oz.)

792. Find the value of 14 cwts. 68 lbs. 7 oz. at \$64.32 per cwt.

793. Find the difference between the Simple and the Compound Interest on \$10,000 for 3 years at 4 per cent.

794. Divide \$115.71 among 11 men, 13 women, and 17 children, a woman receiving twice as much as a child, and a man twice as much as a woman.

795. On a model, the scale of which is 6 inches to a mile, what will represent the height of Scawfell Pike which is 3212 ft. high ? Also what will be the distance between two places which on the model are seen to be 23·13 inches apart ?

796. In running a mile race, *A* can give *B* 20 yards, and *B* can give *C* 88 yards : how many yards can *A* give *C* ?

---

797. The G.C.M. of two numbers is 173, and the L.C.M. is 15224 ; one of the numbers is 1384 : find the other.

798. Find the difference between $1\cdot8\dot{0}\dot{7}$ and $2\cdot\dot{0}1\dot{2}$.
Find the value of $\cdot9\dot{2}9\dot{7}$ of 1 lb. 7 oz. 2 drms.

799. *A*'s rate of working is to *B*'s as $\frac{1}{2}$ to $\frac{1}{3}$; *B*'s to *C*'s as $\frac{1}{4}$ to $\frac{1}{5}$; *C*'s to *D*'s as $\frac{1}{6}$ to $\frac{1}{7}$; how long would they all together take to do a piece of work which *A* can do in 930 hours?

800. The diameters of four spheres are as 3·75 : 5 : 6·25 : 7·5 ; prove that in volume one of them is equal to the other three together.

801. The Compound Interest on a certain sum for 2 years at 4 per cent. exceeds the Simple Interest by $5. What is the sum?

802. Supposing *A* can give *B* 20 yards start in a mile race, and can give *C* 40 yards start, how much start can *B* give *C* in a mile race?

---

803. What must be taken from $\frac{1}{8}+\frac{3}{17}$ of $2\frac{1}{8}$ to leave $3\frac{1}{2}$ of $\frac{2}{11}-\frac{1}{2}$?

804. Express 1 yd. 1 ft. $5\frac{1}{4}$ in. as the decimal of $\frac{9}{20}$ of 2 yds. 1 ft. $4\frac{3}{4}$ in.

805. A rectangular field 176 yards broad is rented for $143.76, being an average rent of $17.97 per acre: find its length.

806. I walk to a certain place and ride back in 5 hrs. 15 min.; I can walk both ways in 7 hrs.; how long do I take to ride both ways?

807. If in a population of a million the births are 1 in 20 and the deaths 1 in 30 annually, what will be the population in 5 years?

808. *A*'s income is $\frac{3}{4}$ of *B*'s income ; *A*'s expenditure is $\frac{1}{2}$ of *B*'s income ; *B*'s expenditure = *A*'s income ; show they each save the same.

809. A greengrocer buys a lot of apples at 3 for 2 cents, and twice as many at 2 for 1 cent ; he sells them at an average price of 6 for 5 cents: find what he gained or lost per cent.

---

810. The populations of five parishes are 1236, 452, 364, 516, and 3430 ; what is the population of a sixth that the average population of the six may be 1256·5?

811. Find the weight of a 13-inch iron shell, two inches thick ; the weight of a cubic foot of iron being 141 lbs.

812. It costs $22 to paper a room 18 ft. by 15 ft., 10 ft. high, with paper $\frac{3}{4}$ yd. wide ; find the price of the paper per piece.

813. Of a certain lot of goods I sell one-fourth at a profit of 5 per cent. ; one-third at a profit of $7\frac{1}{2}$; how must I sell the remainder to gain 15 per cent. on the whole?

814. A cask contains 36 gallons of spirits at $3.30 a gallon: how much water must be added to the spirit to reduce the price by 60c. a gallon?

815. By selling cloth at $2.59 per yard, 11 per cent. is gained: at what price must the same be sold to gain 23 per cent.?

816. If 35 metres are equal to 39 yards, and 17 metres to 9 toises, and 5 plethra to 124 toises, how many yards are there in 525 plethra?

---

817. Divide $\frac{1\frac{1}{15} \text{ of } 2\frac{5}{8}}{5\frac{1}{3} - 4\frac{1}{2}}$ by $\frac{2\frac{1}{2} + 1\frac{2}{3}}{81\frac{2}{3} - 62\frac{1}{2}}$.

818. Find the value of $37 \cdot 2\dot{3} \times \cdot 2\dot{6}$ of \$11.25; and divide ·0030135 by ·1435.

819. How many feet in 150 must a road 10775 feet long rise to be carried from a plain to a hill 431 feet in perpendicular height?

820. *A* gives *B* 5 yds. start in 100 yds. and is beaten by 9 in.: in how many more yds. would *A* have overtaken *B*?

821. If I can make \$3 a year more by investing \$3645 in 3 per cents. at 91⅛ than in 3¼ per cents., find the price of the latter stock.

822. If in a box 3 ft. 6 in. by 1 ft. 7 in. by 1 ft. 10 in. I pack 320 books each 9½ in. by 5¼ by ⅞ in., what will be the average thickness of 114 books each 7 in. long by 4½ wide which just fill the box?

823. *A* and *B* having money in the proportion of 7 to 9, *B* pays ⅓ of his money to *A* and receives back ⅐ of what *A* then has, thus leaving *A* with \$8 more than *B*. How much had each at first?

---

824. Simplify $\frac{3 \cdot 125}{6\frac{1}{4}}$ of 2 yds. 2 ft. 11¼ in.

825. Find the value of 1 ton 11 cwts. 77 lbs. of sugar at \$18.06 per cwt.

826. The average daily number of men passing a house on Sunday, Monday, Friday, and Saturday is 1765; the daily average on Tuesday, Wednesday, and Thursday is 1541: what is the daily average for the whole week?

827. Find, to the nearest cent, the Compound Interest on \$13942.50 for ¾ year at 3 per cent. per annum payable quarterly.

828. A town has a water supply of 104 gals. per day per house; if the number of houses is increased by $\frac{1}{12}$th and the total supply diminished by ⅐th, what is now the supply per house per day?

829. I have 3 jars, *A*, *B*, and *C*, holding 1, 3, and 5 gals. respectively; *A* is empty, *B* full of water, *C* full of wine; I fill *A* from *B*; I fill up *B* from *C* and pour the contents of *A* into *C*; I repeat the process; how much wine is there now in *B*?

830. A coin weighs 3 dwts. 15 grs., of which $\frac{3}{40}$ths is alloy, the rest pure silver: if the value of silver rises 8 per cent. what should be the reduction of silver in the coin?

---

831. Find the value of $2 \cdot \dot{1}0\dot{2} - 1 \cdot 0\dot{0}7\dot{1} + \cdot 5\dot{6}$, marking the recurring period correctly.

832. What date was 40,000 days before Sunday, Jan. 1st, 1871?

833. Find the cost of painting the outside of a box 4 ft. 4 in. deep, 5 ft. long, and 3 ft. 6 in. broad, at 3c. per square foot.

834. If 8 per cent. be gained by selling a piece of ground for $5157, what would be gained per cent. by selling it for $5252.50?

835. What per cent. must be deducted for cash from the credit price of an article when credit is given for 9 months and 5 per cent. simple interest allowed?

836. *A* can do a piece of work in $3\frac{1}{3}$ days, *B* in $2\frac{3}{11}$ days, and *C* in $3\frac{1}{13}$ days: in what time will they be able to do it together?

837. Iron weighs 7·112 times as much as water, and water weighs 1000 oz. per c. ft.: how many cubic inches of iron will weigh a ton?

---

838. Simplify $\left(\frac{2}{5} \text{ of } \frac{3}{7} + \frac{3}{7} \text{ of } 2\frac{1}{3}\right) \times \frac{\frac{2}{3} \text{ of } \frac{5}{6}}{\frac{5}{4} + \frac{4}{5}}$.

839. If 1296 tiles each $9\frac{1}{2}$ in. by $4\frac{1}{2}$ in. are required to pave a yard, how many tiles each 6 in. square would be required for a pavement 4-ninths as large?

840. If 1 lb. of a metal containing copper and zinc in the ratio of ·84 to ·16 be melted up with 2 lbs. containing copper and zinc in the ratio of ·75 to ·25: how much copper will there be in 1 lb. of the resulting metal?

841. *A* and *B* walk 4 miles and 5 miles an hour respectively, they start from the same point to walk round a circle in opposite directions; they meet in 5 minutes: what is the size of the circle?

842. A man possesses $16800 Railway Ordinary Stock: what income does he receive when there is a dividend of $6\frac{1}{2}$ per cent. and a tax of $1\frac{1}{4}$ cents per $1 has been deducted?

843. I buy 3 butts of wine, and after mixing them sell the wine at $7.50 a dozen, making thereby a profit of $12\frac{1}{2}$ per cent.; the number of dozens in each butt is 52 and the price of two of the butts $$366\frac{2}{3}$ and $$273\frac{1}{3}$: find the price of the third.

844. A broker in discounting a bill due in 4 months at 3 per cent. per annum charges $12.25 more than the true discount: what is the amount of the bill?

---

845. Multiply 3901 by ·00025634, and $3\cdot4\dot{7}\dot{8}$ by ·875.
Divide 3901 by 62·457 to three places of decimals.

846. A room 22 ft. 9 in. long, 15 ft. 3 in. wide, 11 ft. 6 in. high, is papered with paper 1 ft. 11 in. wide at 8c. a yard: find the cost of the paper.

847. If 25 men in 15 days of $10\frac{1}{2}$ hrs. each can build a wall 300 ft. long, 1 ft. 9 in. broad, and 7 ft. 6 in. high, how high a wall 849 yds. long, 9 in. broad, could 283 men build in 5 days of 9 hrs. each?

848. An army lost 18 per cent. by sickness, then 14 per cent. of the remainder in battle, and then contained 84,624 men: what was the original number?

849. The whole time occupied by a train 150 yds. long in crossing a bridge at the rate of 25 miles an hour is 20 seconds: find the length of the bridge.

850 The net income of an estate after deducting 12 °/₀ for repairs, 3 °/₀ of the remainder for collection and then \$1 in \$30 on the rest for miscellaneous expenses, is \$1547.15: find the gross income.

851. If 4 oz. of gold 17 carats fine (*i.e.* 17 out of 24 parts) are mixed with 6 oz. of gold 14 carats fine, how much gold will there be in an ornament made from the compound and weighing $3\frac{1}{3}$ oz.?

---

852. Simplify $3\frac{1}{6} \times \frac{8}{57} + \dfrac{6\frac{1}{6} - 5\frac{4}{5}}{6\frac{1}{6} + 5\frac{1}{5}} - 2\frac{1}{3} \div 5\frac{3}{7}$.

853. Find the sum, difference, and product of $\cdot\dot{7}6\dot{1}$ and $1\cdot 0\dot{6}$.

854. If sugar sold at 7c. per lb. give a profit of 12 per cent., at what price per ton must it be sold to gain 15 per cent.?

855. A log of timber is 18 ft. long, 1 ft. 4 in. wide, and 15 in. thick: if a piece containing $2\frac{1}{2}$ cubic feet be cut off from one end, what length will be left?

856. The true discount on \$6472 due in 3 months is \$72: find the rate per cent. per annum; at what rate would the banker's discount for the same sum be \$72.81?

857. The price of diamonds per carat varies as their weight: if a diamond of 2 carats costs \$160, what weight is a diamond worth \$1000?

858. *A*, *B*, and *C* walk 4, $4\frac{1}{2}$, and 5 miles an hour respectively; they start from the same point to walk round a circle, *C* walking in the opposite direction to *A* and *B*; *C* meets *A* one minute after passing *B*; what is the size of the circle?

---

859. Divide 12345 by 252 by short division, explaining how you determine the remainder.

860. Three men whose strides are respectively 2 ft. 9 in., 3 ft., and 3 ft. 3 in., walk a mile together: how often are they in step?

861. The area of the floor of a room 21 ft. long 10 ft. 6 in. high is $\frac{5}{11}$ of the area of the four walls: find its breadth.

862. 13 men working 6 days, 8 women working 5 days, and 5 boys working 4 days earn between them \$122.10; the daily wage of a man, a woman, and a boy are as 5 : 3 : 2: what does a man get?

863. Divide \$105 among 8 men, 7 women, and 23 children so that each man may have as much as a woman and two children together, and each woman may have as much as three children.

864. Which is the better investment, the 10 per cents. at $211\frac{5}{8}$ or the 2 per cents. at $42\frac{1}{4}$? Which is the better investment if $\frac{1}{8}$ per cent. is paid for brokerage?

865. How long would a column of men extending 3420 feet in length take to march through a street a mile long at the rate of 58 paces a minute, each pace being $2\frac{1}{2}$ feet?

---

866. State the rule for determining if a vulgar fraction will reduce to a recurring decimal, and apply it to the fractions $\frac{16}{50}$, $\frac{17}{51}$, $\frac{18}{52}$.

867. If 15 per cent. is lost by selling a picture for $637.50, what should be the price that only 5 per cent. may be lost?

868. Two brothers receive respectively $\frac{8}{19}$ and $\frac{9}{20}$ of an estate; the difference of their shares is $209: find the value of the estate.

869. If 9 oz. of gold 10 carats fine, and 5 oz. 15 carats fine be mixed with 7 oz. of unknown fineness, the mixture is 14 carats fine: what is the unknown fineness? [See Question 941.]

870. *A* runs at the rate of 12 and *B* of 13 miles per hour; in a race *B* wins by 15 secs.: how long is the race?

871. A sphere of lead 3 in. in diameter is recast into three other spheres, two of which are $1\frac{1}{2}$ in. and 2 in. in diameter: find the diameter of the third.

872. *A* and *B* leave Bristol together for London (119 miles), riding 16 and 17 miles per hour respectively; *C* leaves London for Bristol at the same time, riding 19 miles per hour: find how many minutes elapse between the times at which *C* passes *A* and *B*.

---

873. Find the difference between $4\cdot\dot{2}8571\dot{4}$ sq. ft. and $\cdot\dot{1}4285\dot{7}$ sq. yds.

874. If the premium on fire insurance be 13c. per $100, what sum has to be paid on $2162.70?

875. A man invests $21000 in the Russian 6 per cents. at $87\frac{1}{2}$, which he sells out at 85 and invests the proceeds in Railway 6 per cent. Debentures at $127\frac{1}{2}$: find the change in his income.

876. Find how much cardboard is required to make a cubical box and its cover; the edge of the box is 9 in., and the rim of the cover extends 3 in. deep down each side.

877. If a cent be the interest on 48 cents for a month, what is the rate per cent. per annum?

878. If 175 men and 240 boys do in 1330 days the same amount of work as 603 men and 1005 boys in 350 days, compare the average daily work of a man and a boy.

879. *A*, *B*, and *C* ride 16, 17, and 18 miles per hour respectively; *A* and *B* start from London and *C* from Bristol at the same time; *C* passes *A* 6 minutes after passing *B*: how far is it from London to Bristol?

---

880. Find the greatest prime number which, used as divisor of 68130, will leave remainder 27.

881. *A*, *B*, and *C* together purchased a ship; *A* paid for $\frac{2}{9}$ of the ship, *B* paid for $\frac{3}{5}$, and *C* paid $2000: what did *A* pay?

882. On what day of the week will February 10th fall in the years 1960 and 2060?

883. From an account for $32.25 due 18 months hence a tradesman deducts $2.25 for cash payment, at what rate per cent. Simple Interest is the true discount calculated?

884. If 8 men do as much work as 14 women, and 5 women do as much as 9 children, divide $67.50 among 4 men, 6 women, and 9 children.

885. If the rate of exchange between England and Turkey be 51 shillings for 5 ducats, and between England and France be 8*s.* 6*d.* for $10\frac{1}{2}$ francs, find the value of 126 francs in ducats.

886. A man bought apples at the rate of 3 for 1c., and an equal number at 5 for 1c.: what profit per cent. did he make by selling them again at 5 for 2c.?

---

887. Find approximately the length of the diagonal of a square field which contains 8 acres.

888. Five men can do a piece of work in 2 hours which six women could do in $2\frac{1}{2}$ hours, or eight children in 5 hours: how long would a party made up of a man, a woman, and 3 children take to do it?

889. *A* runs a 2-mile race with *B* and loses; had he run a third quicker he would have won by 22 yards; compare their speeds.

890. If 60 c. m. of lead with 54 c. m. of cork weigh as much as $1538\frac{2}{3}$ c. m. of deal, and the weights of equal volumes of lead and deal are as 11·324 : ·45, what number represents an equal volume of cork?

891. Find by duodecimals the height of a room 15 ft. 1 in. 3 pts. by 8 ft. 7 in. 4 pts. which contains 812 cub. ft. $1551\frac{1}{2}$ cub. in.

892. In running a 3-mile race on a circular course of $\frac{1}{3}$ of a mile, *A* overlaps *B* at the middle of the 7th round; by how much will *A* win at the same rate?

893. If 1 candle $8\frac{1}{2}$ in. long cost 3c., and 3 candles of the same size and quality $10\frac{1}{4}$ in. long cost 11c., what is the saving per cent. in buying the one instead of the other?

---

894. Find the H.C.F. and L.C.M. of 111540, 296352, 404352.

895. Find by duodecimals the volume of a stone 3 ft. 4 in. 5 pts. by 4 ft. 3 in. 7 pts. by 1 ft. 1 in.

896. Find to the nearest lb. the error in calculating the weight of 1000 cub. ft. of pure water on the assumption that 1 cub. ft. weighs 1000 ounces.

897. A bag contains 300 coins, sovereigns, half-crowns, and shillings; the value of the half-crowns and shillings = that of the sovereigns; the number of sovereigns = $\frac{2}{3}$ that of the half-crowns; how many shillings are there?

898. I have tea which cost 96c. and some which cost $1.32 per lb.; how must I mix them so that by selling at $1.38 per lb. I may make a profit of 15 per cent.?

899. I want to buy a certain number of apples for a certain sum; if I buy at the rate of 2 for a cent, I shall spend 10 cents too much; if at the rate of 5 for 2 cents, 20 cents too little; what is the sum?

900. Find the ratio of a kilogram to a pound (Avoir.) taking the weight of 36 c. ft. of water to be a long ton and 8 metres = 315 inches.

---

901. A man who receives $3018 interest from a certain amount of 3 per cent. stock, sells out and so realises $90037: at what price is stock selling?

902. *A* can do a certain piece of work alone in 8 days, and *B* the same in 5; after working $2\frac{3}{4}$ days they are joined by *C*, and just finish it by the end of that day: how long would *C* take to do it alone?

903. A clock which gains $2\frac{1}{4}$ minutes in 24 hours is 3 minutes slow at noon on Sunday: when will it indicate correct time, and what will it indicate at 6 o'clock on Monday evening?

904. If 4 men and 14 women can do a piece of work in 5 days which 5 men and 7 children can do in 8 days, or 5 women and 4 children can do in 14 days, find (i) how many men, (ii) how many women, (iii) how many children could do the work in one day.

905. The volumes of spheres are in proportion to the cubes of their radii: the radii of two spheres are in the ratio of 4 to 5; and the weights of equal portions of the smaller and larger spheres are as 12 : 7; given that the weight of the smaller sphere is 256 lbs., find the weight of the larger.

906. Two equal wine-glasses are respectively one-half and one-third full of wine; they are then filled up with water and the contents mixed; half the mixture is then poured back into one of the wine-glasses: what part of the contents of that glass will be wine and what part water?

907. A cubic inch of a certain wood weighs $5\frac{5}{6}$ drams: a piece, measuring $4\frac{1}{16}$ in. long, $2\frac{1}{5}$ in. broad, and 1 in. thick, was cut into matches 2 in. long and $\frac{1}{10}$ in. thick both ways; the waste and sawdust weighed $9\frac{47}{72}$ drams: how many matches were made?

908. One clock gains 2 minutes in 3 days, another loses 6 minutes in 6 days: if they are set right at 12 o'clock to-day, when will their times differ by a quarter of an hour?

909. Supposing the alloy in a half-dollar to be $\frac{1}{18}$ of the mass, and the coin to be worth a cent if it were all alloy, what would be its exact value if it were all pure silver?

910. *A*, *B*, *C* working together can mow a field in 3 days: when *B* and *C* work together they take $4\frac{1}{2}$ days, while *A* and *B* observe that after working together half a day $\frac{1}{10}$ of the field is done: find how long each would be working alone.

911. *A* starts from Cambridge at 3 p.m. to meet a carriage which left London at 1.30 p.m.; *A* walks $4\frac{1}{2}$ miles an hour, and the carriage goes $8\frac{1}{2}$ miles an hour: the distance being $51\frac{3}{4}$ miles, when and where will *A* meet the carriage?

912. In a game at fives, out of 15 points *A* can give *B* 3; also *A* can give *C* 7 points: how many points can *B* give *C* so as to make an even match?

913. *A* and *B* run a mile in opposite directions: *A*'s running is to *B*'s as $6\frac{1}{2} : 5\frac{1}{2}$; *B* gets 4 seconds start, during which time he runs $12\frac{1}{10}$ yards: find when he will pass *A*.

914. Divide $78.65 among *A*, *B*, *C* so that *A*'s share may be to *B*'s as 4 : 3 and *B*'s to *C*'s as 10 : 17.

915. One boy runs 300 yards and another 285 yards in 1 minute. How many yards start must the second have so that they may run a dead heat in a mile race?

916. The premium on my life insurance is 15 per cent. of my income; after deducting this I pay a tax of $3\frac{1}{3}$ cents. per $1, and my net income is then $4930; what is my gross income?

917. Two sums of money are to be divided among 3 persons, the first sum equally, and the second in the proportion of 3 : 5 : 8; the first two receive $67.25 and $84.75 respectively; what does the third receive?

918. If 8 men can sink a well 5 feet in diameter, 48 feet deep in 20 days of 8 hours each, how many nights of 9 hours each will 3 men be in sinking a well 4 feet in diameter, 225 feet deep; the difficulty of working by night being one-eighth greater than by day, the hardness of the ground in the latter case being to that of the former as 9 : 5?

919. Find the Simple Interest on 333.33\frac{1}{3}$ in $3\frac{1}{3}$ years at $3\frac{1}{3}$ per cent.

920. The weights of gold and silver are respectively 15 times and 8 times that of equal volumes of water; a bar of gold a foot long and 1 in. square weighs 8·38 lbs.; find the weight of a bar of silver 10 in. by 4 in. by 3 in.

921. Determine the present value at 4 per cent. Compound Interest of an annuity of $500 payable for three years, the first payment being due in a year's time.

922. A retail dealer adds 10 per cent. of water to the pure spirit he buys and sells the mixture at a price 10 per cent. greater than the cost price of pure spirit; find his profit per cent.

923. Prove that Compound Interest reckoned quarterly at $1.227 per cent. is very nearly equivalent to interest reckoned yearly at 5 per cent.

924. The price of gold is $18.69 per ounce; the value of a mixture of silver and gold weighing 216 ounces is $824.40; but if the weights of the silver and gold were interchanged the value would be $3478.32; find the weight of silver in the mixture and the price per ounce.

925. *A* can do $2\frac{1}{2}$ times and *B* $1\frac{1}{3}$ times as much work as *C* in a day. *A* and *C* are employed together for 10 days on a piece of work, which they could finish in 12 days; but *A* is now taken off and *B* put on; when will the work be finished?

926. Find the smallest sum which can be divided between four persons in proportion to the reciprocals of 4, 5, 6, 7, so that each may receive an exact number of shillings.

927. Show (without actually working it out) that $\frac{1}{49}$ expressed as a recurring decimal cannot have a period of more than 42 digits.

928. A watch has a seconds hand on the same axis as the hour hand; the hour hand, the minute hand, and the seconds hand are together at noon; show that they are never together except at noon.

929. Prove the following rule for determining whether a number is divisible by 99. Separate the digits into pairs beginning at the units' figure, and add together the numbers thus formed; separate the digits of the resulting number as before, and again add, and so on; if the number is divisible by 99 the result is at last 99.

930. A man embarks his whole property in four *successive* ventures; in the first he clears 100 per cent., and in each of the others he loses 20 per cent.; show that there remains to him 2·4 per cent. profit on his original outlay.

931. A number of men can be formed either into a solid square, or into a hollow square 9 deep having 970 men in the front rank of each side; how many men are there in each side of the solid square?

932. The areas of the faces of a rectangular solid are 9 sq. ft., 16 sq. ft., 25 sq. ft. respectively; find the contents of the solid and the length of each edge.

933. Two cogged wheels work together, there being 32 cogs in one and 36 in the other; the larger wheel makes 64 revolutions per second; how often are the same cogs in contact during 6 working days of 10 hours each?

934. I purchase $5440 stock at $126\frac{7}{8}$; the stock is then changed into an annuity of $$5\frac{5}{8}$ for every $100 stock. I sell this annuity at $$24\frac{7}{8}$ for each $1 of annuity; what profit do I make?

935. The work of 3 men is equal to the work of 5 women, or to the work of 7 boys; 3 men, 3 women, and 8 boys together complete two-thirds of a certain work in 7 days, working $8\frac{1}{2}$ hours a day; how long will it take 4 men, 11 women, and 7 boys to finish the work, working 9 hours a day?

936. A man buys a certain number of apples at 3 for 2c. and thrice as many at 2 for 1c.; he sells them at an average price of 10c. per dozen; what does he gain or lose per cent.?

937. I look at my watch between 4 and 5; on looking again between 7 and 8 the hands have exactly changed places; at what time did I first look at my watch?

938. A hare is 500 leaps before a greyhound, and takes 4 leaps to his 3, but 2 of the greyhound's leaps are as much as 3 of the hare's; in how many leaps will the greyhound catch the hare?

939. 1 lb. of tea and 3 lbs. of sugar cost together $1.50; if sugar were to rise 50 per cent. and tea 10 per cent. in price they would cost $1.75; find the price of tea.

940. In a mile race *A* starts at 270 paces of 48 inches per minute, and *B* at 300 paces of 44 in. per min.; after 4 min. *A* quickens his pace to 320 per min.; which wins, by how much, and in what time?

941. If 7 oz. of gold 22 carats fine, 12½ oz. 21 carats fine, and 17 oz. 19 carats fine are melted together, what is the fineness of the compound? [*N.B.*—21 carats fine means 21 parts of pure gold to 3 of alloy; there are always **24** parts, or carats, of compound.]

942. A man bought 76 sheep; he sold a quarter of them at 15 per cent profit, and the remainder at 20 per cent. profit, and realised $902.50; what did he give for each sheep?

943. A train 352 feet long overtakes a man walking in the same direction at the rate of 4 miles an hour, and passes him completely in 15 seconds; when going at the same rate the train passes another man in 9 seconds; at what rate is the second man walking?

944. The dimensions of a rectangular box are as 2 : 3 : 4, and the difference between the cost of covering it with sheet lead at 16c. and at 17c. per square foot is $1.17; find its dimensions.

945. Two men run a 3 mile race round a course of $\frac{3}{10}$ of a mile; their rates are as 4 : 3; how often and where do they pass each other?

946. What quantity of metal will be required to make a hollow spherical ball, the external diameter being 18 in. and the thickness 4½ in.? [See Art. 154.]

947. In Fahrenheit's thermometer the freezing and boiling points are 32° and 212° respectively; in the Centigrade thermometer they are 0° and 100° respectively. What temperature Centigrade corresponds to 65° Fahrenheit?

948. The pressure of compressed air varies inversely as the volume; the pressure on the inner surface of a cylinder fitted with a piston is 20 lbs. on each sq. in., and when the piston is forced in 2 in. the pressure becomes 30 lbs. on each sq. in.: what is the length of the cylinder?

949. The map of a country is drawn on a scale of $\frac{1}{10}$ of an inch to a mile; what area on the map will represent 4000 acres?

950. A hollow circular cylinder stands on a solid cubic pedestal of the same material, whose edge is 6 ft.; the internal and external diameters of the cylinder are 4 and 5 ft.; the weight of the cylinder is the same as that of the pedestal: find its height.

951. A contractor undertakes to execute a certain work in a given time; he employs 55 men, who work 9 hours daily; when $\frac{3}{4}$ of the time has expired he finds that only $\frac{4}{7}$ of the work is done; how many men must he now employ 11 hours a day to fulfil his contract?

952. *A*, *B*, and *C* can do a piece of work in 60 days; after they have worked for 10 days *A* withdraws, and *B* and *C* continue working at the same rate for 20 days; *B* then withdraws, and *C* completes the work in 96 days, working $\frac{1}{3}$ longer each day. Working at his former rate *C* could do the work in 222 days; in how long can *B* alone do it?

953. A coin weighs 87 grains, of which 3 parts out of 40 are alloy, the rest being pure silver; if the value of silver rise 8 per cent., what must be the reduction of pure silver in the coin?

954. Mercury expands $\frac{1}{10000}$ of its volume for every degree (Fahrenheit) of temperature; find the weight of a column of mercury 6 in. high and three-fourths of a sq. in. in section at a temperature of 70°, assuming the weight of a cub. ft. of water at 60° to be 1000 oz., and mercury at the same temperature to weigh 13·6 times water.

955. A man sells an article at 4 per cent. profit; if he had bought it at 4 per cent. less, and sold it for 20 cents more, he would have gained 10 per cent.; what was the cost price?

956. A man has $5000 stock in the 3 per cents. which he invests in the $3\frac{1}{2}$ per cents. at $87\frac{1}{2}$, thus increasing his income by $5; what is the price of the 3 per cents.?

957. A tradesman's prices are 12 per cent. above cost price; and he allows a discount of $2.10 on a bill of $33.60; what is his net profit per cent.?

958. A vessel of 24 gallons is full of water; another of 12 gallons is full of wine; one gallon is taken from each and then poured into the other; this is done three times; how much wine will the vessels then respectively contain?

959. Two clocks strike 9 together on Monday morning; on Tuesday morning one is 8 min. to 11 when the other strikes 11; how much must the first be put on that they may both strike 9 together in the evening?

960. In Reaumur's thermometer the freezing and boiling points are respectively 0° and 80°; when the Centigrade stands at 5° below zero where will (i) the Fahrenheit, (ii) the Reaumur's stand? [See Question 947.]

961. The mean temperature from the 9th to 15th January 1877, both included, was 36·6°, and from the 10th to the 16th it was 39·9°; the mean temperature on the 9th was 30·50°; what was it on the 16th?

962. A train starts full of passengers; at the first station it drops

$\frac{1}{3}$ of these and takes in 96 more; at the next station it drops $\frac{1}{2}$ of the new total and takes in 12 more; on reaching the third station there are found to be 248 left; what number started?

963. A man's income from 5 per cents. after paying a tax of $2\frac{1}{2}$ per cent. on his income is $2925; he sells out at 115, and invests in the 4 per cents. at par. Find the change in his net income (allowing for the tax).

964. A rectangular field contains 6 a. 132 p., its length is to its width as 21 to 13; how many rods of fencing are required for it?

965. A tradesman asked a price for a watch which was 40 per cent. above cost, and gave the purchaser 10 per cent. discount on the price asked, gaining thereby $2.60; what was the cost price?

966. Two rods each a foot long are divided, the one into ten equal parts, the other into 11 equal parts. They are placed side by side so that the seventh points of division in each coincide. What is the distance between their extremities?

967. *A* can do as much work in 3 days as *B* can do in $4\frac{1}{2}$ days; *B* can do as much in 9 days as *C* in 12; and *C* can do in 10 days what *D* can do in 8. How many days' work of *D* are equal to 5 days' work of *A*?

968. *A*, *B*, and *C* can do a piece of work in 12 hours; *A* and *B* can do it in 16 hours; *A* and *C* in 18 hours. What part of the work can *B* and *C* do in $9\frac{1}{7}$ hours?

969. There are 70 bushels of grain in two bins, and in one bin there are 10 bushels less than $\frac{3}{5}$ as much as there are in the other. How many bushels are there in the larger bin?

970. How much per cent. profit must a tradesman charge, so that he may allow 5 % of his sales for bad debts, an average credit of 6 months, and 4 % of the cost of his goods for expenses, and may make a net profit of 10 % on the cost price, money being worth 7 % per ann.?

971. Gold weighs 19·3 times and copper 8·89 times as much as water; how many times as heavy as water is a coin containing 11 parts of gold and 1 of copper?

972. What must be the market price of 3 per cent. stock, that it may give $3\frac{1}{3}$ % interest after deducting 35c. in every $12.

973. An eight gallon cask is full of brandy, and a ten gallon cask is full of water; how much must be transferred from one cask to the other that the mixture may be of equal strength?

974. Determine the depth of a conical glass 2 in. in diameter across the top, that 12 of them may contain a pint. (*N.B.*—The volume of a cone is '*one-third the height* × *area of the base.*')

975. The difference between the Interest for 4 months and Discount on a certain sum due in 4 months, at 4 % is $1; what is the sum?

976. Two clocks begin to strike twelve together; one strikes it in

28 seconds, the other in 25 seconds; what is the interval between their fifth strokes?

977. If 22 oxen eat 33 acres of grass in 54 days, and 17 oxen eat 28 acres of similar pasture in 84 days (the grass growing uniformly, and there being the same amount of grass per acre to begin with); how many oxen will exhaust 40 acres in 24 days?

978. I invest half of a certain sum in the 3 per cents. at 92 and the rest in $3\frac{1}{2}$ per cents. at 98 and receive $55 as income; find the sum.

979. A, B, and C are three towns on a river (which flows uniformly) and B is equidistant from A and C; I can row from A to B and back in $5\frac{1}{4}$ hrs., and I can row from A to C in 7 hrs.; compare the speed of my boat in still water with that of the river.

980. A clock set right at noon indicates 3 minutes past 6 at 6 o'clock, at what time does it indicate 6 o'clock?

981. A is four years more than twice as old as B; if they both live for 10 years more A will be 45 years less than thrice as old as B; find their present ages.

982. Divide $2174 among A, B, and C, so that A may receive $40 more than a third of what B receives, and C may have as much as A and B together.

983. A, B, and C run in a mile race; A beats B by 20 yds., and C by 40 yds.; by how much would B beat C in a half-mile race, supposing the speeds at which each man runs a mile and a half-mile respectively are in the ratio of 8 to 9.

984. I can obtain $6 more per annum by investing a certain sum in 5 per cents. at $128\frac{1}{4}$ than in the 3 per cents. at 81; what is the sum?

985. A spherical balloon contains 28974·25 cub. ft.; find to the nearest yard the number of sq. yds. of silk required to make it. [Art. 154.]

986. A gasholder is to be constructed in the form of a circular cylinder, such that the radius of the circle is to be equal to its height; find its dimensions that it may contain 100,000 cubic feet of gas.

987. A town increases at the rate of 2·5 per cent. every year; it has now 68921 inhabitants; how many had it 3 years ago?

988. A train increases its average speed per minute 10 per cent. every minute; in 4 minutes it travels 1547 yds.; how far did it travel in the first two minutes?

989. I buy two horses for $450, and sell one so as to lose 4 per cent., and the other so as to gain 5 per cent.; and on the whole I neither gain nor lose; what did each horse cost?

990. A cube of lead whose edge is 10 inches is wholly immersed in a cylindrical tub partly filled with water, whose internal diameter is 18 in.; how much will it raise the water in the tub?

991. The time of oscillation of a pendulum varies as the square root of its length, and the pendulum which beats seconds is 39·1375 in. long; how many times per hour will a pendulum 1 yd. long oscillate?

992. Supposing sound to travel 1120 ft. per sec., at what distance from a reflecting obstacle ought I to stand when pronouncing 10 syllables in a second that the echo may repeat the last four syllables?

993. At Compound Interest, the Interest on a certain sum for the first year is $140, and for the second year $147; find the Interest for the third year.

994. Find the surface of a cube which contains 5268·024 cub. in.

995. For what sum should a house worth $9000 be insured at $\frac{1}{10}$ per cent. that in case of fire the owner may receive the value of the house and 1 year's premium?

996. Find to three significant figures the radius of a solid sphere of iron of the same weight as a lead sphere whose radius is 4·25 in., the densities of lead and iron being as 11·3 : 7·2.

997. Given that 27·7274 cub. in. of water weigh 1 lb., and that ice is ·93 times as heavy as water, find to the nearest inch the edge of a cube of ice weighing 1 ton 2 cwt. 40 lbs.

998. A man sold a house at a loss of 20 per cent.; if he had received $300 more for it he would have gained 10 per cent.; what did the house cost him?

999. The ratio of the interest to the true discount on a certain sum for a certain time at a certain rate per cent. per annum is 21 to 20; find the rate per cent.

1000. The population of a certain town increases at a certain rate per cent. per annum; now it is 194,481; four years ago it was 160,000; what will it be 2 years hence?

1001. The published price of a book is 40$\frac{5}{8}$ per cent. greater than the price charged to a bookseller for a single copy; when taking a quantity the bookseller gets 13 copies as 12; find his profit per cent. if he allows to purchasers a discount of 25c. in $1 (i) when he buys a single copy, (ii) when he takes a quantity.

ANSWERS

I. PAGE 6.

1. Hundred, ten, one; 3, 4, 7. **2.** Thousand, hundred, ten, one; 4, 2, 8, 1. **3.** Ten thousand, hundred, ten; 6, 2, 2. **4.** Million, hundred thousand, ten thousand, thousand, hundred, ten; 4, 2, 3, 1, 5, 9. **5.** Ten million, million, thousand, ten; 2, 3, 4, 1. **6.** Hundred million, million, ten thousand, one; 4, 1, 3, 6. **7.** Hundred million, ten million, hundred thousand, thousand, ten; 2, 1, 4, 2, 3. **8.** Ten million, ten thousand, one; 9, 4, 1.

II. PAGE 7.

1. Four thousand seven hundred and twelve. **2.** Five hundred and seventy-six thousand three hundred and twenty-one. **3.** Fifteen thousand two hundred and sixteen. **4.** Eight million nine hundred and seventy-six thousand four hundred and twenty-one. **5.** Six hundred and sixty-six thousand six hundred and sixty-seven. **6.** Eighty-seven million six hundred and fifty-four thousand three hundred and twenty-one. **7.** One hundred and twenty-three million four hundred and fifty-six thousand seven hundred and eighty-nine. **8.** Twelve thousand three hundred and forty-five million six hundred and fifty-four thousand three hundred and twenty-one. **9.** Eleven million one hundred and eleven thousand one hundred and eleven. **10.** Nine hundred and ninety-nine thousand nine hundred and ninety-nine. **11.** Forty-one million three hundred and sixty-five thousand one hundred and twenty-three. **12.** Eight thousand six hundred and fourteen million five hundred and thirty-one thousand two hundred and thirty-six.

III. PAGE 8.

1. 1001. **2.** 900. **3.** 21,006. **4.** 4000. **5.** 500,000. **6.** 50,000. **7.** 405,000. **8.** 5,000,000. **9.** 4,005,000. **10.** 20,000,000. **11.** 70,001,000. **12.** 2,000,000,000,050. **13.** 609,001,200. **14.** 4,000,900,000. **15.** 100,000,010,000. **16.** 900,900,900,090. **17.** Forty. **18.** Eight hundred. **19.** Seven thousand. **20.** Six hundred and one. **21.** Seventy thousand five hundred. **22.** Eight hundred thousand one hundred and two. **23.** Four hundred and ten thousand. **24.** Seven million ninety thousand seven hundred and nine. **25.** Eighty million eighty thousand and eighty. **26.** Forty-five million sixteen thousand. **27.** Eight hundred million one hundred thousand and ten. **28.** Seven hundred and eighty million nine hundred and seven thousand eight hundred and twelve. **29.** Eight thousand one hundred and ten million one hundred and one thousand. **30.** Sixty-six million six hundred thousand and sixty-six. **31.** One thousand and ten million one hundred and one thousand and ten. **32.** Eight thousand and seventy million sixty thousand and ten. **33.** Four hundred million. **34.** Four thousand and fifty million, sixty thousand.

IV. PAGES 11, 12.

1. 143. **2.** 303. **3.** 617. **4.** 2148. **5.** 1,088,064. **6.** 136,559. **7.** 1,715,608. **8.** 4,417,919. **9.** 21,622. **10.** 185,380. **11.** 711,768. **12.** 11,137,929. **13.** 2,550,739. **14.** 17,731,625. **15.** (*a*) 72,720 ; (*b*) 147,720. **16.** 33,884,848. **17.** 8,990,081. **18.** £18,071,192. **19.** 2,148,724,911 dollars.

V. PAGE 14.

1. 40. **2.** 429. **3.** 82. **4.** 689. **5.** 98. **6.** 29. **7.** 355. **8.** 506. **9.** 339,884. **10.** 767,855. **11.** 776,124. **12.** 1. **13.** 47,488. **14.** 8,889,999. **15.** 999,999. **16.** 111101. **17.** 7202. **18.** 951,691. **19.** 4190. **20.** $3640. **21.** 37875 pounds. **22.** 33,691. **23.** $61,950.

VI. PAGES 15, 16.

1. 8352. **2.** 251. **3.** 71. **4.** 628. **5.** 6426. **6.** 27. **7.** 2372. **8.** 65,911. **9.** 48,910. **10.** 513,583. **11.** 8590. **12.** 23,822. **13.** 922,177. **14.** 6132. **15.** 1556. **16.** 17,569. **17.** 499. **18.** $1335. **19.** $14,790.

VII. PAGES 20, 21.

1. 63. **2.** 144. **3.** 729. **4.** 315. **5.** 34,741. **6.** 703,448.
7. 496,120. **8.** 1,051,320. **9.** 141. **10.** 208. **11.** 1212.
12. 3384. **13.** 38,070. **14.** 934,252. **15.** 6,703,400. **16.** 73,000.
17. 471,300. **18.** 88,020. **19.** 8,018,100. **20.** 3,232,000. **21.** 2961.
22. 2655. **23.** 3723. **24.** 1,908,864. **25.** 33,239,080.
26. 48,269,334. **27.** 1,807,616. **28.** 107,731,082. **29.** 15,884,764.
30. 28,451,501. **31.** 327,407,970. **32.** 734,267,385. **33.** 46,288,242.
34. 330,497,032. **35.** 176,862,716. **36.** 77,629,290. **37.** 87,781,368.
38. 58,443,264. **39.** 60,546,748,460. **40.** 288,922,236,841.
41. 3,993,290,527,404. **42.** 213,202,597,608,000. **43.** 479,129,472.
44. 7,623,359,510. **45.** 333,264,634,710. **46.** 75,214,177,695.
47. 202,982,874,624. **48.** 63,698,507,715. **49.** 56,093,959,429.
50. 188,311,316,733. **51.** 9,869,619,144. **52.** 16,789,079,014.
53. 159544 yds. **54.** $5820. **55.** 24,297,000; 1,266,915,000.
56. 289,240; 15,081,800. **57.** 16,632,000,000.

VIII. PAGE 22.

1. 645. **2.** 8547. **3.** 6555. **4.** 8262. **5.** 4440. **6.** 49,893.
7. 11,496. **8.** 58,944. **9.** 13,365. **10.** 81,270. **11.** 2765.
12. 96,228. **13.** 149,688. **14.** 593,712. **15.** 40,095. **16.** 52,218.
17. 378,756. **18.** 1,094,100. **19.** 1,200,240. **20.** 3,856,600.
21. 223,000. **22.** 111,375. **23.** 1,175,328. **24.** 2,899,962.
25. 594,347. **26.** 451,256. **27.** 12,517,442. **28.** 11,633,490.
29. 59,428,016. **30.** 144,145,091. **31.** 8,004,078. **32.** 32,337,202.
33. 9,036,624. **34.** 12. **35.** 57. **36.** 2141. **37.** 17,084. **38.** 20,111.
39. 73,258. **40.** 92,686. **41.** 65,560. **42.** 9144. **43.** 37,376.
44. 77,430. **45.** 37,577. **46.** 20,777. **47.** 43,782.

IX. PAGE 24.

1. 6. **2.** 12. **3.** 12. **4.** 27. **5.** 24. **6.** 25. **7.** 250. **8.** 150.
9. 17. **10.** 157. **11.** 12. **12.** 20. **13.** 7833, *rem* 2. **14.** 544,44, *r* 3.
15. 458,161, *r* 7. **16.** 875,000. **17.** 5,346,680, *r* 5. **18.** 1,556,671, *r* 7.
19. 9347, *r* 37. **20.** 46,321, *r* 95. **21.** 4725, *r* 111. **22.** 91, *r* 178.
23. 13,717, *r* 37. **24.** 18, *r* 3000.

X. PAGE 26.

1. 152, *rem* 19. **2.** 19, *r* 31. **3.** 173, *r* 21. **4.** 13, *r* 32. **5.** 242, *r* 21.
6. 123, *r* 28. **7.** 646, *r* 34. **8.** 645, *r* 85. **9.** 585, *r* 48. **10.** 4927, *r* 22.
11. 8230, *r* 20. **12.** 4592, *r* 76. **13.** 1370. **14.** 5758, *r* 84.
15. 9781, *r* 33. **16.** 8117, *r* 16. **17.** 3243, *r* 41. **18.** 1386, *r* 55.
19. 64, *r* 46. **20.** 52, *r* 16. **21.** 15, *r* 150. **22.** 290, *r* 148.
23. 319, *r* 11. **24.** 93, *r* 64. **25.** 81, *r* 177. **26.** 21, *r* 21.
27. 141, *r* 188. **28.** 126, *r* 233. **29.** 134, *r* 209. **30.** 196, *r* 428.
31. 79, *r* 320. **32.** 114, *r* 649. **33.** 132, *r* 371. **34.** 72, *r* 643.
35. 175, *r* 426. **36.** 71, *r* 20. **37.** 398, *r* 1374. **38.** 239, *r* 12.
39. 552, *r* 1080. **40.** 416, *r* 1560. **41.** 27, *r* 1261. **42.** 416, *r* 1184.
43. 969, *r* 2513. **44.** 148, *r* 1791. **45.** 1006, *r* 4532.
46. 2337, *r* 3618. **47.** 4681, *r* 582. **48.** 128, *r* 4543.
49. 1006, *r* 5538. **50.** 8787, *r* 1988. **51.** 11171, *r* 4593.
52. 12,030, *r* 34,846. **53.** 11,961, *r* 6276. **54.** 30,878, *r* 192,391.
55. 17,767, *r* 360,802. **56.** 27027, 42598. **57.** 80. **58.** 126 boxes.
59. 596. **60.** 656. **61.** 83. **62.** 4681 times.

XI. PAGE 27.

1. 2063. **2.** 492. **3.** 111. **4.** 473. **5.** 738. **6.** 609. **7.** 7065.
8. 7065. **9.** 23072. **10.** 27. **11.** 528. **12.** 7198. **13.** 1111, *r* 6.
14. 201, *r* 20. **15.** 432, *r* 100. **16.** 606, *r* 16. **17.** 5206, *r* 118.
18. 330, *r* 111. **19.** 3651, *r* 72. **20.** 708, *r* 79. **21.** 436, *r* 46.
22. 940, *r* 40. **23.** 595, *r* 40. **24.** 472, *r* 124. **25.** 1086, *r* 41.
26. 2903, *r* 107. **27.** 1583, *r* 191. **28.** 40, *r* 687. **29.** 150, *r* 373.
30. 1834, *r* 246. **31.** 284, *r* 84. **32.** 46, *r* 725. **33.** 1770, *r* 451.
34. 2233, *r* 312. **35.** 26, *r* 750. **36.** 74, *r* 271.

XII. PAGES 28, 29, 30.

1. 1777. **2.** 2488. **3.** 93. **4.** 6237. **5.** 9324. **6.** 3663. **7.** 19.
8. 125, *r* 195. **9.** 19. **10.** 98. **11.** 1544. **12.** 51,600 days.
13. $15. **14.** $323085. **15.** 625. **16.** $2160. **17.** 343 trees.
18. 15 acres. **19.** 259 sheep. **20.** $83700. **21.** $1400.
22. 1,628,728,655. **23.** 3,424,164,225. **24.** $314 gain. **25.** $3450.
26. 1342 lbs. **27.** $15. **28.** 18 pears. **29.** 200 sheep; 100 cows.
30. $208 gain. **31.** $11,000; $6000; $3000. **32.** $75 gain.
33. $3100. **34.** 107, 77, 13. **35.** 8 boxes of each size. **36.** $30.
37. 180; 300; 420. **40.** 35. **45.** 300,937,215,555. **46.** 15,421,771,365.

XIII. PAGE 34.

1. One, five-tenths. **2.** Thirty-six, seven-tenths and five hundredths.
3. One hundred, one thousandth. **4.** Six ten-thousandths.
5. One, one-tenth and seven ten-thousandths.
6. Four hundred and four, four hundredths and four ten-thousandths.
7. One hundred, one ten-thousandth.
8. Five, five-thousandths, and five hundred-thousandths.
9. ·14. **10.** ·000001. **11.** 50·001. **12.** ·0009. **13.** 51·01.
14. 10000·001. **15.** 154·406. **16.** 1079·96. **17.** 6000020·19.

XIV. PAGE 35.

1. (i) 30; (ii) 300; (iii) 3; (iv) ·03; (v) ·16; (vi) 501; (vii) 13; (viii) ·00039; (ix) 310·42; (x) 4000·1; (xi) 303·03; (xii) 4100·2.

2. (i) ·3; (ii) 3; (iii) ·03; (iv) ·0003; (v) ·0016; (vi) 5·01; (vii) ·13; (viii) ·0000039; (ix) 3·1042; (x) 40·001; (xi) 3·0303; (xii) 41·002.

3. (i) 3000; (ii) 30000; (iii) 300; (iv) 3; (v) 16; (vi) 50100; (vii) 1300; (viii) ·039; (ix) 31042; (x) 400010; (xi) 30303; (xii) 410020.

4. (i) ·003; (ii) ·03; (iii) ·0003; (iv) ·000003; (v) ·000016; (vi) ·0501; (vii) ·0013; (viii) ·000000039; (ix) ·031042; (x) ·40001; (xi) ·030303; (xii) ·41002.

5. (i) 2000; (ii) 310; (iii) 4·12; (iv) ·016.

6. (i) 4·3121; (ii) ·001012; (iii) ·3412; (iv) 5; (v) 5012·5.

7. (i) 451; (ii) 3·756; (iii) 4·675; (iv) 45625; (v) ·05675.

XV. PAGE 36.

1. 353·5341. **2.** 34·2475. **3.** 3698·163425. **4.** 36·23236.
5. 4799·87503675. **6.** ·01466. **7.** 41176·17975. **8.** 510·348.
9. 30150·33328. **10.** 81·316. **11.** 378·809265. **12.** 747·57245.
13. 3471·88449 in. **14.** 447·14472 in.

XVI. PAGE 37.

1. 246·302. **2.** 90·26. **3.** 3·35. **4.** ·61325. **5.** 351·49375.
6. 29·999375. **7.** 497·72375. **8.** 526·5. **9.** ·00038235. **10.** ·00849748.
11. 272·4125. **12.** 30·465. **13.** 4494·0880625. **14.** 392·6560855.
15. 2967·24425. **16.** 189·4368. **17.** 5672·45163. **18.** ·66614.
19. 2·7960151. **20.** 2·321.

XVII. Page 37.

1. 21·5625. **2.** 729·75. **3.** 3654·83. **4.** 463125. **5.** ·18525. **6.** ·012984.
7. 1092·08. **8.** 2944·8. **9.** ·211446. **10.** 4·34312. **11.** 5706·25.
12. 241151·2669. **13.** 1112·130201. **14.** 3018036·7.

XVIII. Page 38.

1. 1·5875. **2.** 5·165625. **3.** ·000540125. **4.** 170·5. **5.** 176·25.
6. 4·63. **7.** 229·375. **8.** ·13. **9.** ·000013. **10.** ·0000629.
11. 1·907125. **12.** ·06475. **13.** ·000019459375. **14.** ·0035304.
15. ·2468. **16.** ·00000067625. **17.** 480·625. **18.** ·2525.
19. ·0001412. **20.** ·000026375. **21.** ·0000000212. **22.** 120·033333...
23. 1·403333... **24.** ·99. **25.** 1·145833... **26.** 1·910714...
27. 35·398230... **28.** 11·111111... **29.** ·359673... **30.** ·000409...

XIX. Page 39.

1. 49·44. **2.** 3408·255. **3.** ·11808. **4.** ·0030501. **5.** ·00015129.
6. 90·92869695. **7.** ·0000000368. **8.** 5·658. **9.** 12253·63815.
10. ·058464. **11.** ·18496. **12.** 21632. **13.** ·008. **14.** ·8.
15. ·00008. **16.** ·004624. **17.** ·000015129. **18.** 7·9507.
19. ·001771561. **20.** ·000000027. **21.** 24·727224. **22.** ·1953.
23. 5·4. **24.** ·000013764.

XX. Page 40.

1. 173·05. **2.** 1153·125. **3.** 375000. **4.** ·000035225. **5.** 3625.
6. 415000. **7.** 31·25. **8.** 7·9. **9.** 229·5. **10.** 3·472222... **11.** ·012332...
12. 1280. **13.** 9·974564... **14.** ·000092... **15.** 166666·666666...
16. ·069525... **17.** 401·771749... **18.** 37·137317... **19.** ·149274...
20. 23·201856... **21.** 5690·140845... **22.** 7125890·736342...
23. ·121764... **24.** 625. **25.** 2·4. **26.** ·063535... **27.** 5488.
28. 133336666·666666... **29.** 39824·767741... **30.** 83·33325.
31. ·012. **32.** 790. **33.** 326·7578125. **34.** ·3122. **35.** ·6024.
36. 4·792993... **37.** ·0123. **38.** 3·6125. **39.** 42250. **40.** 6400.
41. ·000625. **42.** 1849. **43.** ·100318... **44.** 630000. **45.** ·512.
46. 4600000. **47.** 1·063783... **48.** 3774045·801526...

XXI. PAGE 41.

1. 2·8828125. **2.** 5314·41. **3.** 900801974·088. **4.** 4208·489631.
5. 81·77750790252. **6.** 9675 and ·005 over. **7.** ·00025.
8. $288.5625. **9.** 145·024 ft. **10.** 23·625 yds. **11.** $57.86.
12. $5.26. **13.** $90. **14.** $15. **15.** $13 ; $26 ; $13. **16.** $3.90.

XXII. PAGE 43

1. 2×3. **2.** 2×5. **3.** 2×7. **4.** 5×7. **5.** 3×3. **6.** $2 \times 2 \times 2$.
7. $2 \times 2 \times 3$. **8.** $2 \times 2 \times 2 \times 3$. **9.** $3 \times 3 \times 3$. **10.** $2 \times 2 \times 3 \times 3$.
11. $2 \times 3 \times 3 \times 5$. **12.** $2 \times 2 \times 3 \times 7$. **13.** $2 \times 2 \times 5 \times 5$. **14.** 11×11.
15. $2 \times 2 \times 2 \times 2 \times 3 \times 3$. **16.** $2 \times 2 \times 3 \times 11$. **17.** $2 \times 2 \times 2 \times 2 \times 7$.
18. 3×17. **19.** $3 \times 3 \times 13$. **20.** $3 \times 3 \times 5 \times 11$. **21.** $2 \times 3 \times 5 \times 5 \times 7$.
22. $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$. **23.** $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5$.
24. $2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 71$. **25.** $2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 11$.
26. $2 \times 5 \times 5 \times 73$. **27.** $3 \times 3 \times 113$.
28. $2 \times 2 \times 2 \times 3 \times 11 \times 13$. **29.** $5 \times 5 \times 11 \times 13$. **30.** $2 \times 5 \times 13 \times 13$.
31. $2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 11 \times 13$. **32.** $2 \times 2 \times 5 \times 5 \times 7 \times 11 \times 13$.

XXIII. PAGE 44.

1. prime. **2.** 3×31. **3.** prime. **4.** 3×37. **5.** prime. **6.** prime.
7. prime. **8.** 7×17. **9.** $3 \times 3 \times 23$. **10.** prime. **11.** prime.
12. prime. **13.** 13×23. **14.** 13×13. **15.** prime. **16.** 7×67. **17.** prime.
18. prime. **19.** $3 \times 13 \times 13$. **20.** prime. **21.** $13 \times 13 \times 13$.
22. 13×179. **23.** 17×269. **24.** 13×233. **25.** prime. **26.** 23×191.
27. 13×547. **28.** 19×409. **29.** prime. **30.** 23×31. **31.** 29×47.
32. 23×433. **33.** 13×503. **34.** 11×163. **35.** 7×83.

XXIV. PAGE 46.

1. 5, 4, 8. **2.** 5, 9. **3.** 11. **4.** 4, 8, 9. **5.** 5, 11.
6. 4, 8. **7.** 9, 11. **8.** 9. **10.** (i) 3 ; (ii) 2 ; (iii) 9.

XXV. PAGE 47.

1. 8. **2.** 15. **3.** 20. **4.** 16. **5.** 27. **6.** 11. **7.** 17. **8.** 6.
9. 91. **10.** 15. **11.** 13. **12.** 17. **13.** 44. **14.** 240. **15.** 32. **16.** 13.
17. 8. **18.** 9. **19.** 15. **20.** 28. **21.** 35. **22.** 11. **23.** 13. **24.** 1.

XXVI. PAGE 49.

1. 13. **2.** 17. **3.** 19. **4.** 29. **5.** 23. **6.** 31. **7.** 53.

8. 46. **9.** 47. **10.** 73. **11.** 13. **12.** 13. **13.** 3. **14.** 22.

15. 29. **16.** 13. **17.** 851. **18.** 7. **19.** 23. **20.** 19.

XXVII. PAGE 49.

7. Yes. **8.** Yes. **9.** Yes. **10.** No. **11.** No. **12.** Yes. **13.** $6.51.

XXVIII. PAGE 51.

1. 180. **2.** 336. **3.** 1125. **4.** 756. **5.** 3300. **6.** 1260. **7.** 1440.

8. 6300. **9.** 360. **10.** 2520. **11.** 2520. **12.** 1120. **13.** 110.

14. 17,424. **15.** 1014. **16.** 792. **17.** 22,176. **18.** 120,120.

19. 8415. **20.** 1260. **21.** 27,720. **22.** 6,340,320. **23.** 2520. **24.** 5040.

XXIX. PAGE 52.

1. $3 \times 5 \times 17 \times 23 \times 29$. **2.** $13 \times 157 \times 652$. **3.** $41 \times 61 \times 211$.

4. $31 \times 149 \times 239$. **5.** $62 \times 1194 \times 1345$. **6.** $13 \times 590 \times 769$.

7. $19 \times 59 \times 149 \times 359$. **8.** $29 \times 109 \times 127 \times 233 \times 10$.

9. $2 \times 5 \times 11 \times 61 \times 107 \times 149$. **10.** $3 \times 8 \times 37 \times 79 \times 79 \times 113$.

11. 49 ; $49 \times 14 \times 85$. **12.** 213 ; $213 \times 25 \times 39$.

13. 157 ; $157 \times 3 \times 8 \times 11$. **14.** 12 ; $12 \times 4 \times 7 \times 63 \times 95$.

XXX. PAGES 52, 53.

1. 3,605,401,800. **2.** £600. **3.** 365. **4.** 50400 m. **5.** 10080 sec.

6. 1188 in. **7.** 360 ft. **8.** 19. **9.** 14467 in. **10.** 30 min.

12. $41 \times 43 \times 11 \times 3 \times 23 \times 47$. **16.** 126 sec. **18.** 7 oz.

XXXI. PAGE 55.

1. 45. **2.** 75. **3.** 50. **4.** 225. **5.** 125. **6.** 300.

7. 150. **8.** $7\frac{1}{2}$. **9.** 25. **10.** 125. **11.** 50. **12.** 200.

13. 150. **14.** 15. **15.** 250. **16.** 280. **17.** 250. **18.** 6500.

XXXII. Page 57.

1. $\frac{3}{7}$. 2. $\frac{13}{23}$. 3. $\frac{5}{17}$. 4. $\frac{4}{25}$. 5. $\frac{2}{63}$. 6. $\frac{37}{45}$. 7. $\frac{7}{9}$. 8. $\frac{14}{29}$.
9. $\frac{33}{56}$. 10. $\frac{14}{15}$. 11. $\frac{97}{101}$. 12. $\frac{71}{97}$. 13. $\frac{71}{121}$. 14. $\frac{134}{437}$. 15. $\frac{26}{225}$.
16. $\frac{3}{4}$. 17. $\frac{70}{1111}$. 18. $\frac{27}{91}$. 19. $\frac{429}{566}$. 20. $\frac{8}{9}$. 21. $\frac{986}{1205}$. 22. $\frac{181}{1200}$.
23. $\frac{334}{421}$. 24. $\frac{101}{113}$. 25. $\frac{5}{6}$. 26. $\frac{13}{17}$. 27. $\frac{19}{23}$. 28. $\frac{31}{37}$. 29. $\frac{31}{41}$.
30. $\frac{29}{37}$. 31. $\frac{701}{751}$. 32. $\frac{811}{997}$. 33. $\frac{241}{661}$. 34. $\frac{911}{991}$. 35. $\frac{6539}{12963}$.

XXXIII. Page 58.

1. $4\frac{3}{7}$. 2. $3\frac{7}{10}$. 3. $7\frac{7}{11}$. 4. $8\frac{11}{12}$. 5. 9. 6. $12\frac{1}{6}$. 7. $25\frac{5}{6}$. 8. 15.
9. $5\frac{16}{31}$. 10. 5. 11. $1\frac{10}{49}$. 12. $8\frac{4}{7}$. 13. $2\frac{37}{41}$. 14. 5. 15. $7\frac{2}{5}$.
16. $9\frac{21}{41}$. 17. $22\frac{171}{365}$. 18. $26\frac{13}{14}$. 19. $20\frac{119}{240}$. 20. $4\frac{6}{11}$. 21. $\frac{22}{7}$. 22. $\frac{41}{9}$.
23. $\frac{181}{10}$. 24. $\frac{212}{11}$. 25. $\frac{353}{31}$. 26. $\frac{365}{31}$. 27. $\frac{365}{29}$. 28. $\frac{2107}{100}$. 29. $\frac{3001}{1000}$.
30. $\frac{1121}{1000}$. 31. $\frac{304}{99}$. 32. $\frac{4157}{99}$. 33. $\frac{2507}{60}$. 34. $\frac{1122}{101}$. 35. $\frac{6243}{201}$.

XXXIV. Page 59.

1. $\frac{75}{15}$, $\frac{105}{15}$, $\frac{315}{15}$, $\frac{1275}{15}$. 2. $\frac{121}{11}$, $\frac{330}{11}$, $\frac{561}{11}$, $\frac{1111}{11}$; $\frac{77}{7}$, $\frac{210}{7}$, $\frac{357}{7}$, $\frac{707}{7}$.
3. $\frac{84}{7}$, $\frac{49}{7}$; $\frac{360}{30}$, $\frac{210}{30}$; $\frac{1200}{100}$, $\frac{700}{100}$. 4. $\frac{693}{21}$, $\frac{210}{21}$, $\frac{1947}{59}$, $\frac{590}{59}$; $\frac{330}{10}$, $\frac{100}{10}$.
5. $\frac{42}{18}$, $\frac{10}{18}$; $\frac{63}{27}$, $\frac{15}{27}$; $\frac{189}{81}$, $\frac{45}{81}$; $\frac{126}{54}$, $\frac{30}{54}$; $\frac{2100}{900}$, $\frac{500}{900}$.
6. $\frac{70}{105}$, $\frac{63}{105}$, $\frac{60}{105}$, $\frac{49}{105}$, $\frac{100}{105}$, $\frac{24}{105}$. 7. No ; 315. 8. $\frac{150}{30}$, $\frac{18}{30}$, $\frac{4}{30}$, $\frac{9}{30}$.
9. $\frac{36}{12}$, $\frac{8}{12}$, $\frac{45}{12}$, $\frac{10}{12}$. 10. 60.

XXXV. Page 61.

1. $\frac{3}{6}$. $\frac{2}{6}$, $\frac{1}{6}$. 2. $\frac{9}{45}$, $\frac{15}{45}$, $\frac{10}{45}$. 3. $\frac{30}{105}$, $\frac{63}{105}$, $\frac{40}{105}$. 4. $\frac{18}{60}$, $\frac{21}{60}$, $\frac{22}{60}$.
5. $\frac{308}{77}$, $\frac{275}{77}$, $\frac{252}{77}$. 6. $\frac{27}{9}$, $\frac{45}{9}$, $\frac{4}{9}$. 7. $\frac{15}{35}$, $\frac{63}{35}$, $\frac{70}{35}$. 8. $\frac{3}{72}$, $\frac{27}{72}$, $\frac{10}{72}$.
9. $\frac{45}{441}$, $\frac{56}{441}$, $\frac{33}{441}$. 10. $\frac{40}{300}$, $\frac{90}{300}$, $\frac{21}{300}$. 11. $\frac{9}{216}$, $\frac{56}{216}$, $\frac{33}{216}$.
12. $\frac{165}{500}$, $\frac{52}{500}$, $\frac{140}{500}$. 13. $\frac{12}{72}$, $\frac{6}{72}$, $\frac{4}{72}$, $\frac{9}{72}$. 14. $\frac{12}{108}$. $\frac{12}{108}$, $\frac{10}{108}$, $\frac{33}{108}$.
15. $\frac{198}{693}$, $\frac{189}{693}$, $\frac{385}{693}$. 16. $\frac{33}{99}$, $\frac{45}{99}$, $\frac{111}{99}$, $\frac{495}{99}$. 17. $\frac{1785}{231}$, $\frac{1320}{231}$, $\frac{671}{231}$, $\frac{300}{231}$.
18. $\frac{143}{143}$, $\frac{39}{143}$, $\frac{44}{143}$.

XXXVI. Pages 62, 63.

1. $\frac{5}{6}$ greatest, $\frac{3}{4}$ least. 2. $\frac{6}{7}$ gr., $\frac{17}{21}$ least. 3. $\frac{2}{3}$ gr., $\frac{7}{12}$ least.
4. $\frac{3}{4}$,, $\frac{8}{11}$,, 5. $\frac{9}{10}$,, $\frac{9}{13}$,, 6. $\frac{13}{21}$,, $\frac{8}{15}$,,
7. $\frac{11}{30}$,, $\frac{13}{36}$,, 8. $\frac{17}{20}$,, $\frac{11}{15}$,, 9. $\frac{3}{8}$,, $\frac{1}{3}$,,
10. $\frac{4}{7}$,, $\frac{4}{9}$,, 11. $\frac{9}{28}$,, $\frac{7}{24}$,, 12. $\frac{29}{30}$,, $\frac{21}{25}$,,
13. $\frac{11}{26}$, $\frac{13}{30}$, $\frac{5}{12}$, $\frac{7}{18}$. 14. $\frac{7}{12}$, $\frac{11}{20}$, $\frac{13}{24}$, $\frac{4}{9}$. 15. $\frac{29}{36}$, $\frac{19}{24}$, $\frac{23}{30}$.
16. $\frac{100}{81}$, $\frac{45}{38}$, $\frac{60}{51}$, $\frac{25}{18}$. 17. $\frac{7}{24}$, $\frac{13}{45}$, $\frac{4}{15}$, $\frac{17}{64}$. 18. $\frac{14}{17}$, $\frac{7}{9}$, $\frac{7}{18}$.
19. $\frac{7}{20}$, $\frac{13}{40}$, $\frac{19}{60}$, $\frac{5}{16}$. 20. $\frac{53}{160}$, $\frac{18}{55}$, $\frac{7}{22}$. 22. $\frac{5}{24}$, $\frac{11}{54}$, $\frac{13}{64}$, $\frac{7}{36}$.

XXXVII. PAGE 63.

1. $2\frac{3}{5}$. 2. $1\frac{1}{11}$. 3. $4\frac{1}{3}$. 4. $2\frac{9}{10}$. 5. $\frac{65}{101}$.
6. $2\frac{2}{5}$. 7. 60 cents. 8. $\frac{3}{16}$. 9. 1 ft. 10. 40 min.

XXXVIII. PAGE 64.

1. $1\frac{1}{6}$. 2. $1\frac{1}{5}$. 3. $\frac{13}{24}$. 4. $\frac{11}{15}$. 5. $\frac{79}{96}$. 6. $\frac{3}{4}$. 7. $2\frac{7}{12}$. 8. $\frac{28}{45}$. 9. $1\frac{3}{4}$.
10. $2\frac{37}{105}$. 11. $1\frac{2}{3}$. 12. $1\frac{19}{25}$. 13. $1\frac{1}{15}$. 14. $1\frac{7}{16}$. 15. $1\frac{211}{504}$.
16. $\frac{27}{35}$. 17. 1. 18. $1\frac{13}{126}$. 19. 1. 20. $1\frac{3}{7}$. 21. $1\frac{29}{130}$.

XXXIX. PAGE 65.

1. 6. 2. 19. 3. $7\frac{1}{2}$. 4. 10. 5. $30\frac{5}{24}$. 6. $7\frac{37}{45}$. 7. $14\frac{1}{12}$. 8. $12\frac{9}{16}$.
9. $10\frac{3}{10}$. 10. $20\frac{25}{56}$. 11. $13\frac{5}{9}$. 12. $6\frac{1}{98}$. 13. $12\frac{53}{196}$. 14. $13\frac{59}{63}$.
15. $4\frac{23}{35}$. 16. $14\frac{4}{75}$. 17. $57\frac{1}{2}$. 18. $17\frac{39}{40}$. 19. $15\frac{11}{14}$. 20. $16\frac{533}{600}$.
21. $31\frac{17}{147}$. 22. $10\frac{1}{4}$. 23. $10\frac{1}{48}$. 24. $12\frac{124}{273}$. 25. $31\frac{226}{231}$. 26. $24\frac{19}{55}$.
27. $4\frac{7}{30}$. 28. $33\frac{13}{20}$. 29. $42\frac{1253}{1495}$. 30. $144\frac{46}{231}$. 31. $115\frac{151}{336}$.
32. $190\frac{1433}{1728}$. 33. $23\frac{403}{880}$. 34. $16\frac{4}{45}$. 35. $7\frac{8}{57}$. 36. $102\frac{5}{26}$.
37. $4\frac{1621}{3960}$. 38. $19\frac{137}{504}$. 39. $11\frac{3513}{3960}$.

XL. PAGE 66.

1. $\frac{1}{6}$. 2. $\frac{1}{12}$. 3. $\frac{7}{18}$. 4. $\frac{17}{60}$. 5. $\frac{11}{72}$. 6. $\frac{11}{75}$. 7. $\frac{1}{8}$. 8. $\frac{154}{225}$.
9. $\frac{11}{84}$. 10. $\frac{1}{6}$. 11. $\frac{4}{385}$. 12. $\frac{35}{192}$. 13. $\frac{1}{3}$. 14. $\frac{5}{36}$. 15. $\frac{7}{60}$.
16. $\frac{13}{60}$. 17. $\frac{3}{8}$. 18. $1\frac{1}{14}$. 19. $\frac{17}{144}$. 20. $\frac{19}{63}$. 21. $\frac{7}{153}$.

XLI. PAGE 67.

1. $\frac{3}{4}$. 2. $\frac{7}{9}$. 3. $\frac{3}{8}$. 4. $1\frac{5}{14}$. 5. $15\frac{73}{84}$. 6. $10\frac{17}{56}$. 7. $1\frac{5}{13}$. 8. $\frac{11}{18}$.
9. $24\frac{169}{192}$. 10. $198\frac{2}{3}$. 11. $\frac{10}{27}$. 12. $9\frac{139}{140}$. 13. $2\frac{8}{19}$. 14. $\frac{2}{3}$. 15. $\frac{8}{11}$.
16. $\frac{49}{60}$. 17. $1\frac{48}{539}$. 18. $\frac{13}{84}$. 19. $10\frac{5}{153}$. 20. $1\frac{1}{264}$. 21. $1\frac{25}{288}$. 22. 0.
23. $11\frac{199}{462}$. 24. $\frac{11}{18}$. 25. $1\frac{5}{16}$. 26. $3\frac{1}{8}$. 27. 1. 28. $3\frac{13}{30}$. 29. $6\frac{6}{7}$.
30. $\frac{7}{24}$. 31. $2\frac{29}{36}$. 32. $1\frac{7}{16}$. 33. $8\frac{23}{36}$. 34. $6\frac{27}{40}$. 35. $\frac{101}{110}$. 36. $\frac{7}{120}$.
37. $5\frac{13}{44}$. 38. $4\frac{1}{9}$. 39. $\frac{3}{8}$. 40. $2\frac{19}{56}$. 41. $1\frac{13}{56}$. 42. $\frac{4}{21}$. 43. $11\frac{9}{10}$.
44. $1\frac{7}{12}$. 45. $28\frac{5}{6}$. 46. $13\frac{19}{66}$. 47. $18\frac{1}{2}$. 48. $\frac{5073}{7000}$.

XLII. PAGE 68.

1. $3\frac{3}{4}$. 2. $2\frac{1}{2}$. 3. $2\frac{1}{3}$. 4. 64. 5. $31\frac{1}{2}$. 6. $8\frac{1}{3}$. 7. $2\frac{23}{30}$. 8. $178\frac{2}{11}$.
9. $21\frac{1}{3}$. 10. $71\frac{2}{3}$. 11. $35\frac{2}{3}$. 12. $115\frac{1}{2}$. 13. $1916\frac{1}{4}$. 14. $2367\frac{7}{9}$.
15. $6335\frac{1}{3}$. 16. $35486\frac{1}{5}$.

XLIII. PAGE 69.

1. 1. **2.** $\frac{3}{40}$. **3.** $\frac{135}{182}$. **4.** $\frac{1152}{4375}$. **5.** 4. **6.** 9. **7.** $1\frac{2}{5}$. **8.** 4.
9. $73\frac{11}{13}$. **10.** $15\frac{39}{64}$. **11.** 26. **12.** $43\frac{5}{16}$. **13.** $\frac{11}{42}$. **14.** 11. **15.** 27.
16. $\frac{3}{106}$. **17.** $\frac{5}{11}$. **18.** $\frac{1}{10}$. **19.** $\frac{70}{99}$. **20.** $\frac{5}{28}$. **21.** $3\frac{1}{5}$. **22.** $3\frac{3}{7}$.
23. $3\frac{17}{23}$. **24.** $3\frac{1}{2}$. **25.** $3\frac{13}{16}$. **26.** $1\frac{1}{45}$. **27.** $1\frac{59}{112}$. **28.** $12\frac{11}{45}$.
29. $2\frac{11}{72}$. **30.** $6\frac{13}{24}$. **31.** $11\frac{43}{60}$. **32.** $126\frac{64}{105}$. **33.** $5\frac{3}{4}$.

XLIV. PAGE 70.

1. $33\frac{1}{3}$. **2.** $4\frac{1}{5}$. **3.** $333\frac{1}{3}$. **4.** $14\frac{13}{16}$. **5.** $142\frac{6}{7}$. **6.** $134\frac{11}{35}$.
7. $\frac{2}{9}$. **8.** $\frac{5}{42}$. **9.** $\frac{11}{48}$. **10.** $\frac{1}{45}$. **11.** $\frac{2}{3}$. **12.** $2\frac{1}{8}$. **13.** $\frac{1}{3}$. **14.** $1\frac{1}{3}$.
15. $25\frac{5}{64}$. **16.** $25\frac{13}{15}$. **17.** $4\frac{41}{55}$. **18.** $10\frac{302}{495}$. **19.** $10\frac{263}{1188}$. **20.** $22\frac{8}{35}$.

XLV. PAGE 71.

1. $3\frac{1}{3}$. **2.** $1\frac{1}{20}$. **3.** $\frac{40}{63}$. **4.** $1\frac{17}{60}$. **5.** 2. **6.** 6. **7.** $\frac{26}{33}$. **8.** 4. **9.** $\frac{1}{4}$.
10. $2\frac{2}{5}$. **11.** 9. **12.** 2. **13.** $\frac{2}{5}$. **14.** $\frac{5}{7}$. **15.** $\frac{10}{11}$. **16.** $\frac{2}{3}$. **17.** $\frac{28}{45}$.
18. $9\frac{4}{7}$. **19.** $1\frac{3}{5}$. **20.** $\frac{17}{28}$. **21.** $14\frac{1}{2}$. **22.** $3\frac{95}{123}$. **23.** $1\frac{37}{199}$. **24.** $3\frac{8}{15}$.
25. $\frac{1}{3}$. **26.** $\frac{45}{88}$. **27.** 144. **28.** $\frac{5}{32}$. **29.** 1.

XLVI. PAGE 72.

1. $\frac{6}{7}$. **2.** $2\frac{1}{6}$. **3.** $\frac{1}{6}$. **4.** $\frac{3}{4}$. **5.** $1\frac{3}{7}$. **6.** $3\frac{3}{10}$. **7.** $4\frac{1}{10}$. **8.** $3\frac{1}{18}$.
9. 1. **10.** $3\frac{4}{57}$. **11.** $1\frac{2}{3}$. **12.** 2. **13.** $1\frac{2}{23}$. **14.** $\frac{1}{7}$. **15.** $1\frac{14}{181}$.
16. $\frac{127}{8428}$. **17.** 10. **18.** $\frac{2}{3}$. **19.** $1\frac{21}{229}$. **20.** $\frac{81}{250}$. **21.** $5\frac{12}{13}$.

XLVII. PAGE 73.

1. $\frac{3}{7}$. **2.** $\frac{11}{28}$. **3.** $1\frac{4}{7}$. **4.** $3\frac{2}{3}$. **5.** $3\frac{21}{68}$. **6.** $\frac{92}{475}$. **7.** $3\frac{136}{421}$.
8. $4\frac{267}{320}$. **9.** $9\frac{21}{214}$. **10.** $\frac{818}{1511}$. **11.** $1\frac{4}{101}$. **12.** $\frac{225}{574}$.

XLVIII. PAGES 74, 75.

1. $5\frac{9}{16}$. **2.** 12. **3.** $1\frac{3}{8}$. **4.** $2\frac{1}{6}$. **5.** $15\frac{3}{70}$. **6.** $16\frac{19}{21}$. **7.** $6\frac{31}{35}$.
8. $26\frac{1}{4}$. **9.** $1\frac{19}{42}$. **10.** $\frac{16}{35}$. **11.** $20\frac{7}{12}$. **12.** $21\frac{377}{420}$. **13.** $3\frac{5}{8}$.
14. $12\frac{115}{144}$. **15.** $4\frac{1}{8}$. **16.** $5\frac{9}{16}$. **17.** $4\frac{5}{64}$. **18.** $12\frac{7}{22}$. **19.** $\frac{6}{35}$.
20. $2\frac{124}{133}$. **21.** 5. **22.** $\frac{3}{7}$. **23.** 8. **24.** 2. **25.** $\frac{2}{9}$. **26.** $\frac{21}{64}$.
27. $5\frac{73}{84}$. **28.** $\frac{1}{3}$. **29.** $9\frac{17}{40}$. **30.** $13\frac{148}{183}$. **31.** $2\frac{17}{648}$. **32.** 5.
33. $36\frac{43}{88}$. **34.** $5\frac{2}{3}$. **35.** $\frac{25}{32}$. **36.** $\frac{45}{64}$. **37.** $\frac{1}{12}$. **38.** $43\frac{5}{16}$. **39.** 1.
40. $2\frac{61}{177}$. **41.** 14. **42.** $6\frac{781}{864}$. **43.** $\frac{2}{3}$. **44.** $573\frac{3}{11}$. **45.** $\frac{8}{9}$.
46. $\frac{1}{9}$. **47.** $3\frac{7}{80}$. **48.** 1. **49.** $\frac{83}{90}$. **50.** 1.

XLIX. PAGE 76.

1. $\frac{1}{4}$. **2.** $\frac{1}{50}$. **3.** $\frac{1}{8}$. **4.** $\frac{1}{80}$. **5.** $6\frac{7}{16}$. **6.** $10\frac{1}{16}$. **7.** $\frac{166547}{4000000}$.
8. $\frac{321}{800}$. **9.** $367\frac{3}{8}$. **10.** $70\frac{1}{250000}$. **11.** $81\frac{1}{20000}$. **12.** $4\frac{73}{80}$. **13.** $\frac{321}{445}$.
14. $1\frac{59}{375}$. **15.** $\frac{117}{711}$. **16.** $7\frac{4}{9}$.

L. PAGES 76, 77.

1. ·5. **2.** ·25. **3.** ·375. **4.** ·3125. **5.** ·45. **6.** 5·65625. **7.** 7·025.
8. ·11375. **9.** 9·875. **10.** ·109375. **11.** ·444444... **12.** ·384615...
13. ·380952... **14.** ·714285... **15.** ·219780... **16.** 3·727272...
17. 4·642857... **18.** 11·777777... **19.** 20·428571... **20.** 3·989130...
21. $\frac{5}{6}$; $\frac{3}{4}$. **22.** $\frac{6}{7}$; $\frac{17}{21}$. **23.** $\frac{2}{3}$; $\frac{7}{12}$. **24.** $\frac{4}{7}$; $\frac{11}{21}$. **25.** $\frac{11}{13}$; $\frac{7}{9}$. **26.** $\frac{7}{12}$; $\frac{4}{9}$.
27. $·75. **28.** ·625 cwt. **29.** ·1875 lbs. **30.** 4·09375 tons.
31. $27.15. **32.** 4·4375 cwt. **33.** 3·075 yds. **34.** 21·35 hours.

LI. PAGES 77, 78, 79.

1. $\frac{7}{90}$. **2.** 630. **3.** twice. **4.** $34\frac{43}{162}$. **5.** $41\frac{6}{35}$. **6.** $3\frac{33}{40}$. **7.** $\frac{25}{146}$.
8. $50\frac{2}{5}$. **9.** $14\frac{113}{168}$. **10.** $14\frac{6}{61}$. **11.** ·833 etc. **12.** 43·333. **13.** $2.50.
14. $3\frac{153}{200}$, or 3·765. **15.** 64. **16.** ·826086. **18.** ·03493. **19.** $7\frac{7}{12}$.
20. 3. **21.** $\frac{8}{15}$. **22.** $840. **23.** $33\frac{3}{5}$. **24.** 783 ft. **25.** $1692.90.
26. $3125. **27.** \$$4977\frac{7}{9}$. **28.** $7500. **29.** $5488. **30.** $2\frac{3}{4}$ miles.
31. $25631.41. **32.** 240. **33.** $459\frac{3}{8}$ ft. **34.** $\frac{2}{45}$. **35.** 75. **36.** 420.
37. 135. **38.** $2\frac{3}{10}$ ft.

LII. PAGE 81.

1. 432 c. **2.** 30000 c. **3.** 140 c. **4.** 40 f. **5.** 2·63 f. **6.** ·045 f.
7. $36.41. **8.** $1·035. **9.** 236 cents. **10.** 1010 cents. **11.** $4·875.
12. $38·025. **13.** 12·675 cents. **14.** 500 fr. **15.** 30 fr. **16.** 700 fr.

LIII. PAGE 84.

1. 1760. **9.** 1856.

LIV. PAGES 85, 86.

1. 3886 in. **2.** 65958 in. **3.** 31680 in. **4.** 95040 in. **5.** 738 in.
6. 261 in. **7.** 168 hrs. **8.** 744 hrs. **9.** 1080 hrs. **10.** 167000 lbs.
11. 93000 lbs. **12.** 214510 lbs. **13.** 37136 oz. **14.** 24336 oz.
15. 97936 oz. **16.** 133344 oz. **17.** 154016 drs. **18.** 153654272 drs.
19. 179 in. **20.** 63018 in. **21.** 633600 in. **22.** 198024 in.

23. 510884 in. **24.** 44460 sq. ft. **25.** 5227650 sq. ft.
26. 5728140 sq. ft. **27.** 19613070 sq. ft. **28.** 2178900 sq. ft.
29. 368 pints. **30.** 492 pints. **31.** 702 pints. **32.** 28 pints.
33. 23328000 c. in. **34.** 8156160 c. in. **35.** 46691661 c. in.
36. 549504 c. in. **37.** 1059900 secs. **38.** 31557600 secs.
39. 1250784 secs. **40.** 2741247 secs. **41.** 1296000 secs.
42. 433243 secs. **43.** 217038 secs. **44.** 150600 secs. **45.** 1693647 secs.
46. 30720 grs. **47.** 61440 grs. **48.** 125208 grs. **49.** 6342000 grs.
50. 36750 grs.

LV. PAGE 87.

1. 12612 ft. **2.** 43989 ft. **3.** 111687 ft. **4.** 528648 ft. **5.** 659340 in.
6. 83502 in. **7.** 1198134 in. **8.** 405258 in. **9.** 2045286 in.
10. 65321 in. **11.** 102955 yds. **12.** 144657 yds. **13.** 95138 yds.
14. 513219564 sq. in. **15.** 76643820 sq. in. **16.** 628908948 sq. in.
17. 6266844 sq. in. **18.** 761694516 sq. in. **19.** 105658668 sq. in.
20. 7355 sq. yds. **21.** 28052 sq. yds. **22.** 44464 sq. yds.
23. 1087392 sq. yds. **24.** 339624 sq. yds.

LVI. PAGE 88.

1. £4. 3*s.* 4*d.* **2.** £51. 8*s.* 4*d.* **3.** £13. 12*s.* 7½*d.* **4.** £12. 17*s.* 2½*d.*
5. £10416. 13*s.* 4*d.* **6.** $7.44. **7.** $10000. **8.** $76.53.
9. 5 tons. **10.** 2 cwt. 85 lbs. 4 oz. **11.** 2 tons 12 cwt. 1 oz.
12. 2 cwt. 67 lbs. 11 oz. 12 drs. **13.** 1 ton 6 cwt. 41 lbs.
14. 4 tons 13 cwt. 33 lbs. **15.** 52 tons 18 cwt. 57 lbs. 1000 grs.
16. 14 tons 5 cwt. 49 lbs. 5 oz. **17.** 7 tons 4 cwt. 53 lbs. 8 oz. 6 drs.
18. 1111 yds. 4 in. **19.** 243 yds. 2 in. **20.** 1522 yds. 1 ft.
21. 63 mi. 231 yds. 5 in. **22.** 745 yds. 6 in. **23.** 58 m. 879 yds. 5 in.
24. 31 m. 995 yds. 1 ft. 8 in. **25.** 132 m. 1013 yds. 1 ft.
26. 12 m. 1046 yds. 3 in. **27.** 13 m. 1310 yds. 2 ft. 6 in.
28. 685 sq. yds. 7 sq. ft. 120 sq. in.
29. 1 a. 2199 sq. yds. 5 sq. ft. 136 sq. in.
30. 333 sq. yds. 4 sq. ft. 26 sq. in. **31.** 2 a. 3756 sq. yds.
32. 5 a. 2430 sq. yds. 2 sq. ft. 108 sq. in. **33.** 21 c. yds. 11 c. ft. 1216 c. in.
34. 16888 c. yds. 24 c. ft. **35.** 174 c. yds. 22 c. ft. 580 c. in.
36. 102 c. yds. 18 c. ft. 107 c. in. **37.** 716 c. yds. 23 c. ft. 126 c. in.
38. 1250 gals. **39.** 6250 gals. **40.** 78 bus. 4 qts.
41. 696 bus. 1 pk. 3 qt. 1 pt. **42** 3 wks. 6 da. 18 hrs. 40 m

43. 1 wk. 4 da. 13 hrs. 46 m. 40 sec.
44. 16 wks. 3 da. 17 hrs. 46 m. 40 sec. **45.** 74 wks. 2 da. 13 hrs. 20 m.
46. 14 wks. 5 da. 19 hrs. 40 m. **47.** 64 lbs. 3 oz. 15 dwt. 11 grs.
48. 137 lbs. 9 oz. 10 dwt. 4 grs. **49.** 64 lbs. 10 oz. 10 dwt. 20 grs.
50. 72 lbs. 9 oz. 2 dwt. 11 grs. **51.** 637 lbs. 11 oz. 4 dwt. 14 grs.
52. 68 cwt. 44 lbs. 2000 grs.; 8317 lbs. (Troy) 8 oz. 10 dwt.
53. 128 cwt. 27 lbs. 2000 grs.; 15588 lbs. (Troy) 8 oz. 11 dwt. 16 grs.
54. 5 cwt. 28 lbs. 1971 grs.; 642 lbs. (Troy) 2 dwt. 3 grs.
55. 128 cwt. 42 lbs. 4900 grs.; 15607 lbs. (Troy) 5 oz. 7 dwt. 12 grs.
56. 11° 27′ 14″. **57.** 15° 46′. **58.** 3 rt. angles 7° 46′ 40″.
59. 1 rt. angle 48° 14′ 3″. **60.** 3° 25′ 45″.

LVII. Page 89.

1. 2 m. 1 fur. 15 p. 2 yds. 1 ft. 6 in.
2. 138 m. 7 fur. 4 p. 2 yds. 1 ft. **3.** 2 m. 7 fur. 15 p. 1 yd. 1 ft. 6 in.
4. 49 m. 3 fur. 18 p. 1 yd. **5.** 157 m. 6 fur. 25 p. 10 in.
6. 28 m. 3 fur. 10 p. 5 yds. **7.** 3 fur. 15 p. 2 yds. 2 ft.
8. 58 m. 3 f. 39 p. 4 yds. 1 ft. 11 in. **9.** 82 m. 1 f. 21 p. 1 yd. 1 ft. 7 in.
10. 2 m. 3 fur. 3 p. 2 yds. **11.** 72 m. 30 p. 2 ft. 6 in.
12. 107 m. 1 fur. 7 p. 5 yds. 6 in. **13.** 14 m. 2 p. 3 yds. 1 ft. 9 in.
14. 5 m. 7 fur. 34 p. 4 yds. 7 in.
15. 759 a. 1 r. 25 p. 29 sq. yds. 2 sq. ft. 108 sq. in.
16. 1 a. 1 r. 9 p. 23 sq. yds. 1 sq. ft. 112 sq. in.
17. 2 r. 22 p. 8 sq. ft. 40 sq. in.
18. 8 a. 1 r. 39 p. 1 sq. yd. 3 sq. ft. 36 sq. in.
19. 2 a. 3 r. 4 p. 5 sq. yds. **20.** 5 a. 2 r. 10 sq. yds. 2 sq. ft. 108 sq. in.
21. 224 a. 2 r. 26 p. 25 sq. yds. 7 sq. ft. 72 sq. in.
22. 9 a. 29 p. 26 sq. yds. 6 sq. ft. 82 sq. in. **23.** 1 a. 2 r. 3 p. 4 sq. yds.
24. 1 a. 22 p. 28 sq. yds. 2 sq. ft. 13 sq. in.
25. 47 a. 3 r. 19 p. 28 sq. yds. 4 sq. ft.

LVIII. Page 89.

1. 2 cwt. 5 lbs. 5000 grs. **2.** 6 cwt. 85 lbs. 5000 grs. **3.** 21875.
4. 121 lbs. 6 oz. 6 dwts. 16 grs. **5.** 1 cwt. 42 lbs. 6000 grs.
6. 239 gals. **7.** 100 ch. 16½ yds. **8.** 29 tons 3 cwt. 9 lbs. 12·6 oz. (nearly).
9. 4000 p. **10.** 125 m. **11.** 364 oz. 11 dwts. 16 grs.
12. 24 years 48 wks. 4 days. **13.** 1240 grs. **14.** 132.

LIX. PAGES 90, 91, 92, 93.

1. $2.57. **2.** 2*s.* 5¾*d.* **3.** 1 yd. 3 in. **4.** 2 yds. 9 in. **5.** 39 lbs. 10 oz.
6. 8 yds. 1 ft. 6 in. **7.** 10 yds. 1 ft. 10 in. **8.** 18 yds. 2 ft. 3 in.
9. 28 lbs. 6 oz. **10.** £1. 17*s.* 0¼*d.* **11.** 40 days 10¾ hrs.
12. 18 yds. 6¾ in. **13.** 28 lbs. 6¾ oz. **14.** 67 lbs. 8 oz. 19 dwt.
15. 186 tons 6 cwt. 39 lbs. **16.** £98. 14*s.* **17.** £24. 10*s.* 10*d.*
18. £152. 2*s.* 11¾*d.* **19.** £37. 6*s.* 7½*d.* **20.** 5072 hrs. 11 min. 47¾ sec.
21. 610 tons 9 cwt. 37½ lbs. **22.** 943 days 4 hrs. 40 min.
23. 958 days 14 hrs. 36¾ min. **24.** 1243 days 3 hrs. 40¼ min.
25. 1220 mi. 70 yds. 3¼ in. **26.** £10898. 5*s.* 9½*d.* **27.** 17506° 34′ 56″.
28. 16263 cwt. 20 lbs. 10 oz. **29.** 44626 tons 5 cwt. 83 lbs.
30. 48019 days 4 hrs. 17 min. **31.** 1917 tons 10 cwt. 31 lbs. 10 oz.
32. 27065 tons 132 lbs. 4 oz. **33.** 34715 days 32 mins.
34. 35045 days 21 hrs. 29 min. **35.** 52 lbs. 8 drs.
36. 10 cwt. 70 lbs. 10 oz. **37.** 1237 tons 13 cwt. 3 qr. 18 lbs.
38. 3 tons 10 cwt. 2 qr. 11 lbs. **39.** 14 tons 18 cwt. 7 lbs.
40. 582 tons 10 cwt. 3 lbs. **41.** 1 lb. 7 oz. 1 dwt. 10 gr.
42. 2 lbs. 4 oz. 10 dwt. 4 gr. **43.** 10 lbs. 3 oz. 8 dwt. 3 gr.
44. 14 p. 4 yds. 2 ft. 11 in. **45.** 117 m. 3 f. 26 p. **46.** 19 m. 3 f. 3 p. 2 yds.
47. 2 p. 14 sq. yds. 6 sq. ft. 1 sq. in. **48.** 297 a. 1 r. 6 p.
49. 185 a. 23 sq. p. 25½ sq. yds. **50.** 168 cu. yds. 6 cu. ft. 718 cu. in.
51. 65 cu. yds. 2 cu. ft. 600 cu. in. **52.** 149 cu. yds. 20 cu. ft. 1116 cu. in.
53. 24 gals. 2 qts. **54.** 20 bus. 5 qts. **55.** 18334 bus. 1 pk. 3½ qts.
56. 2 days 3 hrs. 8 m. 16 sec. **57.** 155 days 17 hrs. 39 min.
58. 25 weeks 6 days 13 hrs. **59.** 59° 13′ 17″.
60. 54° 34′ 51″. **61.** 68° 12′ 42″.

LX. PAGE 94.

1. 2 ft. 11 in. **2.** 11 lbs. 11 oz. 19 dwt. **3.** $148.92. **4.** $223.19.
5. 2 yds. 1 ft. 9 in. **6.** 2 yrs. 3 days 21 hrs. 56 min. **7.** £25. 18*s.* 2*d.*
8. 6 sq. yds. 2 sq. ft. 74 sq. in. **9.** £87. 17*s.* 5¾*d.* **10.** $7.92.
11. 87 lbs. 6 oz. **12.** 6 tons 15 cwt. 79 lbs. **13.** 13 cwt. 99 lb.
14. 1 ft. 3 in. **15.** 1 fur. 20 p. **16.** 1584 yds. 1 ft.
17. 78 sq. yds. 4 sq. ft. 61 sq. in. **18.** 1 sq. yd. 5 sq. ft. 117 sq. in.
19. 4719 sq. yds. 4 sq. ft. 72 sq. in. **20.** 1 ac. 24 po. 28½ yds.
21. 4 c. yds. 16 c. ft. 1584 c. in. **22.** 6 gals. 1 qt. 1 pt.
23. 2 pks. 7 qts. **24.** 2 hrs. 38 m. 30 secs. **25.** 2 days 2 hrs. 30 m.
26. 5 days 7 hrs. 25 min. **27.** 86° 39′ 30″. **28.** 1° 39′ 58″.

LXI. PAGE 95.

1. 1 day 2 hrs. 42 min. **2.** $16.13. **3.** 90 yds. 2 in.
4. 262 days 3 hrs. 40 min. **5.** $2335.99. **6.** 15*s.* 4*d.* **7.** 2 qts. 1 pt.

LXII. PAGE 96.

1. 18 yds. **2.** 66 lbs. 4 oz. **3.** £39. 15*s.* **4.** £67. 16*s.*
5. 67 days 12 hrs. 51 min. **6.** 14 mi. 596 yds. 13 in.
7. 25 cwt. 71 lbs. 8 oz. **8.** $364.15. **9.** 37 yds. 1 ft. 3 in.
10. 128 cwt. 16 lbs. **11.** 57 wks. 3 days 2 hrs. **12.** 405° 57′ 51″.

LXIII. PAGE 97.

1. 134 wks. 6 days. **2.** 255 yds. 4 in. **3.** 519 lbs. 12 dwt.
4. 553 yds. 1 ft. **5.** 116 wks. 4 days 18 hrs. **6.** 228 qrs. 2 lbs. 10 oz.
7. $1183.84. **8.** 221 days 16 hrs. **9.** 99 sq. yds. 8 sq. ft. 16 sq. in.
10. 17640 cwt. **11.** £88333. 6*s.* 8*d.* **12.** 27843 miles 6 fur.
13. 56061 sq. yds. 6 sq. ft. 80 sq. in. **14.** 5195 sq. mi. 200 acres.
15. £18054. 3*s.* 4*d.* **16.** 19050 bus. **17.** 41 tons 9 cwt. 44 lbs.
18. 194 tons 13 cwt. 40 lbs. **19.** 682 tons 3 cwt. 20 lbs.
20. 232 tons 17 cwt. 80 lbs. **21.** 1 cwt. 16 lbs. 4 drs.
22. 107 cwt. 20 lbs. 15 oz. **23.** 28 lbs. 5 oz.
24. 250 lbs. 10 oz. 10 dwts. **25.** 30 m. 1 f. 20 yds.
26. 105 m. 1266 yds. 2 ft. **27.** 418 acres 2 r.
28. 796 acres 3 r. 20 p. **29.** 309 sq. yds. 5 sq. ft. 96 sq. in.
30. 3052 sq. yds. 1 sq. ft. 64 sq. in. **31.** 1235 gals.
32. 1147 gals. 2 qts. **33.** 751 bus. 3 pk. 4 qts.
34. 260 bus. 2 pks. 4 qts. **35.** 37 days 19 hrs. 48 m. 20 secs.
36. 7927 yrs. 345 days.

LXIV. PAGE 98.

1. $2281.25. **2.** $33318.03. **3.** $10126.53. **4.** $11313.82.
5. $29597.04. **6.** $321476.11. **7.** $237149.74. **8.** $763736.61.
9. 8009 yds. 7 in. **10.** $3009288.94. **11.** 1256 tons 8 cwt. 54 lbs.
12. 6951 tons 6 cwt. 51 lbs. **13.** 6798 m. 120 yds.
14. 545 m. 6 f. 210 yds. **15.** 636 c. yds. 2 c. ft. 724 c. in.
16. 1214 acres 2020 sq. yds.

LXV. PAGE 98.

1. $4. **2.** $6.48. **3.** $8.40. **4.** $9.24. **5.** $103.14.

6. $195.93. **7.** $618.31. **8.** $175. **9.** $693. **10.** $410.40.

11. £7. **12.** $538.37. **13.** £174. 18*s.* **14.** $656.04.

15. £82. 2*s.* 6*d.* **16.** $5907.11. **17.** £283. 16*s.* 8*d.* **18.** $5524.38.

19. $7894.17. **20.** £8487. 16*s.* 8*d.* **21.** $80. **22.** $8.64.

23. 343 tons 1 cwt. 12 lbs. **24.** $2321.40. **25.** $281.25.

26. $1186.25. **27.** $1703.70.

LXVI. PAGE 99.

1. $2.45. **2.** $11.89. **3.** $6.86. **4.** $3.62. **5.** $15.27.

6. $9.57. **7.** $5.22. **8.** $8.37. **9.** $5.37. **10.** $69.31.

11. $3.36. **12.** $7.57. **13.** $84.75. **14.** $21.05.

15. 7 yds. 2 ft. 11 in. **16.** 1 ft. 10 in. **17.** 3 yds. 9 in.

18. 8 yds. 2 ft. 11 in. **19.** 8 yds. 2 ft. 9 in. **20.** 9 yds. 1 ft. 9 in.

LXVII. PAGE 100.

1. $4.92. **2.** $796.74. **3.** 11*s.* 4*d.* **4.** £5. 4*s.* 9½*d.*

5. 24 ft. 5 in. **6.** 3 yds. 1 ft. 10·87 in. **7.** $76.54. **8.** $96.24.

9. $932.18. **10.** 70 c. **11.** $8.64. **12.** $19.29. **13.** $92.70.

14. $92.65. **15.** 1 c. **16.** 0 c. **17.** 3 cwt. 16 lbs. 5 oz.

18. 54 yds. 3 in. **19.** 157 sq. yds. 4 sq. ft. 72 sq. in.

20. 10 a. 117 sq. yds. 126 sq. in.

LXVIII. PAGE 101.

1. $1.51. **2.** $4.80. **3.** $8.76. **4.** $1.13.

5. 3 cwt. 37 lbs. 8 oz. (24 oz. *rem.*) **6.** 2 cwt. 99 lbs. 2 oz. (nearly).

7. 4 mi. 780 yds. **8.** 1 mi. 142 yds. 1 ft. 2 in. (nearly).

9. 2 cu. yds. 3 cu. ft. 100 cu. in. **10.** 3*s.* 6*d.* **11.** $321.89.

12. $6.96. **13.** $20.19. **14.** $42.76. **15.** $7.90. **16.** $17.83.

17. $106.50. **18.** $92.52. **19.** $102.02. **20.** $1.53. **21.** 92 c.

22. $10.16. **23.** $15.68. **24.** 9*d.* **25.** 3 cwt. 57 lbs. 2 oz.

26. 7 cwt. 70 lbs. 8 oz. **27.** 3 tons 26 lbs. 14 oz.

28. 3 tons 24 lbs. 10 oz. **29.** 70 yds. 2 ft. 10 in.

30. 15 acres 1 r. 31 po. **31.** 5 fur. 78 yds. 2 ft. 7 in.

32. 11 acres 3 r. 27 po. **33.** 5 sq. yds. 8 sq. ft. 131 sq. in.

34. 38 gal. 2 qts. 1 pt. **35.** 2 days 17 hrs. 39 m. 20 sec. **36.** 14′ 35″.

LXIX. Page 102.

1. (i) 40 mi. 2 f. 37 p. 1 yd. 1 ft. 6 in., (ii) 193 mi. 6 f. 2 p. 5 yds.
2. (i) 684 mi. 20 p. 2 yds., (ii) 491 mi. 5 f. 14 p. 3 yds. 1 ft. 6 in.
3. (i) 5423 mi. 4 f. 6 p. 2 yds. 2 ft. 6 in., (ii) 21563 mi. 1 f. 34 p. 2 yds. 2 ft. 6 in.
4. (i) 17 mi. 31 p. 5 yds. 2 ft. 9 in., (ii) 656 mi. 5 f. 37 p. 1 yd. 1 ft. 9 in.
5. (i) 48 a. 3 r. 1 p. 19 sq. yds. 6 sq. ft. 108 sq. in., (ii) 351 a. 11 p. 27 sq. yds. 2 sq. ft. 36 sq. in.
6. (i) 1519 a. 2 r. 26 p. 3 sq. yds. 7 sq. ft. 56 sq. in., (ii) 994 a. 10 p. 3 sq. yds. 5 sq. ft. 64 sq. in.
7. (i) 6565 a. 1 r. 14 p. 27 sq. yds. 4 sq. ft. 72 sq. in., (ii) 21735 a. 1 r. 2 p. 11 sq. yds. 6 sq. ft. 108 sq. in.
8. (i) 8048 a. 2 r. 39 p. 23 sq. yds. 6 sq. ft. 72 sq. in., (ii) 64903 a. 3 r. 38 p. 7 sq. yds. 7 sq. ft. 108 sq. in.
9. (i) 1 mi. 4 p. 3 yds. 10 in.; rem. 8 in., (ii) 3 f. 27 p. 3 yds.
10. (i) 1 f. 2 yds. 1 in.; rem. 4 in., (ii) 1 f. 4 p. 3 yds.
11. (i) 1 mi. 7 f. 24 p. 3 yds. 10 in.; rem. 22 in., (ii) 1 mi. 6 f. 32 p. 4 yds. 1 ft. 10 in.; rem. 4 in.
12. (i) 5 f. 25 p. 2 yds. 1 in.; rem. 60 in., (ii) 2 f. 10 p. 2 yds. 11 in.
13. (i) 2681 a. 1 r. 31 p. 6 sq. yds. 79 sq. in.; rem. 5 sq. in., (ii) 372 a. 1 r. 27 p. 20 sq. yds. 1 sq. ft. 74 sq. in.; rem. 3 sq. in.
14. (i) [illegible]56 a. 2 r. 32 p. 8 sq. yds. 6 sq. ft. 125 sq. in.; rem. 15 sq. in., (ii) 350 a. 1 r. 11 p. 29 sq. yds. 7 sq. ft. 64 sq. in.
15. (i) 3 r. 25 p. 12 sq. yds. 2 sq. ft. 3 sq. in.; rem. 19 sq. in., (ii) 1 r. 2 p. 10 sq. yds. 8 sq. ft. 87 sq. in.; rem. 111 sq. in.
16. (i) 1 a. 3 r. 27 p. 19 sq. yds. 1 sq. ft. 65 sq. in.; rem. 45 sq. in., (ii) 2 r. 30 p. 8 sq. yds. 5 sq. ft. 107 sq. in.; rem. 21 sq. in.

LXX. Page 103.

1. 61. **2.** 69. **3.** 49. **4.** 13. **5.** 11. **6.** 106.
7. 10; 47 yds. 1 ft. 4 in. **8.** 408; 4 yds. **9.** 36; 307 days 6 hrs.
10. 5326; 19 hrs. **11.** 11; 5 lbs. **12.** 51; 1 m. 7 fur. 134 yds.
13. 9. **14.** 168. **15.** 28. **16.** 110. **17.** 9. **18.** 48; 8 lbs.

LXXI. Page 104.

1. 31. **2.** 100. **3.** 55. **4.** 34. **5.** 22. **6.** 15. **7.** 53. **8.** 27.

LXXII. PAGES 104, 105, 106.

1. 56; 112 grs. rem. **2.** $30.88. **3.** $970.66. **4.** 7 oz. **5.** $328.
6. 12 shillings. **7.** £57. 12*s.* **8.** 9*s.* 2*d.* **9.** $465; $15 loss.
10. 60 cents. **11.** 288. **12.** 11912. **13.** Loss of $9.40. **14.** $68.20.
15. $7.31; $7.71. **16.** $5.41; $10.82. **17.** $65.21; $80.21.
18. 25 yds. 1 ft. 11 in.; 51 yds. 10 in. **19.** 29 yds. 2 ft. 11$\frac{1}{2}$ in.; 89 yds. 2 ft. 10$\frac{1}{2}$ in.; 59 yds. 2 ft. 11 in. **20.** $4170.10; $2085.05.
21. £4. 0*s.* 11$\frac{3}{4}$*d.* **22.** $95.94. **23.** $219.50. **24.** 57640000.
25. 135 gals. **26.** $4235. **27.** $98.83. **28.** 11th April. **29.** Saturday.
30. 883. **31.** 1·82. **32.** 31645. **33.** Thursday. **34.** 29th March 1882.
35. Tuesday. **36.** 7296 hrs. **37.** 4 tons 13 cwt. 4 lbs. **38.** 60°; 5°.
39. *A*, $458.40; *B*, $229.20; *C*, $76.40. **41.** 1° 15′.
42. (i) 7 d. 3 hrs.; (ii) 3 min. 48 sec. **43.** 18,000 turns.
44. 2 hrs. 40 min.

LXXIII. PAGES 107, 108.

1. 2 ft. $\frac{4}{9}$ in. **2.** 4.56\frac{2}{3}$. **3.** 10 cwt. 3 lbs. 2 oz.
4. 33 days 8 hrs. 24 min. **5.** $14.98 nearly. **6.** 63 yds.
7. 4 tons 16 cwt. 38 lbs. 2 oz. **8.** 60 tons 10 cwt. 45 lbs. 11 oz.
9. $1209.26 (nearly). **10.** $71.65. **11.** $418.75. **12.** $14.05.
13. 4 mi. 266 yds. 2 ft. **14.** 183 yds. 6 in. **15.** $115.82.
16. $100.95. **17.** 1 yd. 2 ft. 7$\frac{9}{14}$ in. **18.** 17 yds. 2 ft. $\frac{4}{7}$ in.
19. $34.70. **20.** 15 hrs. 21 min. 25$\frac{5}{7}$ sec. **21.** 1 ton 13 cwt. 20 lbs.
22. 1 cwt. 25 lbs. 10 oz. **23.** 2 ac. 1120 sq. yds. **24.** 79 yds. 1 ft. 4$\frac{4}{5}$ in.
25. 579 c. yds. 2 c. ft. 1369 c. in. **26.** 42 bus.
27. 18 ac. 7 p. 15 sq. yds. 1 sq. ft. 18 sq. in. **28.** 25 ac. 1128 sq. yds.
29. 2 m. 381 yds. 1 ft. **30.** 58 bus. 2 pks. **31.** 22.14\frac{2}{3}$.
32. $4022.99. **33.** £9. 12*s.* 9$\frac{1}{7}$*d.* **34.** £2. 7*s.* 3*d.*
35. 1044 yds. 2 ft. $\frac{23}{24}$ in. **36.** £6. 1*s.* 1$\frac{7}{11}$*d.* **37.** 116 d. 13 h. 6 m. 40 s.
38. $54.04 ($54.036). **39.** $34.30. **40.** $205.60. **41.** $14.15.
42. $164.64. **43.** 5 yds. 1 ft. 11·7 in. **44.** 3 m. 6 f. 22 yds.

LXXIV. PAGE 110.

1. 7\frac{1}{2}$. **2.** 18\frac{3}{5}$. **3.** 3$\frac{1}{2}$ of $5. **4.** 2$\frac{2}{5}$. **5.** 16$\frac{9}{25}$. **6.** $\frac{52}{217}$.
7. 1$\frac{7}{48}$ yd. **8.** 3$\frac{233}{3200}$ cwt. **9.** 21$\frac{7}{8}$ yd. **10.** 1$\frac{13}{16}$ yd. **11.** 5$\frac{113}{160}$ cwt.
12. 2$\frac{19}{32}$ tons. **13.** 3$\frac{23}{240}$ mi. **14.** 3$\frac{11}{12}$ yd. **15.** 1$\frac{241}{324}$ cu. yd.
16. 3$\frac{25}{144}$ days. **17.** 3$\frac{59}{1760}$ acres. **18.** 3$\frac{29}{80}$°. **19.** $\frac{1}{37}$. **20.** 2$\frac{1}{14}$.

21. $\frac{173}{320}$. **22.** $\frac{2971}{9240}$. **23.** $\frac{11}{156}$. **24.** $\frac{961}{16940}$. **25.** $\frac{13}{31696}$. **26.** $\frac{8}{575}$.
27. $\frac{4}{5}$. **28.** $2\frac{1}{14}$. **29.** $\frac{173}{352}$. **30.** $22\frac{10}{321}$. **31.** $\frac{1}{5}$. **32.** $1\frac{105}{286}$.
33. $7.50. **34.** 27 lbs. 2 oz. 10 dwt. **35.** $\frac{23}{800}$. **36.** $\frac{800}{23}$. **37.** $\frac{231}{304}$.
38. $3725. **39.** $195\frac{1}{4}$. **40.** $\frac{47}{31120}$. **41.** $\frac{475}{586}$.

LXXV. PAGE 111.

1. $\frac{3292}{4565}$. **2.** $1\frac{17}{20}$. **3.** $\frac{613}{5280}$. **4.** $1\frac{3125}{4661}$. **5.** 1280. **6.** $\frac{5}{4032}$.
7. $\frac{155}{1096}$. **8.** $102\frac{3}{5}$. **9.** $\frac{4}{105}$. **10.** $\frac{20676}{105299}$. **11.** $1\frac{769}{1377}$. **12.** 4.
13. $34\frac{15}{22}$. **14.** $1\frac{1}{4}$. **15.** 2. **16.** $\frac{32}{121}$. **17.** 14 tons 4 cwt. $66\frac{2}{3}$ lb.
18. $\frac{125}{244}$. **19.** $\frac{175}{192}$. **20.** $21\frac{143}{320}$; 932 yds. 2 ft. $9\frac{129}{160}$ in. **21.** $203.55.
22. $39960.05.

LXXVI. PAGE 112.

1. 3·742 kilometres. **2.** 40 kilometres. **3.** ·00342 kilometres.
4. 1230 centimetres. **5.** 40336 centimetres. **6.** 9 centimetres.
7. 100000 centimetres. **8.** 3100 centimetres. **9.** 30 centimetres.

LXXVII. PAGE 113.

1. $729\frac{3}{8}$ oz. **2.** $8\frac{1}{16}$ sq. in. **3.** 531 min. **4.** $157\frac{1}{2}$ d. **5.** $8745\frac{5}{8}$ lbs.
6. $646\frac{7}{8}$ in. **7.** 1222 p. **8.** $9218\frac{3}{4}$ lbs. **9.** $46\frac{7}{8}$ lbs. **10.** 318060 secs.
11. 4245·648 yds. **12.** 16514·08 sq. yds. **13.** 3·16875 days.
14. £1·5218375. **15.** 17·2837 tons. **16.** ·0035 tons.
17. 1·4290625 days. **18.** 6·25 miles. **19.** 147·05 lbs. troy (nearly).
20. 36·640625 bus. **21.** ·245 tons. **22.** ·003125 days.
23. ·00818… tons. **24.** ·0002457 deg.

LXXVIII. PAGES 113, 114.

1. £3. 9*s.* **2.** 1 yd. 1 ft. 29·85 in. **3.** 21 miles 649 yds.
4. 13 acres 70 p. **5.** 2 tons 7 cwt. 50 lbs. **6.** 1 yd. 2 ft. 5·8125 in.
7. 4 tons 18 cwt. 37 lbs. 8 oz. **8.** 3 tons 8 cwt. 43 lbs. 12 oz.
9. 1 cwt. 31 lbs. 10 oz. **10.** 43 days 2 hrs. 59 min. 25·44 secs.
11. 121 days 12 hrs. **12.** 2 yrs. 173 days 1 hr. 6 min. 57·6 secs.
13. 2 tons 19 cwt. 16 lbs. 12 oz. **14.** 3·5 yds. **15.** 16 times $2.31.
16. 3·55 of $\frac{5}{8}$ yd. **17.** ·0105 of 1 ton. **18.** ·0043103… of 9 yds. 2 ft.
19. ·0025 of 1 day. **20.** 1·21875 tons. **21.** 3·44375 miles.
22. 1·375 weeks. **23.** 1·1315625 days. **24.** 1·359375 bus.
25. 10·39375 miles. **26.** 11·601. **27.** 1·375 poles. **28.** 2·53125 gals.
29. 1·8465625 acres.

LXXIX. Pages 115, 116.

1. ·375. **2.** ·625. **3.** ·3. **4.** 1·25. **5.** ·0310546875. **6.** 1. **7.** ·309.
8. 2·56. **9.** 3·983... **10.** ·308 (nearly). **11.** ·211... **12.** ·625.
13. 1·155. **14.** 3·4375. **15.** 1·5543. **16.** ·252. **17.** 15·0447.
18. 3·73424629... **19.** 30 inches. **20.** 15 days 14 hrs. **21.** 2 lbs. 10 oz.
22. 119 tons 12 cwt. **23.** 2 days 3 hrs. 38 min.
24. 12 sq. yds. 7 sq. ft. 132½ sq. in. (nearly).
25. 3 tons 6 cwt. 78 lbs. 10·32 oz. **26.** 6 cwt. 54 lbs. 13·6 oz.
27. 7 yds. 2 ft. 8·7975 in. **28.** 7 days 40 min. 29·55 secs.
29. 1570 days 19 hrs. 3·54 min. **30.** 4 tons 81 lbs. 4 oz.
31. 6 cwt. 50 lbs. **32.** 17 lbs. 4 oz. **33.** 2 fur. 31 p. 1 yd. 1 ft. 9·024 in.
34. 2 mi. 6 fur. 28 p. **35.** 82 lbs. 8 oz. **36.** 31 gals. 0·763 pts.
37. 39 p. 4 yds. 1 ft. 2·4 in. **38.** 2 lbs. 8 oz. **39.** 3 cwt. 37 lbs. 10·4 oz.
40. 71 days 9 hrs. 36 min. **41.** 2 tons 16 cwt. 77 lbs.
42. 4 tons 3 cwt. 97 lbs. **43.** 24 mi. 4 fur. 18 p.
44. 41 yds. 2 ft. 2·08 in. **45.** ·3. **46.** 14·4. **47.** ·0024350649.
48. ·15625.

LXXX. Pages 116, 117, 118.

1. 831600. **2.** 4989·6 oz. **3.** 206·25 yds. **4.** 1690·4408 oz.
5. ·001. **6.** ·0955753. **7.** 360 hrs. **8.** 1 ft. 7 in. **9.** $2\frac{1}{24}$ of $4.
10. ·9390507936... **11.** $\frac{13}{648}$. **12.** 13 in. **13.** 4 hrs. 18 min.
14. 1254 days 16 hrs. 54 min. **15.** $\frac{1}{20}$. **16.** (i) £·038875;
(ii) £·0008125. **17.** 14 tenths of an inch. **18.** 3 days 5 hrs. 2 min. 9 secs.
19. $7.70. **20.** $21.11. **21.** 96 c. **22.** ·1 crown. **23.** ·00386 sq. mi.
24. 95 p. 15 yds. 0 ft. 5·184 in. **25.** 49 acres. **26.** 25·67 francs.
27. 7699·52 + ... metres. **28.** 1609·344. **29.** $27.18. **30.** 4·6 cu. yds.
31. 631, and ·033 in. over. **32.** 1103, and ·65 p. over.
33. 146, and $\frac{1}{16}$ inch over. **34.** 38, and $3\frac{62}{143}$ qts. over. **35.** 471·2385.
36. $1.20. **37.** 11·30976 ft. **38.** $8.90. **39.** 150 lbs. **40.** $10\frac{35}{59}$.
41. $\frac{5}{8}$ of $\frac{1}{4}$ of 1 ton. **42.** $\frac{1}{3}$ of 15 cwt. 78 lbs. **43.** $\frac{1111}{7137}$.
44. 3 oz. 7 dwt. 12·37 grns. **45.** $\frac{33}{245}$. **46.** $5\frac{3}{8}$ gal.; 195.
47. 1 min. 57 sec. **48.** 480, 700, 909; unit = $\frac{1}{64}$ inch.

LXXXI. Page 119.

1. 1 ft. 6 in. **2.** 8 hrs. **3.** 25 lbs. **4.** 20 c. **5.** 6 in. **6.** 2 days.
7. 1 pint. **8.** 3 cu. ft. **9.** 10 lbs. **10.** 160 yds. **11.** 3 in.
12. 4 min. **13.** 9 sq. inches. **14.** 5 c. **15.** 4 c. **16.** 48 min.
17. 50 lbs. **18.** 2 c. **19.** 24 minutes. **20.** 36 seconds

21. 30 seconds. **22.** 2 oz. **23.** 2 hrs. 24 min. **24.** 12 seconds.
25. 10*s.* **26.** 6*s.* 8*d.* **27.** 5*s.* **28.** 4*s.*

LXXXII. PAGES 120, 121.

1. $100. **2.** $93. **3.** $11.87½. **4.** $100. **5.** $18.60. **6.** $73.
7. 1 cwt. 5 lbs. 4 oz. **8.** 88 cwt. 60 lbs. **9.** 5 cwt. 42 lbs. 4 oz.
10. 138 cwt. **11.** 20 cwt. 44 lbs. 6 oz. **12.** 4 cwt. 58 lbs. 7 oz.
13. 112 cwt. 37 lbs. 8 oz. **14.** 208 cwt. 32 lbs. 8 oz.
15. 58 cwt. 80 lbs. 14 oz. **16.** 7 cu. yds. 11 cu. ft.
17. 5 cu. yds. 5 cu. ft. 864 cu. in. **18.** 5 cu. yds. 15 cu. ft. 1296 cu. in.
19. 3 cu. yds. 4 cu. ft. 864 cu. in. **20.** 20 cu. yds. 12 cu. ft. 864 cu. in.
21. 218 cu. yds. 16 cu. ft. 144 cu. in. **22.** 64 cu. yds. 11 cu. ft. 864 cu. in.
23. $7128. **24.** $4875.

LXXXIII. PAGES 122, 123.

1. 1091 tons 5 cwt. **2.** 955 tons. **3.** 4669 tons 8 cwt. 75 lbs.
4. 1476 tons 2 cwt. 12 lbs. 8 oz. **5.** 11395 tons 14 cwt. 70 lbs. 12 oz.
6. 3962 tons 10 cwt. 60 lbs. **7.** 10196 tons 3 cwt. 43 lbs. 12 oz.
8. 31577 tons 5 cwt. 60 lbs. **9.** 55350 tons 10 cwt. 12 lbs. 8 oz.
10. 9651 tons 2 cwt. 18 lbs. 12 oz. **11.** 15015 tons 9 cwt. 5 lbs.
12. 60103 tons 9 cwt. 22 lbs. **13.** 105 cu. yds. 24 cu. ft. 1536 cu. in.
14. 375 cu. yds. 25 cu. ft. **15.** 4716 cu. yds. 864 cu. in.
16. 888 cu. yds. 18 cu. ft. **17.** 623 gals. 3 qts. 1 pt.
18. 12701 gals. 2 qts. **19.** 63555 gals. 3 qts. 1 gi. **20.** 95406 gals. 1 qt.
21. 23034 bus. 1 pk. 4 qts. **22.** 10745 gals. 2 qts. 1 pt.
23. 603 miles 1728 yds. **24.** 530 yrs. 135 days 7 hrs. 30 min.

LXXXIV. PAGE 124.

1. $6637.05. **2.** $9707.10. **3.** $11187.66. **4.** $655.88.
5. $48166.56. **6.** $3501.50. **7.** $59.32½. **8.** $87.57. **9.** $542.88.
10. $2192.33. **11.** $51.28. **12.** $160.46. **13.** $78.17. **14.** $581.95.
15. $527.14. **16.** $92.08. **17.** $323.40. **18.** $866. **19.** $293.
20. 693 bus. 0·5625 pks. **21.** 7189 bus. 2·46875 pks.
22. 530 cwt. 50·7 lbs.

LXXXV. PAGES 125, 126.

1. $13050.96. **2.** $34922.12½. **3.** $83.89½. **4.** $9906.85.
5. 136 tons 12 cwt. 62 lbs. 8 oz. **6** 52 tons 13 cwt. 75 lbs

7. $409659.25. **8.** $68710.55. **9.** $63456.25. **10.** $2195.39.

11. $466.72½. **12.** $8341.79. **13.** $7983. **14.** 14173 tons 18 cwt. 1·6 lbs.

15. $1359.71. **16.** $1887.27. **17.** £267. 17*s.* 5½*d.* **18.** £31. 15*s.* 3*d.* gain.

19. $25626.04. **20.** $5.82. **21.** $836.46. **22.** 559·8613671875 q.

23. $587.16. **24.** $16.01. **25.** $35377.42. **26.** $6464.91.

LXXXVI. PAGE 128.

1. $52.01. **2.** $19.25. **3.** $6.41. **4.** $79.47. **5.** $11.88.

6. $19.49. **7.** $617.41. **8.** $56.86. **9.** $137.88. **10.** $45.12.

11. $233.46. **12.** $188.50.

LXXXVII. PAGE 130.

1. $\cdot\dot{6}$. **2.** $\cdot\dot{2}8571\dot{4}$. **3.** $\cdot\dot{4}2857\dot{1}$. **4.** $\cdot\dot{5}7142\dot{8}$. **5.** $\cdot\dot{7}1428\dot{5}$. **6.** $\cdot\dot{8}5714\dot{2}$.

7. $\cdot 0\dot{6}$. **8.** $\cdot\dot{6}\dot{3}$. **9.** £$\cdot\dot{1}$. **10.** $34\cdot\dot{0}\dot{9}$*d.* **11.** $65\cdot\dot{1}4285\dot{7}$ lbs.

12. $7\cdot 8\dot{3}$ in. **13.** $3\cdot\dot{8}5714\dot{2}$ gal. **14.** $52\cdot\dot{1}4285\dot{7}$ dys. **15.** $\cdot\dot{6}$.

16. $\cdot\dot{2}8571\dot{4}$. **17.** $\cdot\dot{4}2857\dot{1}$. **18.** $\cdot\dot{5}7142\dot{8}$. **19.** $5\cdot\dot{1}$. **20.** $20\cdot\dot{2}3076\dot{9}$.

21. $2\cdot\dot{6}\dot{3}$. **22.** $451\cdot\dot{8}5714\dot{2}$. **23.** $50\cdot 2\dot{6}$. **24.** $19\cdot 458\dot{3}$. **25.** $5\cdot 1041\dot{6}$.

26. $200\cdot\dot{0}1234567\dot{9}$. **27.** $27\cdot 09\dot{3}$. **28.** $4\cdot\dot{4}$. **29.** $\cdot 556\dot{8}\dot{1}$.

30. $11\cdot\dot{2}4691358\dot{0}$. **31.** $\cdot\dot{1}$. **32.** $\cdot\dot{0}\dot{1}$. **33.** $\cdot\dot{0}0\dot{1}$. **34.** $\cdot\dot{0}000\dot{1}$.

LXXXVIII. PAGE 131.

1. $\frac{7}{9}$. **2.** $\frac{1}{90}$. **3.** $\frac{700}{999}$. **4.** $\frac{7}{999}$. **5.** $2\frac{13}{990}$. **6.** $10\frac{1}{90}$. **7.** $14\frac{131}{9990}$.

8. $9\frac{601}{990}$. **9.** $\frac{302}{2775}$. **10.** $\frac{6397}{9990000}$. **11.** $\frac{527}{4950000}$. **12.** $4\frac{1453}{3300}$. **13.** $\frac{1}{30}$.

14. $\frac{9}{110}$. **15.** $60\frac{997}{1100}$. **16.** $\frac{991}{1100000}$. **17.** $\frac{34}{275}$. **18.** $\frac{53}{550}$. **19.** $\frac{1019}{2220}$.

20. $\frac{7003}{11100}$. **21.** $676\frac{80}{111}$. **22.** $30\frac{669}{740}$. **23.** $710\frac{17}{1375}$. **24.** $\frac{17}{18750}$.

25. $60\frac{61}{66}$. **26.** $10\frac{1}{44}$.

LXXXIX. PAGE 132.

1. ·1. **2.** 30·48. **3.** 8·1237. **4.** ·006. **5.** 4. **6.** 3·7. **7.** 71.

8. ·0001. **9.** 1·679. **10.** 1·25. **11.** $4\frac{108}{875}$. **12.** $7\frac{1}{56}$. **13.** $\frac{71}{150}$.

14. $\frac{2408}{2397}$. **15.** $\frac{11}{7}$. **16.** 35. **17.** $\frac{220}{331}$.

XC. PAGE 133.

1. 373·1740... **2.** 376·1403... **3.** 3·454033... **4.** 21·70958...

5. 448·4084... **6.** 8047·894... **7.** 3·088809... **8.** 566·9099...

9. 296·6555... **10.** 45·22552... **11.** ·0599459... **12.** 1·779777...

13. 334·0332... **14.** ·2332816... **15.** 2·522923... **16.** 11·11102...

17. 213·2617... **18.** 8·226464... **19.** ·6650650... **20.** 7·222222...

21. 919·0160... **22.** 1·129338...

XCI. PAGE 134.

1. 7·06$\dot{8}$. **2.** 4·16$\dot{4}\dot{8}$. **3.** 1·4581$\dot{4}$7856$\dot{9}$. **4.** 1·58$\dot{4}$6355$\dot{4}$. **5.** ·08$\dot{2}\dot{3}$.
6. 79·9$\dot{1}$. **7.** 2·2$\dot{5}$. **8.** ·96. **9.** 37·13$\dot{9}\dot{8}$. **10.** 2·0$\dot{5}$8704$\dot{1}$. **11.** 1·$\dot{3}\dot{8}$.
12. ·8$\dot{3}$71$\dot{9}$. **13.** ·08$\dot{5}$9949238167$\dot{0}$. **14.** 2·0$\dot{4}$5936$\dot{8}$. **15.** ·3$\dot{1}$5$\dot{9}$.
16. ·5$\dot{6}$. **17.** ·179. **18.** ·017$\dot{1}$3022$\dot{1}$. **19.** 3·5$\dot{6}$0$\dot{0}$. **20.** 3·97$\dot{2}$00$\dot{0}$.
21. 2·18726$\dot{2}\dot{5}$. **22.** 1·290$\dot{0}$0191$\dot{1}$. **23.** 84·44$\dot{0}$3958$\dot{5}$.
24. ·27$\dot{1}$1791990009$\dot{8}$. **25.** 1. **26.** 14·53$\dot{1}$1144177480$\dot{8}$.
27. 21·2457$\dot{7}$3675475655$\dot{6}$. **28.** 3·5635$\dot{8}$4548364746$\dot{3}$.
29. 180·0258$\dot{3}$8356448257$\dot{4}$. **30.** 297·75$\dot{8}$6813983843$\dot{6}$.

XCII. PAGE 135.

1. 25·4. **2.** 9·2. **3.** 1·230$\dot{6}\dot{9}$. **4.** 6·64$\dot{8}$. **5.** ·09. **6.** 1·70$\dot{1}\dot{8}$.
7. 133·47541$\dot{6}\dot{3}$. **8.** ·00000456$\dot{5}\dot{4}$. **9.** 72·19946$\dot{3}$. **10.** 861·6207$\dot{8}$8$\dot{5}$.
11. 7·24838$\dot{8}\dot{2}$. **12.** 337·5408$\dot{1}$02$\dot{5}$. **13.** 12·340$\dot{4}$7$\dot{4}$.
14. 603·735341088$\dot{3}$. **15.** 1209·8782$\dot{5}$12$\dot{2}$. **16.** ·00362$\dot{6}$7$\dot{0}$. **17.** ·7$\dot{3}$.
18. ·3$\dot{9}$7461$\dot{0}$. **19.** 5·$\dot{0}$2168835$\dot{5}$. **20.** 1111·$\dot{1}$. **21.** $\dot{0}$6172839$\dot{5}$.
22. 392·25696284519... **23.** ·318$\dot{2}$3543$\dot{1}$. **24.** 856·$\dot{9}$85$\dot{6}$.
25. $\frac{200}{49}$=4·$\dot{0}$81632...42 figs. **26.** 142·$\dot{8}$5714$\dot{2}$. **27.** ·12$\dot{1}$8$\dot{5}$.
28. 10·8739569609134... **29.** ·0625. **30.** 11·$\dot{9}$0476$\dot{1}$. **31.** ·005.
32. 3·$\dot{6}$0$\dot{3}$. **33.** ·$\dot{1}$236858...42 figs. **34.** 1562·5.

XCIII. PAGE 136.

1. ·00$\dot{2}$. **2.** ·0$\dot{0}\dot{4}$. **3.** ·11$\dot{2}\dot{7}$. **4.** 1·04$\dot{3}$2$\dot{7}$. **5.** 1·$\dot{7}$6$\dot{9}$. **6.** ·28$\dot{1}\dot{3}$.
7. 161·$\dot{8}$5$\dot{1}$. **8.** 3·0$\dot{4}\dot{6}$. **9.** 18·6$\dot{5}\dot{9}$. **10.** ·32$\dot{7}$. **11.** 1·4$\dot{8}$5714$\dot{2}$.
12. 1·$\dot{4}$963503$\dot{6}$. **13.** 15·$\dot{0}$7$\dot{4}$. **14.** 8·2930$\dot{5}$. **15.** ·$\dot{5}$80381471...
16. ·0000446$\dot{4}$2857$\dot{1}$. **17.** 306·1. **18.** 3375. **19.** ·0001$\dot{3}$. **20.** 1·22$\dot{6}$.
21. ·1$\dot{0}\dot{9}$. **22.** ·02$\dot{5}$8$\dot{5}$. **23.** 1·5. **24.** 100÷17=5·$\dot{8}$8235...15 figs.
25. 6·$\dot{3}\dot{6}$÷83=·07667031763 4173...

XCIV. PAGE 137.

1. recurs; 12. (12 is a *limit;* the actual period has 6 digits).
4. recurs; 1. **6.** recurs; 1. **7.** recurs; 6. **9.** recurs; 1.
11. recurs; 2. **13.** recurs; 30. **14.** recurs; 9. **15.** recurs; 6.

XCV. PAGE 137.

1. 2. **2.** $\frac{1}{21} = \cdot\dot{0}4761\dot{9}$. **3.** 1. **4.** 4. **5.** ·125. **6.** $\cdot\dot{3}$.
7. $\frac{576}{41} = 14\cdot\dot{0}487\dot{8}$. **8.** 9. **9.** $\cdot91\dot{1}4285\dot{7}$. **10.** $\cdot192\dot{9}0123456\dot{7}$.
11. $\cdot9791\dot{6}$. **12.** $3\cdot\dot{9}520\dot{2}$. **13.** 1. **14.** $\frac{3}{280} = \cdot010\dot{7}1428\dot{5}$.

XCVI. PAGE 138.

1. 58·2552 pts. **2.** 532·746 in. **3.** ·62963 yds. **4.** 33·10 lbs.
5. $\cdot0079\dot{5}\dot{4}$. **6.** $\cdot172013\dot{8}$. **7.** $\cdot0118591\dot{6}$. **8.** $\cdot20\dot{4}$. **9.** $\cdot155\dot{9}5238\dot{0}$.
10. $7.20. **11.** $134.50. **13.** 5 tons 19 cwt. 67 lbs. 8 oz.
14. 19·544 lbs. **15.** 188·848. **16.** $11\cdot6\dot{9}7\dot{2}$ inches.
17. 6 yds. 1 ft. $2\cdot850\dot{0}16\dot{2}$ in. **18.** $7\cdot\dot{1}3\dot{7}$. **19.** $25. **20.** $\frac{37}{45}$.

XCVII. PAGE 144.

1. 122·96. **2.** 2·4901. **3.** 56·783. **4.** 718·95. **5.** 11359.
6. 61·768. **7.** 17029. **8.** 4303·8. **9.** ·60941. **10.** 396·154.
11. 54398 yds. **12.** 1·9999... = 2. **13.** 2·9999... = 3. **14.** 213·4 litres.
15. 1924·79 yds. **16.** 383000 kilometres. **17.** 275999·08 ft.
18. 5788·8 sq. yds.

XCVIII. PAGE 145.

1. 6·3277. **2.** ·10774. **3.** 8470·4. **4.** ·31831. **5.** 7·07108.
6. 257·99. **7.** ·7448. **16.** ·99412.

XCIX. PAGES 145, 146.

1. 141 yds. 1 ft. 3 in. **2.** 1244·511 yds. **3.** 403 times. **4.** 62·35 in.
5. 314·16 sq. ft. **6.** 4·7746 ft. **7.** 13·768 ft. **8.** 336 revolutions.
9. 9168·3 sq. ft. **10.** 2489·0096 yds. **11.** 1244·511 yds.
12. 20·78 in. nearly.

C. PAGE 147.

1. $6227060.62½. **2.** $515103.57½. **3.** $11794.66. **4.** $936.56.
5. $78.75. **6.** $27316.73. **7.** $243.95. **8.** $21534.98. **9.** $10319.62.
10. $53.83. **11.** $98.45. **12.** $30.67. **13.** $2074.17. **14.** $41.99;
$189.33. **15.** $1.65; $7984.60. **16.** $5691.90. **17.** $1404.47.

CI. PAGE 148.

1. 5 hrs. $13\frac{1}{3}$ min. **2.** ·07183. **3.** 3·141592. **4.** ·35416.
5. 113·001 grs. **6.** 1·71828, etc. **7.** 1·41068, etc. **8.** 3·14159, etc.
9. 1·15356, etc. **10.** 50 acres 4499 sq. yds. **11.** $5089·39.
12. 64·83 miles. **14.** 202·36 fr. **15.** 14·7735 kilograms.
16. 764·55 litres. **17.** 4969·6 kilograms.

CII. PAGES 151, 152.

1. 1 sq. yd. **2.** 90 sq. in. **3.** 1 sq. ft. 64 sq. in. **4.** 15 sq. yds. 5 sq. ft.
5. 43 sq. yds. 3 sq. ft. **6.** 2022 sq. yds. 5 sq. ft.
7. 433 sq. yds. 3 sq. ft. **8.** 1 sq. yd. 8 sq. ft. 72 sq. in.
9. 4 sq. ft. 91 sq. in. **10.** 6 sq. ft. 60 sq. in. **11.** $25\frac{1}{8}$ sq. yds.
12. $36\frac{3}{4}$ sq. yds. **13.** $56\frac{1}{3}$ sq. yds. **14.** 27 sq. yds. 2 sq. ft.
15. 57 sq. yds. 7 sq. ft. **16.** 25 sq. yds. **17.** 7 ft. **18.** 12 ft.
19. 5 ft. 4 in. **20.** 7 ft. 4 in. **21.** 3 acres. **22.** 3080 sq. yds.
23. 34 sq. yds. 36 sq. in. **24.** $16\frac{1}{4}$ sq. ft. **25.** 13200 sq. ft.
26. 6 sq. ft. **27.** (i) 1 ac. 860 sq. yds.; (ii) 1 ac. 347 sq. yds. 7 sq. ft.;
(iii) 512 sq. yds. 2 sq. ft. **28.** (i) $38\frac{1}{2}$ sq. yds.; (ii) 14 sq. yds. 7 sq. ft.;
(iii) 23 sq. yds. 6 sq. ft. 72 sq. in. **29.** $239.13. **30.** 50 c.
31. $29.17. **32.** $32.40. **33.** $110.01.
34. 10 sq. yds. 4 sq. ft. 108 sq. in. **35.** $21\frac{1}{3}$ ft. **36.** 36 ft.
37. $907\frac{1}{2}$ ft.

CIII. PAGE 153.

1. 28 yds. **2.** $26\frac{2}{3}$ yds. **3.** $37\frac{1}{3}$ yds. **4.** $44\frac{1}{3}$ yds. **5.** 20 yds. $2\frac{3}{7}$ ft.
6. 52 yds.

CIV. PAGE 154.

1. 66 sq. yds. 6 sq. ft. **2.** 80 sq. yds. 6 sq. ft.
3. 55 sq. yds. 3 sq. ft. 108 sq. in. **4.** 77 sq. yds. 4 sq. ft.
5. 413 sq. yds. **6.** 87 sq. yds. 5 sq. ft.

CV. PAGE 155.

1. $205\frac{5}{7}$ yds. **2.** $186\frac{3}{8}$ yds. **3.** 264 yds. **4.** 270 yds.
5. 220 yds. **6.** $122\frac{1}{8}$ yds.

CVI. PAGES 156, 157.

1. 72. **2.** 1900. **3.** 8. **4.** $18\frac{4}{9}$. **5.** $15\frac{17}{25}$. **6.** $13.86. **7.** $1.04
8. $128. **9.** $8.53. **10.** $61.76. **11.** $249.08. **12.** $35.10.
13. 13 ft. 6 in. **14.** 45 ft. **15.** 24 ft. **16.** 60 ft. **17.** $71.56.
18. $74.02. **19.** $$13.82\frac{1}{2}$. **20.** 324 bricks.

CVII. PAGE 158.

1. 2 c. yds. 6 c. ft. **2.** 45 c. yds. **3.** 2 c. yds. 18 c. ft.
4. 6 c. yds. 6 c. ft. **5.** 480 c. in. **6.** 5 c. yds. 15 c. ft.

CVIII. PAGE 159.

1. 19 tons 5 cwt. **2.** 8640 bricks. **3.** 108 times. **4.** $91\frac{647}{693}$ times.
5. 1 hr. 4 m. **6.** 6 min. 28·5 sec.

CIX. PAGES 160, 161.

1. 20 ft. **2.** 1680 sq. ft. **3.** $9.39. **4.** 24 ft. per sec. **5.** 768 yds.
6. 32·4 ft. **7.** $19.57. **8.** $175. **9.** 3 ft. 3 in. **10.** $823.68.
11. $1628. **12.** 4 inches. **13.** 1829 tons 10 cwt. 40 lbs.
14. 63063 tons ; 15080889·6 gals. **15.** $56\frac{23}{32}$ tons. **16.** 217800 tons.
17. $1.01. **18.** 2 inches.

CX. PAGE 162.

1. 4 ft. 3 in. **2.** 21 ft. $11\frac{1}{4}$ in. **3.** 3 sq. ft. $41\frac{1}{2}$ sq. in.
4. 8 sq. ft. $132\frac{1}{3}$ sq. in. **5.** 37 sq. ft. $129\frac{7}{12}$ sq. in. **6.** 3 c. ft. 486 c. in.
7. 5 c. ft. 1544 c. in. **8.** $619\frac{2}{3}$ c. in. **9.** 41 ft. 9′. **10.** 3 ft. 11′.6″.
11. 3 sq. ft. 3′.0″.3‴. **12.** 4 c. ft. 0′.9″.0‴.2⁗. **13.** 45 ft. 8′.8″.
14. 38 sq. ft. 10′.6″.3‴. **15.** 6 c. yd. 20 c. ft. 6′.3″.6‴.6⁗.

CXI. PAGE 163.

1. 72 sq. ft. 11′. **2.** 274 sq. ft. 6‴.2⁗. **3.** 62 sq. ft. 10′.10″.
4. 315 sq. ft. 8′.11″.0‴.10⁗. **5.** 170 c. ft. 4′.7″.7‴.2⁗ ($662\frac{1}{8}$ c. in.)
6. 812 c. ft. 10′.9″.3‴.6⁗ ($1551\frac{1}{2}$ c. in.)
7. 450 c. ft. 5′.3″.10‴.8⁗.6′′′′′ ($766\frac{17}{24}$ c. in.) **8.** 10 ft. 10 in. (nearly).

CXII. PAGE 165.

1. 66 c. **2.** 40 c. **3.** 84 c. **4.** 3 c.; 60 c. **5.** $8. **6.** (i) 900; (ii) 6300. **7.** 24 acres. **8.** 77 acres. **9.** 9 hrs. **10.** 12 lbs.

CXIII. PAGES 166, 167.

1. 21 days. **2.** 375 days. **3.** 5 days. **4.** 8 hours. **5.** 5 hours. **6.** 275 mths. **7.** 8 d. **8.** 20 d. **9.** 40. **10.** 10.

CXIV. PAGES 167, 168, 169, 170.

1. 105 days. **2.** 15 hrs. **3.** 8 cwt. **4.** $5. **5.** 27 acres. **6.** $650. **7.** 6 men. **8.** 400 days. **9.** 36 loaves. **10.** 12 horses. **11.** 18 bus. **12.** 35 days. **13.** 49 days. **14.** 200 bus. **15.** 49 yds. **16.** $530.40. **17.** 42 yds. **18.** $192. **19.** $2800. **20.** $3750.

CXV. PAGES 171, 172, 173.

1. $4.68. **2.** $1\frac{1}{8}$ bush. **3.** $20\frac{5}{11}$ yds. **4.** $10.53. **5.** $9.82. **6.** $102.08. **7.** $1619.50. **8.** 45 hrs. **9.** $76.32. **10.** $6593.92. **11.** 410 bus. **12.** $515.70. **13.** $2.14½. **14.** 3 hrs. 3 m. 36 sec. **15.** 26·18 yds. **16.** 7 hrs. **17.** 69 yds. **18.** 20 days. **19.** 4 days. **20.** $42\frac{2}{15}$ days. **21.** $\cdot 999893\dot{0}7936\dot{5}$. **22.** $201.96. **23.** $674.80. **24.** $1222.98. **25.** $3742.80. **26.** $312.62. **27.** 142½ lbs. **28.** 10 lbs. **29.** 2 hrs. **30.** 36 men. **31.** 24604 + ... **32.** $400.83. **33.** 75 yds. **34.** $16.50. **35.** 166½ days. **36.** $2287896.92. **37.** $48.16. **38.** 10. **39.** 195 yds. 10 in. **40.** 50 days. **41.** 40 d. **42.** 10. **43.** $24\frac{3}{4}$. **44.** $62500. **45.** $111111. **46.** 60 days.

CXVI. PAGES 174, 175, 176, 177.

1. 8. **2.** $270. **3.** 25. **4.** 21 weeks. **5.** 56. **6.** $700. **7.** 50 hours. **8.** 13½. **9.** 35. **10.** $92.75. **11.** 16. **12.** 14. **13.** 5. **14.** $20. **15.** 8. **16.** $7\frac{1}{5}$ c. **17.** 32 mi. **18.** 110 min. **19.** 495. **20.** 30. **21.** 4. **22.** 10. **23.** 108½ tons. **24.** $3.20. **25.** $251.10. **26.** $5\frac{1}{3}$ oz. **27.** 25. **28.** 13½ wks. **29.** 60. **30.** 25. **31.** 4. **32.** 4. **33.** 72. **34.** 264 yds. **35.** 6. **36.** 4 days. **37.** $15.30. **38.** 5. **39.** 42 ft. 8 in. **40.** 7 days. **41.** $5\frac{1}{3}$. **42.** $6\frac{36}{345}$.

CXVII. PAGES 177, 178.

1. $13\frac{1}{3}$. **2.** 504. **3.** 26419·2. **4.** 267·3. **5.** 749 lire 72·3 c. **6.** 715. **7.** 5 fr. 10 c. nearly. **8.** 4343 reis. **9.** 75 c. **10.** 56. **11.** £12. **12.** $4\frac{2202}{2502}$ cents. **13.** The merchant in paying through Amsterdam loses £$12\frac{27}{29}$. **14.** He should remit by London.

CXVIII. PAGES 180, 181.

1. At 2 h. 30 m. P.M., 75 m. fr. London. **2.** $1\frac{7}{11}$ hrs. after dep. fr. London, $49\frac{1}{11}$ m. fr. London. **3.** In 6 hrs. 40 m.; $\frac{5}{6}$ hr. earlier. **4.** $3\frac{51}{74}$ m. per hr. **5.** 1 d. 16 hrs. from time the second was put right. **6.** 1 hr. **7.** (i) 120 days; (ii) 11 h. 46′; (iii) 12 h. 16′. **8.** (i) $21\frac{9}{11}$′ past 7, $54\frac{6}{11}$′ past 7; (ii) $27\frac{3}{11}$′ past 2; (iii) $16\frac{4}{11}$′ past 12; $49\frac{1}{11}$′ past 12; (iv) $32\frac{8}{11}$′ past 3. **9.** (i) $21\frac{9}{11}$′ past 4, $5\frac{5}{11}$′ past 1; (ii) $38\frac{2}{11}$′ past 1, $54\frac{6}{11}$′ past 4. **10.** 10 h. 6 m. $13\frac{41}{48}$ sec. **11.** $10\frac{10}{143}$ min. gain. **12.** $27\frac{8}{11}$ min. **13.** $6\frac{41}{44}$ hrs. **14.** *A* wins by $1\frac{5}{17}$ secs. **15.** *B* wins by 50 yds. $\frac{5}{36}$ min. **16.** $39\frac{17}{22}$ yds. **17.** $20\frac{20}{17}$ yds. **18.** 60″.

CXIX. PAGE 183.

1. 6 days. **2.** 6 days. **3.** $8\frac{4}{7}$. **4.** $14\frac{2}{5}$ min. **5.** $\frac{11}{15}$. **6.** $8\frac{4}{7}$ m. **7.** $11\frac{2}{3}$. **8.** 32. **9.** $37\frac{1}{2}$. **10.** 40 c. **11.** $12\frac{1}{2}$ dys. **12.** $7\frac{4}{7}$ dys. **13.** 5.

CXX. PAGE 185.

1. $4\frac{1}{2}$. **2.** 40. **3.** $6.75. **4.** 5 cwt. 25 lbs. **5.** $202.50. **6.** 9 men. **7.** 2 acres. **8.** 50 c. **9.** 5 c. **10.** $1.38. **11.** $16.19. **12.** $49. **13.** $119.86. **14.** $43.61. **15.** $382.12. **16.** $45.59. **17.** $427.70. **18.** $56.46. **19.** $648.75. **20.** 120 sheep. **21.** $191.25. **22.** $5.70. **23.** 125 eggs. **24.** 48 miles. **25.** $8.75. **26.** $250. **27.** $14.38.

CXXI. PAGES 186, 187.

1. 5. **2.** $7\frac{1}{2}$. **3.** $37\frac{1}{2}$. **4.** 25. **5.** $16\frac{2}{3}$. **6.** $13\frac{1}{3}$. **7.** 50. **8.** $58\frac{1}{3}$. **9.** $7\frac{1}{2}$. **10.** $13\frac{1}{3}$. **11.** $3\frac{3}{4}$. **12.** $\frac{25}{48}$. **13.** 35. **14.** $4\frac{11}{16}$. **15.** 10. **16.** $41\frac{2}{3}$. **17.** $26\frac{2}{3}$. **18.** 2400. **19.** 20000. **20.** $4000. **21.** $7800. **22.** $7200. **23.** (i) 65600; (ii) 67240; (iii) 68921. **24.** (i) 130050; (ii) 127500; (iii) 125000.

CXXII. PAGE 188.

1. $10. **2.** $70. **3.** $270. **4.** $2100. **5.** $962.50. **6.** $159.84.
7. $1248. **8.** $1400. **9.** $719.25. **10.** $206.25. **11.** $1023.
12. $179.85. **13.** $448.92. **14.** $39.65. **15.** $79.45. **16.** $5.10.
17. $73.80. **18.** $165.66. **19.** $48.10. **20.** $13.41. **21.** $26.40.
22. $54.13. **23.** $7.33. **24.** $763.56. **25.** $2430.

CXXIII. PAGE 189.

1. $292.44. **2.** $187.28. **3.** $15.44. **4.** $145.05. **5.** $792.35.
6. $174.14. **7.** $4288.57. **8.** $3743.60. **9.** $9533.34.
10. $39499.52. **11.** $44351.01. **12.** $85523.13.

CXXIV. PAGE 191.

1. $624.32. **2.** $1092.56. **3.** $2522. **4.** $942.39. **5.** $478.83.
6. $468.34. **7.** $260.21. **8.** $258.89. **9.** $305.29. **10.** $306.83.
11. $782.36. **12.** $1164.36.

CXXV. PAGE 192.

1. $1711.50. **2.** $587.01. **3.** $1373.55. **4.** $111.08. **5.** $111.54.
6. $423.85. **7.** $5701.72. **8.** $916.76.

CXXVI. PAGE 193.

1. $464.62. **2.** $522.53. **3.** $904.67. **4.** $33763.09. **5.** $4322.89.
6. $19. **7.** $7247.63. **8.** $387.86. **9.** $553.68. **10.** $1719.49.
11. $91865.33. **12.** $2028.12. **13.** 15144·3475 frs. **14.** 82792·22 frs.
15. $5064.90. **16.** $8129.25. **17.** $10459.30. **18.** 503156·69(318) frs.

CXXVII. PAGES 194, 195.

1. 4. **2.** 4. **3.** 5. **4.** 7. **5.** 4½. **6.** 4½. **7.** 4. **8.** 5.
9. 3¼ yrs. **10.** 13. **11.** 3¾ yrs. **12.** 7½. **13.** $350.
14. $4100. **15.** $2300. **16.** $956. **17.** $3001. **18.** $21160.

CXXVIII. PAGES 196, 197.

1. $1846. **2.** 3¾ per cent. **3.** 7½ yrs. **4.** $26000. **5.** 8¾ yrs.
6. 10. **7.** $400. **8.** $1000. **9.** 3½. **10.** $17.96. **11.** May 29th.
12. $613.50. **13.** $22.77. **14.** $76321. **15.** $42.56. **19.** June 6th.
20. $1.10. **21.** 24 c. **22.** $3125. **23.** $8006.34. **24.** $8008.71.

CXXIX. PAGE 197.

1. $1. **2.** (i) $4.25; (ii) $2.97½; (iii) $19.12½; (iv) $6.51.
3. (i) 6*s.* 3*d.*; (ii) 3*s.* 9*d.*; (iii) 5*s.*; (iv) 10*s.* 5*d.* **4.** 20 per cent.

CXXX. PAGE 199.

1. $250. **2.** $3000. **3.** $4000. **4.** $1375. **5.** $1440. **6.** $1356.48.
7. $651.35. **8.** $309.79. **9.** $17.15. **10.** $264.55. **11.** $13.04.
12. $10.71.

CXXXI. PAGE 199.

1. $909.09; $90.91. **2.** $4166.67; $833.33. **3.** $7954.55; $795.45. **4.** $4481.96; $239.04. **5.** $7883.50; $236.50.
6. $13682.11; $592.89.

CXXXII. PAGES 200, 201.

1. $8.42. **2.** $15.89. **3.** $17.68. **4.** $2523.15. **5.** $266.70.
6. $120.13. **7.** $33150. **8.** $14579.06. **9.** $3308.89. **10.** $365000.
11. $3389.63. **12.** $3458.82. **13.** $3475.75. **14.** $983.03.
15. $856.69.

CXXXIII. PAGES 201, 202.

1. 15 c. **2.** $26775. **3.** $1.33. **4.** $17166.67. **5.** $70000.
6. $5197.63. **7.** $138.60. **8.** $15.70. **9.** $580.14. **10.** $6437.24.
11. $6395.77. **15.** $24000. **16.** $31.25.

CXXXIV. PAGE 204.

1. $42.98. **2.** $33.57. **3.** $1346.45. **4.** $3.56. **5.** $1486.52. **6.** $10.67.

CXXXV. Page 207.

1. 3175·20 fr. **2.** £1456. 8*s*. $2\frac{1}{2}$*d*. **3.** $716.$62\frac{1}{2}$. **4.** $221. **5.** £625. **6.** $403.10. **7.** 5547 florins. **8.** 651·42 fr. **9.** £200. **10.** $591.46. **11.** £166. 16*s*. 8*d*. **12.** 4173·16 florins.

CXXXVI. Page 209.

1. Yes. **2.** No. **3.** No. **4.** $40\frac{1}{2}$. **5.** 35. **6.** 2*s*. 6*d*. **7.** $42\frac{2}{3}$ men. **8.** $20\frac{5}{8}$ acres. **9.** 2 hrs. 48 min. **10.** 12 lbs. **11.** 21 days. **12.** 375 days. **13.** 5 days. **14.** 8 hrs. **15.** 5 hrs. **16.** $12\frac{1}{2}$ min.

CXXXVII. Pages 209, 210.

1. $2.25. **2.** 7112 oz. **3.** 9·425 ft. nearly. **4.** 1·91... ft. **5.** $67\frac{1}{11}$. **6.** $187.50. **7.** 1·8 in. **8.** $540.

CXXXVIII. Pages 211, 212, 213, 214.

1. 26 ; 65 ; 21. **2.** $13 ; $26 ; $39 ; $39 ; $52. **3.** $29.50 ; $59 ; $88.50 ; $118. **4.** $66 ; $77 ; $110. **5.** $876 ; $852. **6.** $192 ; $240 ; $288. **7.** 9 tons ; 6 tons ; 4 tons 10 cwt. **8.** $7.50 ; $12 ; $5. **9.** $320 ; $360 ; $384. **10.** 132 lbs. ; 28 lbs. ; 20 lbs. **11.** $30 ; $7.50 ; $3 ; $1.50 ; $1. **12.** $1760 ; $2112 ; $2464. **13.** 20 lbs. **14.** 270 lbs. ; 198 lbs. (nitre) ; 42 lbs. (charcoal). **15.** 720 lbs. ; 80 lbs. **16.** $28.95 ; $67.55. **17.** 7 cwt. 82 lbs. ; 19 cwt. 55 lbs. **18.** 13 of each coin. **19.** $8 ; $80 ; $84. **20.** $120 ; $60 ; $15. **21.** $135 ; $45 ; $15. **22.** $60 ; $105 ; $30. **23.** (Child's share) 42 tons 4 cwt. 32 lbs. ; (Woman's share) 84 tons 8 cwt. 64 lbs. ; (Man's share) 126 tons 12 cwt. 96 lbs. **24.** (Child's share) $658.95 ; (Woman's share) $1317.90 ; (Man's share) $1976.85. **25.** $260. **26.** $7288.41. **27.** $24.05 ; $48.10 ; $72.15 ; $120.25 ; $168.35. **28.** 40 c. ; 40 c. ; 80 c. ; $1.60 ; $3.20 ; $6.40. **29.** 1 ac. 4760 sq. yds. **30.** 8 yds. **31.** 2 ft. ; 2 ft. 3 in. **32.** $189 ; $243. **33.** $120. **34.** $8316.90. **35.** $1000. **36.** A, $1500 ; B, $2800 ; C, $2000. **37.** A, $600 ; B, $600 ; C, $450 ; D, $400. **38.** A, $16 ; B, $24 ; C, $40. **39.** A, $24 ; B, $25. **40.** D, $3042. **41.** $1527.21. **42.** $115. **43.** $2625. **44.** A, $142.50 ; B, $112.50 ; C, $90. **45.** $121\frac{5}{7}$ ft. **46.** $5700. **47.** 200. **48.** 176 3*d.'s* ; 48 half-sovs. ; 96 half-crowns. **49.** 50. **50.** A man, $1.25 ; a woman, 75 c. ; a child, 60 c. **51.** A man, $367.50 ; a woman, $122.50 ; a child, $73.50. **52.** $384. **53.** The diameters are $\frac{1}{3}\sqrt{3}$ ft. ; $\frac{1}{3}\sqrt{6}$ ft. **54.** 6 inches ; 8 inches. **55.** 4·2426 in. ; 5·6568 in.

CXXXIX. PAGE 215.

1. 3 bus. oats, 2 bus. peas. **2.** 3 lbs. at 56 c., 1 lb. at 88 c. **3.** 5 lbs. at 68 c., 2 lbs. at 82 c. **4.** 12 cwt. at $6.25, $12.50, $15.50, and 37 cwt. at $17.50. **5.** (i) 1 oats, 1 barley, 2 wheat; (ii) 1 oats, 4 barley, 4 wheat. **6.** 7 cwt. at $22, 7 cwt. at $27.50, 2 cwt. at $18.75. **7.** 3 parts of water to 10 of milk.

CXL. PAGES 217, 218, 219.

1. 20 per cent. **2.** $22\frac{22}{49}$ per cent. **3.** 20 per cent. **4.** $180. **5.** 25 per cent. **6.** $40. **7.** $132. **8.** $123. **9.** $300. **10.** $9.07½. **11.** $2.50. **12.** 86 c. **13.** 5 per cent. **14.** $11. **15.** Gain $18\frac{3}{4}$ per cent. **16.** $428. **17.** $7.35. **18.** 10 per cent. **19.** $9932.25. **20.** $26.67. **21.** $7.29. **22.** $5\frac{1}{68}$ per cent. **23.** $450. **24.** 740 marks. **25.** 115·923... per thousand. **26.** 175000. **27.** (i) 40,000; (ii) 1320; (iii) 800. **28.** Loss 80 per cent. **29.** 62½ per cent. **30.** $50. **31.** 25 per cent. **32.** 96 cents per 1000. **33.** $200. **34.** $6 per gal. **35.** (i) $14\frac{7}{12}$ per cent.; (ii) in equal parts. **36.** 50 per cent. **37.** 30 cents per lb. **38.** 78 cents per lb. **39.** $8000000. **40.** 1 lb. at 72 c., 2 lbs. at 84 c. **41.** 42 cents per lb. **42.** 36 cents per lb. **43.** 132½ per cent. **44.** 2 for 3 cents; 512. **45.** 15 for 21 cents, 40 dozen.

CXLI. PAGES 220, 221.

1. 81 yrs. **2.** 19500. **3.** 10 st. 10⅓ lbs. **4.** 50°. **5.** 3½ ft. **6.** 9 miles. **7.** 36 miles per hour. **8.** 9 st. 6½ lbs. **9.** 13 st. 2 lbs. **10.** $8.50. **11.** $6.53. **12.** $6.40. **13.** $14{\cdot}6\dot{3}$ yrs. **14.** 16·8 yrs. **15.** $17{\cdot}\dot{7}\dot{2}$, or, $17\frac{8}{11}$ yrs. **16.** $$9\frac{11}{12}$. **17.** 51°.

CXLII. PAGE 223.

1. $980; $30. **2.** $3360; $120. **3.** $1856.25; $68.75. **4.** $4698; $217.50. **5.** $5712; $190.40. **6.** $17028; $516.

CXLIII. PAGES 224, 225.

1. $50000; $2500. **2.** $70000. **3.** $243790.31. **4.** $910; $22181.25. **5.** $650; $20000. **6.** $750. **7.** $119700. **8.** $14000. **9.** $20000 stock; $22800. **10.** $43800.

CXLIV. PAGES 226, 227, 228.

1. \$1099.25. **2.** \$75. **3.** \$114.75. **4.** \$525. **5.** \$28.94.
6. 4·32 per cent. **7.** 3·0769... per cent. **8.** 8·536... per cent.
9. 4·762... per cent. **10.** 3·669... per cent. **11.** 3·286... per cent.
12. 4·777... per cent. **13.** 4 per cents. **14.** 4·4 per cent stock.
15. 5 per cents. **16.** 6 per cents. **17.** Great Northern.
18. $3\frac{1}{2}$ per cents. **19.** \$150. **20.** \$22200 ; \$14985. **21.** \$80.
22. \$23. **23.** $77\frac{4}{5}$. **24.** \$7.19. **25.** \$2136. **26.** 190 per cent.
27. \$65275. **28.** \$1800. **29.** 2 years. **30.** \$3944 in 3 per cents., \$156 in 5 per cents. **31.** \$480 in 3 per cents., \$2520 in 6 per cents.
32. \$15000. **33.** \$1380. **34.** \$108. **35.** 117. **36.** 50 per cent.
37. $104\frac{6}{13}$. **38.** \$37800. **39.** $66\frac{1}{3}$. **40.** $2\frac{68}{91}$ %. **41.** \$22960.
42. A ; \$15854. **43.** \$6000 stock, $114\frac{1}{2}$.

CXLV. PAGE 230.

1. 9. **2.** 81. **3.** 243. **4.** 128. **5.** 625. **6.** 343. **7.** 512.
8. 441. **9.** 1225. **10.** 9261.

CXLVII. PAGE 233.

1. 27. **2.** 32. **3.** 42. **4.** $10\frac{1}{5}$. **5.** $7\frac{10}{11}$. **6.** 63. **7.** 93. **8.** 69.
9. 153. **10.** 366. **11.** 339. **12.** 307. **13.** 619. **14.** 804.
15. 919. **16.** 971. **17.** 3579. **18.** 4607. **19.** 1679. **20.** 9070.
21. 568·04. **22.** 80·047. **23.** 7080·69. **24.** 2·236. **25.** 3·162.
26. 2·646. **27.** ·3162. **28.** ·5774. **29.** ·8165. **30.** ·5345.
31. 1·773. **32.** ·3944. **33.** ·6129. **34.** 1·772. **35.** ·5877.
36. 2·003. **37.** 31·62. **38.** 5·774.

CXLVIII. PAGE 235.

2. 3·1622. **3.** 1·5537. **4.** ·409684. **5.** 14·142. **6.** ·65465.
7. ·74535. **8.** 4·4721. **9.** 69·5701. **10.** 1·7724. **11.** 1·6487.
12. ·65901. **13.** 1·6818. **14.** 1·196836.

CXLIX. PAGE 235.

1. $1\frac{1}{4}$ miles. **2.** 311·1 yds. **3.** 852 yds. **4.** 1470·9 ft. nearly.
5. 141·42. **6.** 39·25... $=39\frac{1}{4}$ yds. (very nearly). **7.** 6 ft. 8·1 in.
8. 5·01... ft.

CL. PAGE 238.

1. 12. **2.** 31. **3.** 44. **4.** 81. **5.** 29⅓. **6.** 43½. **7.** 171. **8.** 174.
9. 337. **10.** 317. **11.** 247. **12.** 305. **13.** 3·57. **14.** ·824.
15. 70·7. **16.** 11·81. **17.** 280·1. **18.** 1077·2. **19.** 53·29. **20.** 13·05.
21. $3\cdot\dot{6}$. **22.** ·05609. **23.** 1·259. **25.** 1·259... **26.** ·584...
27. ·271... **28.** 2·154... **29.** 4·641... **30.** 9 ft. 6 inches. **31.** 15¾ in.
32. 27 in. (edge), 27$\sqrt{3}$ in. (diagonal). **33.** 36 $\sqrt[3]{10}$ in. $= 36 \times 2\cdot154$...in.
34. 10 $\sqrt[3]{2}$ in. $= 10 \times 1\cdot259$...

CLI. PAGES 240, 241.

	7	8	2	3	twelve
1.	2626,	1750,	1111101000,	1101001,	6**e**4.
2.	1221,	710,	111001000,	121220,	320.
3.	1660,	1247,	1010100111,	221011,	487.
4.	2020,	1274,	1010111100,	221221,	4**t**4.
5.	2205,	1425,	1100010101,	1002020,	559.

	ten	5	2	twelve
6.	100000,	11200000,	11000011010100000,	49**t**54.
7.	40000,	2240000,	1001110001000000,	1**e**194.
8.	210,	1320,	11010010,	156.
9.	172800,	21012200,	101010001100000000,	84000.
10.	1441728,	332113403,	101011111111111000000,	596400.

CLII. PAGE 243.

	ten	7	twelve	3
1.	$\cdot\dot{3}$,	$\cdot\dot{2}$,	·4,	·1.
2.	·2,	$\cdot\dot{1}25\dot{4}$,	$\cdot\dot{2}49\dot{7}$,	$\cdot\dot{0}12\dot{1}$.
3.	$\cdot8\dot{3}$,	$\cdot\dot{5}$,	·**t**,	$\cdot2\dot{1}$,
4.	$\cdot\dot{4}2857\dot{1}$,	·3.	$\cdot\dot{5}186\mathbf{t}\dot{3}$,	$\cdot\dot{1}0212\dot{0}$.
5.	$\cdot30\dot{5}$,	$\cdot\dot{2}0654\dot{3}$,	·38,	$\cdot02\dot{2}\dot{0}$.

	6	8
6.	·4,	$\cdot\dot{5}\dot{2}$,
7.	$\cdot\dot{4}$,	$\cdot\dot{6}31\dot{4}$,
8.	·444043,	·63,
9.	·55,	$\cdot7\dot{6}\dot{1}$,
10.	242103...	·3505642...

	eleven	2
6.	$\cdot\dot{7}\dot{3}$,	$\cdot1\dot{0}$.
7.	$\cdot\dot{8}$,	$\cdot\dot{1}10\dot{0}$.
8.	·884...	·110011.
9.	·**t**77...	$\cdot11\dot{1}1100\dot{0}$.
10.	·5,	$\cdot\dot{0}11101000\dot{1}$.

	7	9	twelve
11.	$\cdot\dot{1}\dot{5}$,	$\cdot\dot{2}$,	·3
12.	$5\cdot\dot{2}\dot{4}$,	$5\cdot\dot{3}$,	5·46.
13.	2·060...	2·108...	2·158...
14.	4·041...	4·070...	4·106...
15.	$\cdot\dot{2}$,	·3,	·4.

	ten	2	7
16.	$\cdot\dot{7}0\dot{3}$ or $\frac{19}{27}$,	$\cdot\dot{1}0110100001001011\dot{1}$,	$\cdot\dot{4}6324101\dot{5}$.
17.	·928 or $\frac{116}{125}$,	·111011011001000 etc.,	·633206...
18.	·703125 or $\frac{45}{64}$,	·101101,	$\cdot\dot{4}631126\dot{4}$.
19.	·1484375 or $\frac{19}{128}$,	·0010011,	$\cdot\dot{1}01625344352620\dot{1}$.

22. $197\frac{1223}{2352}$. **23.** $377\frac{35}{448}$. **24.** $4\frac{103773}{248688}$.

The scales mentioned are those in which the fractions can be expressed in a Terminating form; in the remaining scales they recur.

27. 3, 6, 9, twelve. **28.** 5, ten. **29.** 6, twelve. **30.** 6, twelve.
31. 6, twelve. **32.** none. **33.** none. **34.** 2, 4, 6, 8, ten, twelve.
35. 3, 6, 9, twelve. **36.** none.

EXAMPLES FOR EXERCISE. CLIII.

1. One billion three hundred thousand five hundred. **2.** $4379.43.
3. $2\frac{1}{2}$ cents. **4.** 720. **5.** 1483 yds. 1 ft. 10 in. **6.** $3\frac{13}{30}$. **7.** $\frac{593}{880}$.
8. ·031875. **9.** 3 tons $18\frac{3}{4}$ lbs. **10.** $81\frac{7}{9}$ sq. yds.

11. $\frac{25}{1673}$. **12.** $22\frac{85}{96}$. **13.** $\frac{7}{8}$. **14.** $14.40. **15.** $\frac{3}{16}$. **16.** ·14; $\frac{7}{25000}$.
17. 16. **18.** ·0028125. **19.** 13 tons 3 cwt. 75 lbs. **20.** $566.18.

21. 3,050,372,424. **22.** 12 m. 5 fur. 2 yds. 8 in. **23.** 3 cwt. 37 lbs. 8 oz.
24. $\frac{5}{7}$; $\frac{9}{16}$. **25.** $21\frac{2}{3}$. **26.** $\frac{5}{23}$. **27.** 8 days 21 hrs. 12 min. 30 secs.
28. 153·41134. **29.** 3 cwt. 50 lbs. **30.** 18 days.

31. 181749. **32.** 31264. **33.** 9240. **34.** 12; $\frac{19}{24}$. **35.** $6\frac{7}{8}$. **36.** $\frac{223}{462}$.
37. Five hundred; five tenths; five ten-thousandths. **38.** 1·59964; ·00236. **39.** ·511. **40.** $860.

41. 1607266. **42.** 107 sq. yds. 6 sq. ft. **43.** $\frac{13}{22}$. **44.** $58\frac{19}{143}$. **45.** $\frac{1}{6}$.
46. 2 hrs. $4\frac{97}{147}$ min. **47.** 53·875; ·08. **48.** ·56875. **49.** $3\frac{1}{2}$ days.
50. 4 min. 47 secs.

51. 1 ton 15 cwt. $71\frac{3}{7}$ lbs. **52.** 924. **53.** $41\frac{3}{4}$. **54.** $1\frac{13}{16}$.
55. 1108 yds. 2 ft. $4\frac{3}{7}$ in. **56.** $1.54. **57.** $\frac{9}{125}$; 1·096.
58. 59 days 13 hrs. 30 min. **59.** 12 acres. **60.** 4200 half-crowns.

61. 245800 oz. **62.** 5720. **63.** 2202 sq. yds. 3 sq. ft. 18 sq. in.
64. $\frac{5}{7}$; $\frac{13}{17}$. **65.** $9\frac{13}{16}$. **66.** $1\frac{7}{10}$. **67.** $6.87. **68.** 1024·38; 73730.
69. ·088125. **70.** $153.12.

71. $8565.78. **72.** 370. **73.** 5400. **74.** $\frac{1}{2}$. **75.** $1\frac{22}{35}$.
76. 20 tons 13 cwt. $33\frac{1}{3}$ lbs. **77.** 42 miles. **78.** ·0875 ; $\frac{349}{400}$.
79. 5 wks. 1 day 11 hrs. **80.** 19*s.* 6*d.*

81. 12451. **82.** $1812.78. **83.** 720. **84.** 1432 tons 2 cwt. 80 lbs.
85. $3\frac{13}{30}$. **86.** $\frac{503}{560}$. **87.** ·00016. **88.** ·16. **89.** ·5225. **90.** $394.

91. 4896 yds. 2 ft. 5 in. **92.** 113 sq. yds. 7 sq. ft. **93.** $\frac{1234}{4321}$.
94. $12\frac{43}{96}$. **95.** $2\frac{23}{60}$. **96.** $405.50. **97.** $\frac{15}{44}$. **98.** 1685·4412.
99. 74 tons 9 cwt. $6\frac{1}{4}$ lbs. **100.** $383.25.

101. Two billions seventy-one millions three thousand six hundred and one. **102.** 37 yds. 2 ft. 11 in.
103. (i) 34 sq. yds. 6 sq. ft. 72 sq. in.; (ii) 25 ft. **104.** $\frac{5}{8}$.
105. $\frac{13}{30}$, $\frac{3}{7}$, $\frac{2}{5}$, $\frac{27}{70}$. **106.** $2\frac{3}{5}$. **107.** $59.45. **108.** $\frac{18}{625}$; ·0808.
109. £17. 6*s.* $10\frac{1}{2}$*d.* **110.** (i) 1000 times ; (ii) 1,000,000 times ; (iii) one denotes that there are no hundreds, the other no tenths.

111. 183261 grs. ; 6 cwt. 28 lbs. 9 oz. **112.** $18.88. **113.** 120.
114. 5 yds. 2 ft. $6\frac{36}{49}$ in. **115.** $22\frac{41}{45}$. **116.** $\frac{251}{1600}$. **117.** $10500.
118. 488·75 ; ·025. **119.** ·14375. **120.** One receives $37.50 ; each of the others $12.50.

121. 83448. **122.** 506 tons 15 cwt. 31 lbs. 4 oz. **123.** $\frac{57}{83}$.
124. $2\frac{14}{15}$. **125.** $\frac{478}{795}$. **126.** $\frac{1612}{6597}$. **127.** 4·25.
128. ·7644 ; 2·5. **129.** ·8. **130.** 45 min.

131. 2 cwt. 4 lbs. **132.** 40 sq. yds. 4 sq. ft. **133.** 1008.
134. 5 tons 10 cwt. 36 lbs. $14\frac{10}{11}$ oz. **135.** $9\frac{8}{15}$. **136.** $7.31.
137. $15.12. **138.** ·625 ; $4\frac{8}{625}$. **139.** 273 yds. 2 ft. 3·75 in.
140. 20 yds.

141. One hundred and fourteen thousand nine hundred and ninety-six.
142. 1111 wks. 4 days 11 hrs. **143.** 97 francs. **144.** $\frac{2}{5}$. **145.** $1\frac{79}{91}$.
146. 1 ton 12 cwt. $1\frac{3}{7}$ lbs. **147.** 2 remainder ; 38 divisor.
148. 30·7622. **149.** 38·25. **150.** 44 tons 2 cwt. 75 lbs.

151. 60 tons 5 cwt. 73 lbs. **152.** 58 sq. yds. 8 sq. ft. **153.** 840.
154. $8\frac{2}{55}$. **155.** $15\frac{55}{63}$. **156.** $\frac{19}{320}$. **157.** $\frac{29}{1600}$. **158.** 72·7272.
159. ·0125. **160.** $26.26.

161. 1386 sq. yds. 8 sq. ft. 96 sq. in. **162.** $\frac{9}{11}$. **163.** $\frac{53}{122}$.
164. 15·836271. **165.** $124.6875. **166.** $325.60 (+ a fraction of a cent).
167. $2117.49. **168.** 113 ; 339 ; 678 ; 791.

169. 651 tons 1 cwt. 20 lbs. 13$\frac{1}{3}$ oz. **170.** 18$\frac{25}{28}$.
171. 39 yds. 2 ft. 0$\frac{1}{7}$ in. **172.** 491·401. **173.** $28800.
174. $1726.67. **175.** $66.72. **176.** $423.39.

177. 1772 tons 10 cwt. 87 lbs. 2 oz. **178.** 5$\frac{31}{72}$. **179.** $\frac{2}{3}$.
180. ·05403 ; ·$0\dot{5}4\dot{5}$. **181.** $21.75. **182.** $72. **183.** $567.77.
184. 42 sq. yds. 2$\frac{1}{4}$ sq. ft.

185. 22$\frac{1}{2}$. **186.** $\frac{21}{25}$. **187.** ·1565955 ; ·000000000862.
188. 23·9342229. **189.** 6 cwt. 18 lbs. 12 oz. **190.** $797.
191. 130 lbs. **192.** $40000.

193. 2 a. 133 po. 14 sq. yds. 108 sq. in. **194.** 16$\frac{15}{17}$.
195. ·$822\dot{6}1904\dot{7}$. **196.** ·3679344 ; 67·36. **197.** 90 lbs.
198. 18 days. **199.** $3.75. **200.** $58406.25.

201. 7$\frac{49}{60}$. **202.** 5 days 20 hrs. **203.** ·$\dot{1}4285\dot{7}$.
204. 10 cwt. 93 lbs. 12 oz. **205.** $545710. **206.** $1658.88 ; 36 days.
207. $936.48. **208.** A, 375 ; B, 150 ; C, 225.

209. 37 sq. yds. 4$\frac{1}{2}$ sq. ft. **210.** 12$\frac{21}{80}$. **211.** 1$\frac{13}{75}$. **212.** 15$\frac{3}{8}$; 2$\frac{28}{75}$.
213. 1181·5625 ; 5$\frac{1}{2}$. **214.** 170 yds. **215.** $347.73.
216. A, $1100 ; B, $220 ; C, $27.50.

217. 35 cwt. 49 lbs. 5 oz. **218.** 2 tons 18 cwt. 81$\frac{19}{21}$ lbs. **219.** 3$\frac{1}{5}$.
220. ·62512 ; ·002. **221.** 25 sq. ft. 27 sq. in. **222.** 162.
223. 64,050,000 miles. **224.** $3820.32.

225. 7$\frac{91}{225}$. **226.** 8$\frac{2}{3}$. **227.** ·$1\dot{3}$; $\frac{9}{25}$; 3$\frac{7}{11}$. **228.** $\frac{1}{16}$ of $1 (= 6 cents +);
·03125. **229.** $257.61. **230.** 6 hours. **231.** $1508.25.
232. $306.60 ; $383.25 ; $587.65.

233. $\frac{3}{4}$. **234.** $\frac{4}{7}$. **235** 4·625 ; 11·$0\dot{6}$. **236.** ·06 ; 48. **237.** ·311875.
238. $32.19. **239.** $975.47 ; $7218.47. **240.** 66 sq. yds. 1 sq. ft.

241. $\frac{1}{2}$. **242.** $5.83. **243.** 1$\frac{7}{40}$; 4$\frac{1}{6}$. **244.** $19.66.
245. 11 tons 17 cwt. 79$\frac{1}{3}$ lb. **246.** 67$\frac{1}{2}$ acres. **247.** $1324.05.
248. $2.14.

249. 1,456,975 *f*. **250.** 1. **251.** $1{\cdot}\dot{5}7142\dot{8}$. **252.** $20\frac{6411}{22500}$. **253.** $10.
254. 275 acres 2 r. **255.** $10\frac{2}{3}$ miles. **256.** *A*, $210; *B*, $200; *C*, $190.

257. $12\frac{4}{7}$. **258.** $\frac{121}{1440}$. **259.** ·000145; $10556{\cdot}467\dot{2}8571\dot{4}$.
260. 1 ton 3 cwt. 42 lbs. 3 oz.; ·117109375. **261.** $807.95.
262. 1960.87\frac{1}{2}$. **263.** 2984.62\frac{1}{2}$. **264.** $465.52.

265. 171,675 guas. **266.** $14\frac{17}{60}$. **267.** 2 yds. 1 ft. $7\frac{16}{35}$ in.
268. ·115625. **269.** 33·6. **270.** $181.56. **271.** $12,000. **272.** 19 acres.

273. 4 hours 33 m. $9\frac{1}{2}$ secs. **274.** $\frac{3}{8}$. **275.** ·0645. **276.** $143.82.
277. $\cdot 0347\dot{2}$. **278.** $6555.36. **279.** 80 men. **280.** $2400.

281. $22\frac{157}{240}$. **282.** 266 days 11 hrs. 3 m. $42\frac{2}{9}$ sec.
283. 45 days 9 hrs. $18\frac{6}{7}$ min. **284.** 31·25. **285.** $236.08.
286. $451.05. **287.** 285 days. **288.** $3840.

289. 18 times; 1 a. 74 p. **290.** $1\frac{1}{8}$. **291.** ·008; 12·365.
292. $\cdot 8\dot{3}$. **293.** $25{\cdot}69\dot{4}$. **294.** $130 × 26 × 26, or $87880.
295. 172239 c. yds. 26 c. ft. 288 c. in. **296.** 417 miles.

297. 143; 85800. **298.** 6 c. yds. 500 c. in. **299.** $9.55.
300. 2·114; 30·16. **301.** 2*s*. 6·8*d*. **302.** $878.90.
303. $35.10. **304.** 17 in.

305. 23 m. 220 yds. 7·2 in. **306.** $19\frac{4}{9}$. **307.** $\frac{2}{7}$. **308.** $\frac{9}{40}$.
309. $26{\cdot}2\dot{7}$; $\frac{1}{120}$. **310.** 45 yds. **311.** $487.75. **312.** $26\frac{2}{3}$ days.

313. 131670. **314.** 311 yds. $1\frac{1}{2}$ in. **315.** $7\frac{31}{80}$. **316.** 52·6875; $\cdot\dot{6}$.
317. ·0375832; ·016. **318.** $640.64. **319.** $648.90 (Int.);
$630 (Disc.). **320.** *A*, $656; *B*, $287.

321. 4464 tons 5 cwt. 2 qrs. 24 lbs. **322.** $1\frac{7}{36}$. **323.** $1\frac{13}{42}$.
324. $\frac{13}{8000}$; $\frac{54}{55}$. **325.** 19 m. 1004 yds. 1 ft. 4 in. **326.** 10,821.
327. $20\frac{1}{2}$ yds. by 41 yds. **328.** $30.

329. $84\frac{3}{7}$ lbs. **330.** 4125 cu. ft. **331.** $4\frac{13}{28}$. **332.** $1\frac{19}{252}$.
333. 2·631; ·23870476; ·00003125. **334.** $2595.94. **335.** $27\frac{1}{2}$ days.
336. $24048; $20040; $15030.

337. 3; 375921. **338.** $\frac{5}{12}$. **339.** $\frac{69}{200000}$; $2\frac{71}{900}$. **340.** $687.96.
341. $2854.20. **342.** $17230.25. **343.** $22\frac{1}{2}$ %. **344.** $\cdot 005\dot{4}$ sq. in.

345. 6600 yds. **346.** 19511 yds. $2\frac{1}{4}$ in. **347.** $6\frac{577}{1800}$. **348.** $\frac{7}{20}$.
349. $54\cdot0000\dot{6}5\dot{8}$. **350.** $\frac{16573}{505050}$. **351.** $58\frac{1}{3}$ yds. **352.** \$$233\frac{1}{3}$ (\$233.33).

353. 1 yd. 1·875 in. **354.** 18 cwt. $77\frac{43}{45}$ lbs. **355.** ·048. **356.** $6\frac{2}{3}$.
357. \$104.75. **358.** \$5581.95. **359.** \$$2453.89\frac{1}{2}$. **360.** 15 sovs. ; 45 half-crowns ; 105 shillings.

361. 60. **362.** \$1004.06. **363.** \$18.45. **364.** $9\cdot99\dot{1}1884\dot{6}$.
365. 66 m. **366.** \$315858.75. **367.** \$5.06.

368. $\frac{3}{4}$. **369.** 105 wks. 16 hrs. $14\frac{46}{101}$ min. **370.** $\cdot\dot{1}$.
371. 567 tons 3 cwt. 71 lbs. 4 oz. **372.** ·504. **373.** $10\frac{1}{2}$ cu. ft.; 84 c.
374. 32 days.

375. 31. **376.** \$9248.51. **377.** ·1918 ; ·0016. **378.** \$1193.85.
379. 25 c. ft. 1440 c. in. **380.** 5 °/o. **381.** \$29.

382. $1094\frac{804}{1019}$. **383.** 553 yds. 1 ft.
384. ·53191[4893617021276595744...] **385.** $6\frac{2}{3}$ days. **386.** \$22.50.
387. \$165. **388.** 4 °/o. **389.** 5 m. 30 secs.

390. $\frac{13}{20}$. **391.** ·000648. **392.** 900 yds. **393.** \$3220.
394. \$134744.40. **395.** \$13.86. **396.** $3\frac{1}{2}$ °/o at 89.

397. $\frac{101}{144}$. **398.** $2\cdot0\dot{8}\dot{2}$. **399.** \$1296.59. **400.** 66 ; \$75.90.
401. *A*, \$4000 ; *B*, \$1200 ; *C*, \$1800. **402.** \$7.89.
403. 128 tons 5 cwt.

404. 47 yds. 1 ft. $2\frac{4}{7}$ in. **405.** $\frac{141}{15625}$; $\frac{79}{165}$. **406.** 6 hours.
407. 70 sq. yds. **408.** \$35. **409.** A man, \$10 ; a woman, \$6 ; a child, \$4. **410.** \$15,000.

411. $20\frac{14}{45}$. **412.** ·009928 ; $\cdot\dot{5}2\dot{5}$. **413.** \$10.32. **414.** 4 years.
415. 60 °/o gain. **416.** \$4000. **417.** 2024 sq. ft. ; $\cdot0\dot{4}\dot{6}$.

418. $11\frac{34}{35}$. **419.** ·108125. **420.** 21 hours. **421.** 107 sq. yds. 3 sq. ft.
422. \$164.85. **423.** $4\frac{1}{2}$ years. **424.** \$1062.50. **425** 9680 c. yds.

426. 20·48. **427.** 5·565. **428.** \$7.014. **429.** \$237.91.
430. \$29.37 ; \$58.74 ; \$88.11. **431.** $37\frac{1}{2}$ cents. **432.** 25 minutes.

433. 0. **434.** 17 yds. 1 ft. $10\frac{3}{4}$ in. **435.** $10\frac{1}{11}$ miles.
436. 24 ft. 10 in. **437.** *A*, $63 ; *B*, $45 ; *C*, $54. **438.** $29.50.
439. $9571.25. **440.** $\frac{1}{2}$ hour.

441. ·00063865. **442.** $\cdot 2\dot{1}\dot{8}$. **443.** $37\frac{1}{2}$ days. **444.** 25.37\frac{1}{2}$.
445. 7 miles. **446.** $3.75. **447.** 3 days.

448. $15\frac{1}{10}$. **449.** $62\frac{1}{2}$ cents. **450.** 89.37\frac{1}{2}$. **451.** $28\frac{14}{27}$ yds.
452. $8\frac{16}{21}$. **453.** $2388. **454.** $1824.35. **455.** 4500 ducats.

456. ·00000019375. **457.** $\cdot\dot{0}1234567\dot{9}$. **458.** $500. **459.** $16.13.
460. $933.20. **461.** $300. **462.** $183.75. **463.** $38\frac{2}{11}$.

464. 3 lbs. 7 oz. 10 dwts. 15 grs. **465.** ·00390625. **466.** 5·75 versts.
467. $14\frac{1}{4}$ yds. **468.** 171 yds.; 150 yds.; 237 yds.; 102 yds.
469. $184.55. **470.** $78.75. **471.** $7\frac{1}{7}$ %.

472. 1600. **473.** $\cdot 09\dot{5}6\dot{8}$. **474.** 1609.62\frac{1}{2}$. **475.** $3746.25.
476. 2940. **477.** $9\frac{11}{21}$ %. **478.** 14 minutes.

479. $\frac{83}{336}$. **480.** $7\cdot 53\dot{3}\dot{7}$. **481.** $1813.43. **482.** $3322.01.
483. 7 hours. **484.** 16 %. **485.** $4500 stock.

486. 34 wks. 1 day 19 hrs. 45 min. **487.** $17\frac{1}{2}$ miles.
488. $2\frac{19}{108}$. **489.** $94.50 ; $117.60 ; $140.70. **490.** 7 months.
491. 77\frac{3}{4}$. **492.** $353.85.

493. 1. **494.** 1 yd. 1 ft. **495.** $56. **496.** $330 ; $220 ; $165 ;
$132. **497.** $26.25. **498.** $9\frac{1}{11}$ %. **499.** 12 days.

500. 655·36. **501.** 774. **502.** 20 ft. **503.** $144 ; $198 ; $216.
504. $70. **505.** $4150 stock. **506.** $127\frac{1}{2}$.

507. 5 wks. 3 hrs. $20\frac{8}{11}$ min. **508.** 60·5. **509.** $114.75. **510.** $68.25.
511. $30331.20. **512.** $15. **513.** $87.50 (men) ; $82.50 (women) ;
$35 (children).

514. ·0372. **515.** $549675.21. **516.** 135 cu. yds.
517. 100 yds. 1 ft. 3 in.; 66 yds. 2 ft. 10 in.; 53 yds. 1 ft. 8 in.
518. $150. **519.** 1232 yds. **520.** $29000.

521. 2 wks. 6 days $20\frac{1}{2}$ hrs. **522.** 143 yds. 2 ft. 9 in. **523.** $1\frac{99}{259}$.
524. $11.45. **525.** $10\frac{1}{2}$ hours. **526.** 14 years. **527.** $1\frac{7}{18}$ per cent.

528. 4 cwt. 29 lbs. **529.** $77\frac{3}{8}$. **530.** ·010625, 2, $\cdot 019\dot{4}$.
531. $32276\frac{3}{4}$ yds. **532.** \$194.88. **533.** \$252.75.
534. *C* runs 7225 yds. to *A*'s 7216 yds.

535. 10 lbs. 4 oz. 11 dwts. $7\frac{13}{16}$ grs. **536.** $1\frac{17}{22}$. **537.** 1 ton 12 cwt. 50 lbs.
538. $8\frac{1}{8}$ days. **539.** \$200. **540.** \$35.20. **541.** 12.30 P.M.

542. 99. **543.** ·4265625. **544.** 32·580612, $\cdot 000\dot{0}\dot{2}$, ·01356.
545. 48 oz. **546.** 5 per cent. **547.** 9 miles 110 yds.

548. $\frac{36}{55}$. **549.** $\frac{186}{307}$. **550.** 2 tons 9 cwt. 50 lbs.; 5 cwt.; 2 cwt.
551. 14. **552.** \$942.75. **553.** *A*, \$28; *B*, \$32; *C*, \$40.
554. $4\frac{16}{121}$ sq. inches.

555. $11\frac{17}{21}$. **556.** $\frac{23}{320}$. **557.** $\cdot 41\dot{6}$, $\cdot\dot{3}8461\dot{5}$, $\frac{13}{800}$, $\frac{6803}{19800}$.
558. 34 yds. $9\frac{1}{2}$ inches. **559.** \$2960. **560.** \$1950. **561.** 21 c.

562. $\frac{3}{8}$. **563.** $11\frac{74}{99}$. **564.** $\cdot 2\dot{9}\dot{8}$, $\cdot 3\dot{4}6\dot{9}$. **565.** 798 sq. ft.
566. \$36.15. **567.** 9 days. **568.** \$10 (man), \$5 (woman), \$3 (child).

569. 1051 tons 4 cwt. **571.** 7 tons 11 cwt. 81 lbs. 4 oz.; $\cdot 351736\dot{1}$.
572. \$10. **573.** $\$9.82\frac{1}{2}$. **574.** 80 c.

575. ·718, ·00002. **576.** \$81.54. **577.** *A*, \$10; *B*, \$18; *C*, \$19.80.
578. *A*, \$120; *B*, \$60; *C*, \$720. **579.** \$1084.85. **580.** At 3.30 P.M.; 50 miles from London.

581. $\$2620.22\frac{1}{2}$. **582.** 660 men. **583.** 48 larger, 8 smaller.
584. \$21.05. **585.** \$850. **586.** $12\frac{4}{5}$ c.

587. 1. **588.** 4·25, ·9. **589.** \$8.40. **590.** $\frac{1}{2}$ cent. **591.** 1296 c. in.; \$1.38. **592.** $5\frac{2}{3}$ yrs. **593.** *A*, \$4.20; *B*, \$7.70; *C*, \$7.20.

594. ·09975. **595.** 366 yds. 1 ft. $2\frac{5}{7}$ in. **596.** 7. **597.** 144 men.
598. \$27.50. **599.** \$3500. **600.** \$7000.

601. $3\frac{5}{24}$. **602.** ·071875, $13\cdot\dot{9}13043$ etc. (22 places). **603.** $\cdot 7\dot{7}\dot{0}$.
604. \$11.95. **605.** \$343.83; \$19.83. **606.** \$10500.

607. $\frac{23}{320}$. **608.** ·2776, 1388. **609.** ·393375; \$232.68.
610. \$3159. **611.** \$235.54. **612.** \$12.

613. 2 fur. 14 po. 3 yds. 2 ft. 6 in. **614.** $\frac{4}{17}$. **615.** $11\frac{1}{4}$ days.
616. 32 miles. **617.** $8\frac{1}{3}$ per cent. **618.** \$220. **619.** \$25.34.

620. $11\frac{7}{16}$. **621.** \$7.28 per ton. **622.** $\cdot 00759\ldots$; $\cdot 171875$.
623. 5383 tons 17 cwt. 50 lbs. **624.** \$936.48 ; \$1105.21.
625. \$62.50. **626.** \$98.80.

627. 1. **628.** 95 yds. $3\frac{1}{5}$ in. **629.** \$31.50. **630.** \$3329.75.
631. \$1685 (man) ; \$421.25 (woman) ; \$84.25 (boy). **632.** \$2710.
633. 9*s*.

634. $\cdot 0428125$, $8{\cdot}\dot{3}\dot{9}$. **635.** \$4.69½. **636.** $3\frac{5}{8}$. **637.** ½ day.
638. $14\frac{7}{12}$ per cent. **639.** $1\frac{1}{3}$ per cent. **640.** 1 hr. 42 min. after my friend had started ; $5\frac{19}{20}$ miles from starting place.

641. $23\frac{14}{25}$. **642.** 75 po. 1 yd. 2 ft. 6 in. **643.** $17{\cdot}7108\dot{2}$.
644. 15 days. **645.** $5\frac{2}{3}$ years. **646.** 5 per cent. **647.** 2 days.

648. 113 yds. 2½ in. **649.** $\cdot 016$, $2{\cdot}234375$. **650.** \$11.37½. **651.** \$300.
652. \$632.50. **653.** Gain \$$66\frac{2}{3}$. **654.** 39 yards.

655. $2\frac{2}{5}$. **656.** $\frac{5}{16}$. **657.** 126 yds. $0\frac{3}{4}$ in. **658.** $263{\cdot}\dot{6}\dot{3}$.
659. 1310 tiles. **660.** \$236.25. **661.** 125 per cent.

662. $\frac{1}{39}$. **663.** $14{\cdot}603\dot{0}86450\dot{0}$. **664.** $6{\cdot}09743576$, $\cdot 0187$.
665. 87 miles nearly, 1,188,374 m. nearly. **666.** 7½ days.
667. \$8050. **668.** \$27.

669. $2\frac{2}{39}$. **670.** \$2.50. **671.** $5\frac{1}{5}$, $300{\cdot}03$. **672.** 3 years.
673. \$4088. **674.** 4½ miles per hour. **675.** 216 c. ft.

676. 10 ac. 151 po. **677.** \$56.25. **678.** \$429.82. **679.** \$1320.
680. $4\frac{4}{5}$ miles per hour. **681.** 343 c. ft. **682.** \$960.24.

683. $1{\cdot}43648$. **684.** $2\frac{1777}{3700}$. **685.** $13{\cdot}2093$ gallons. **686.** \$528.
687. $\cdot\dot{3}0864197\dot{5}$, $\cdot 7\dot{6}$. **688.** \$56.03. **689.** $35\frac{13}{22}$ miles per hour.

690. 1. **691.** 15. **692.** \$881.79½. **693.** $2{\cdot}41$ etc. **694.** \$504.
695. 81 c. yds. **696.** \$374. **697.** It loses $\frac{1440}{1441}$ min. per day.

698. $2\frac{97}{189}$. **699.** 95 lbs. **700.** 17. **701.** A fourth. **702.** $3\frac{3}{4}$ per cent.
703. *A*, \$6 ; *B*, \$5.25 ; *C*, \$2.75. **704.** The latter. **705.** *A*, $12\frac{3}{8}$ sec. ; *B*, 13½ sec. ; *C*, 15 sec.

706. 1. **707.** $1813{\cdot}67$. **708.** \$450. **709.** $3\frac{3}{4}$ per cent.
710. 21 c. **711.** $1{\cdot}965$ etc.

215. There is always some remainder in the process of finding $\sqrt{2}$; hence it is not possible to find a decimal number whose square is equal to 2 exactly.

A number which cannot be exactly expressed as a decimal number is said to be **incommensurable** with decimal numbers; and is called *an incommensurable number.*

Two quantities, whose ratio is an incommensurable number, are said to be incommensurable.

The length of the diagonal and of a side of a square are incommensurable.

Cube Root.

216. The cube of a number contains either thrice as many digits, or one less or two less than thrice as many digits as the number.

For the cube of a number less than 1000 and not less than 100 must be less than 1000000000 and not less than 1000000; so that it must contain either 9 or 8 or 7 digits.

217. To find the cube root of a given cube number we proceed as follows:—

I. When the cube number is less than 1000 its cube root is given by the Table on page 231.

II. When the cube number is greater than 1000, we first consider the **form** of the cube of a number expressed by two digits.

Example i.

$$\begin{aligned}(76)^3 &= (70+6) \text{ times } (70+6)^2\\ &= (70+6) \text{ times } \{(70)^2 + \text{twice } 70 \times 6 + 6^2\}\\ &= (70)^3 + \text{thrice } (70)^2 \times 6 + \text{thrice } 70 \times 6^2 + 6^3.\end{aligned}$$

Example ii. [illegible]

$$+ \text{thrice } (800)^2 \times 50 + \text{thrice } 800 \times (50)^2 + (50)^3.$$

[illegible] 'cube root' is based upon the form of [illegible] which may be stated thus:—

The cube of the sum of two numbers = the cube of the first number + thrice the product of the square of the first by the second + thrice the product of the first by the square of the second + the cube of the third number.

This is the algebraic formula $(a+b)^3=a^3+3a^2b+3ab^2+b^3$.

Example i. *Find the* cube root *of* 438796.

$$\begin{array}{rr|r}
7^2 = & 49) & 438`976(70+6 \\
 & & 343 \\ \cline{3-3}
\text{thrice }(70)^2 = & 14700 & 95\,976 \\
\text{thrice }70\times 6 = & 1260 & \\
6^2 = & 36 & \\ \cline{2-2}
 & 15996 & 95\,976 \\ \cline{3-3}
\end{array}$$

The digits are marked off in threes [Art. 216].

We observe that

$$(70)^3=343000,$$

and that $(80)^3=512000$; so that the required cube is $(70+\text{some digit})^3$. Accordingly we commence the subtraction of $(70+\text{some digit})^3$ by subtracting $(70)^3$.

To complete the subtraction of $(70+\text{some digit})^3$ we must also subtract $\{\text{thrice }(70)^2\times\text{this digit}+\text{thrice }70\times(\text{this digit})^2+(\text{this digit})^3\}$, that is, $\{\text{thrice }(70)^2+\text{thrice }70\times\text{this digit}+(\text{this digit})^2\}\times\text{the digit}$. Hence our new 'divisor' is the above bracket.

Accordingly we find that *three times* $(70)^2=14700$; and we *judge from the size of the remainder* that the digit required is 6 : on trial it is found to be so.

Example ii. *Find the* cube root *of* 233744896.

We first mark off the digits in *threes*, thus 233`744`896, and subtract the largest possible cube number as in Example (i) from 233`744 ; thus

$$\begin{array}{rr|r}
6^2 = & 36) & 233`744(61 \\
 & & 216 \\ \cline{3-3}
\text{thrice }(60)^2 = & 10800 & 17\,744 \\
\text{thrice }60\times 1 = & 180 & \\
1^2 = & 1 & \\ \cline{2-2}
 & 10981 & 10\,981 \\ \cline{3-3}
 & & 6\,763
\end{array}$$

We have really subtracted $(610)^3$, that is, $(61)^3\times1000$ from 233744896. The remainder is 6763896.

We must now *complete* the subtraction of $(610+\text{some digit})^3$ from 233744896 ; thus

$$\begin{array}{rr|l}
\text{thrice }(610)^2 = & 1116300 & 6763896(610+6 \\
\text{thrice }610\times 6 = & 10980 & \\
6^2 = & 36 & \\ \cline{2-2}
 & 1127316 & 6763896 \\ \cline{3-3}
\end{array}$$

thus $(610+6)^3$ subtracted from 233744896 leaves *no remainder.*

713. 3 min. 46·8 sec. **714.** $\frac{1}{2}$. **715.** ·0000000020350...
716. 18·6 ft. **717.** 13$\frac{1}{2}$ feet. **718.** 3$\frac{1}{3}$ per cent.
719. $4200 stock, $14 increase.

720. $\frac{17}{21}$. **721.** 12$\frac{113}{567}$. **722.** 26970000. **723.** $6.57.
724. $174.20. **725.** $24960. **726.** 4.25\frac{3}{4}$.

727. $\cdot 0019\dot{2}8571\dot{4}$. **728.** About 91827 c. ft. **729.** 10.
730. $4862·02$\frac{1}{2}$. **731.** $274.50, $183, $137.25, $76.25.
732. 58$\frac{16}{33}$ per cent. **733.** 2$\frac{1}{7}$ hrs.

734. 5$\frac{53}{200}$. **735.** 2·17533 – ... **736.** $750.24.
737. At 15 min. past 5 o'clock. **738.** $63.25. **739.** *B* by 4$\frac{28}{75}$ yds.
740. 4950 yds. per hour.

741. $2 \times 5 \times 11^2 \times 13^2$. **742.** $\frac{19}{36}$. **743.** $445.14. **744.** 135.
745. $102.50. **746.** $776.45. **747.** $200.

748. 1109889. **749.** $6.70. **750.** 6$\frac{4}{5}$ hours. **751.** $11.68.
752. $277.15. **753.** $40905. **754.** 4 women.

755. 3$\frac{3}{4}$. **756.** $\cdot 0\dot{2}\dot{4}$. **757.** 362. **758.** 567. **759.** 6$\frac{1}{4}$ %.
760. No change. **761.** 101 yds.

763. 160. **764.** $4347. **765.** $2.52. **766.** $440.
767. 245 miles ; 2 hrs. 20 min. A.M. **768.** 151$\frac{1}{4}$ ac.

769. 10 cwt. 50 lbs. **770.** 69 c. **771.** 1 hr. 10 m. 13·5 sec. nearly.
772. 22 ft. 5 in. ; 19 ft. ; 14 ft. 1 in. **773.** 7$\frac{4}{5}$ per cent.
774. 128$\frac{1}{4}$ sq. ft. **775.** 1$\frac{1}{2}$ hours.

776. $\frac{1}{2}$. **777.** $20 \times 300 \times 49$ sq. units. **778.** 33$\frac{1}{3}$ %. **779.** 43$\frac{1}{3}$.
780. 168 c. ft. 305 c. in. $7''''\ 1'''''\ 3''''''$. **781.** $2174 ; $967 ; $241.75.
782. $2240.70.

783. 12 c. **784.** $\cdot\dot{9}=1$. **785.** 8$\frac{1}{2}$ hrs. per day. **786.** 8.30 P.M.
787. 74$\frac{2}{27}$ per cent. **788.** $9150 ; $5820. **789.** 16 a. 3 r. 8$\frac{8}{9}$ p.

790. 3 yds. 3 qrs. **791.** 10 ft. 9·7 in. **792.** $944.50. **793.** $48.64.
794. $5.32 (man) ; $2.66 (woman) ; $1.33 (child). **795.** 3$\frac{13}{20}$ in.,
3·855 miles. **796.** 107 yds.

797. 1903. **798.** $\cdot 2\dot{0}4941\dot{3}$; 1 lb. 5 oz. 8 drms. **799.** 350 hrs.
801. $3125. **802.** 20$\frac{29}{37}$ yds.

803. $\frac{4}{11}$. **804.** $1\cdot\dot{3}$. **805.** 220 yds. **806.** 3 hrs. 30 min.
807. 1086158. **809.** Gain 50 per cent.

810. 1541. **811.** 62·71 lbs. **812.** $2.70 (per 12 yds.)
813. At a profit of 27 °/ₒ. **814.** 8 gallons. **815.** $2.87.
816. 27404 yds.

817. $18\frac{2}{5}$. **818.** $111.70 ; ·021. **819.** 6 feet. **820.** 5 yds.
821. $101\frac{1}{4}$. **822.** 1 in. **823.** *A*, $49 ; *B*, $63.

824. 1 yd. 1 ft. $5\frac{5}{8}$ in. **825.** $573.77. **826.** 1669. **827.** $316.06.
828. $82\frac{2}{7}$ gals. **829.** $1\frac{7}{15}$ gals. **830.** $5\frac{173}{180}$ grs.

831. $1\cdot6\dot{6}1579\dot{7}$. **832.** Friday June 25, 1761. **833.** $3.26.
834. 10 per cent. **835.** $3\frac{51}{83}$ °/ₒ. **836.** 1 day. **837.** $8708\frac{28}{889}$ c. in.

838. $\frac{20}{63}$. **839.** 684. **840.** ·78 lbs. **841.** $\frac{3}{4}$ mile. **842.** $1078.35.
843. $400. **844.** $123725.

845. ·99998234, $3\cdot04\dot{3}\dot{9}$, 62·459. **846.** $12.16. **847.** 6 ft. 8 in.
848. 120000. **849.** $94\frac{4}{9}$ yds. **850.** $1875. **851.** $2\frac{1}{9}$ oz.

852. $\frac{49}{558}$. **853.** $1\cdot8\dot{2}8\dot{4}$; $\cdot3\dot{0}4\dot{9}$; $\cdot8\dot{1}2545879\dot{2}$. **854.** $143.75.
855. $16\frac{1}{2}$ ft. **856.** $4\frac{1}{2}$ % ; the same. **857.** 5 carats. **858.** $2\frac{17}{20}$ miles.

859. 48, rem. 249. **860.** 12. **861.** $17\frac{1}{2}$ ft. **862.** $1.11.
863. Men, $50 ; women, $26.25 ; children, $28.75. **864.** 2 per cents ;
10 per cents. **865.** One hour.

866. The last two recurring. **867.** $712.50. **868.** $7220. **869.** $18\frac{3}{7}$.
870. 1144 yds. **871.** $2\frac{1}{2}$ in. **872.** 6 minutes.

873. 3 sq. ft. **874.** $2.81 (nearly). **875.** $480. **876.** $4\frac{1}{8}$ sq. ft.
877. 25. **878.** 3 : 2. **879.** 119 miles.

880. 47. **881.** $2500. **882.** Wednesday, Tuesday. **883.** 5 per cent.
884. $26.25 ; $22.50 ; $18.75. **885.** 10 ducats. **886.** 50 per cent.

887. 278·3 yds. nearly. **888.** $4\frac{4}{25}$ hours. **889.** 40 : 53. **890.** ·24.
891. 6 ft. 3 in. **892.** $\frac{6}{13}$ miles. **893.** $1\frac{131}{369}$ %.

894. 12 ; $11 \times 2^7 \times 5 \times 3^5 \times 13^2 \times 7^3$. **895.** 15 c. ft. $1182\frac{107}{144}$ c. in.
896. 179. **897.** 260. **898.** 1 to 2. **899.** 300 apples, $1.40.
900. 11 to 5 nearly.

901. $89\frac{1}{2}$ %. **902.** 10 days. **903.** At 8 P.M. on Monday; 5 hrs. 59 min. $48\frac{3}{4}$ sec. **904.** 60 ; 105 ; 168. **905.** $291\frac{2}{3}$ lbs. **906.** $\frac{5}{12}$ wine, $\frac{7}{12}$ water. **907.** 360 matches. **908.** In 9 days. **909.** $54\frac{1}{12}$ c. **910.** *A*, 9 days ; *B*, $11\frac{1}{4}$ days ; *C*, $7\frac{1}{2}$ days. **911.** $13\frac{1}{3}$ m. from Cambridge ; at 6 P.M. **912.** 5 points. **913.** 4 min. $28\frac{5}{6}$ sec. after *B* started. **914.** *A*, \$26 ; *B*, \$19.50 ; *C*, \$33.15. **915.** 88 yds. **916.** \$6000. **917.** \$111. **918.** 288. **919.** \$37.04. **920.** 44·69...lbs. **921.** \$1386.75. **922.** 21 %. **924.** 32 oz. gold, 184 oz. silver ; \$1.23. **925.** 3 dys. **926.** £15. 19*s*. **931.** 186. **932.** 60 c. ft. ; 80 in., 45 in., $28\frac{4}{5}$ in. **933.** 1728000. **934.** \$709.75. **935.** 2 days. **936.** $53\frac{11}{13}$ %. **937.** 4 h. $36\frac{12}{13}$ min. **938.** Hare 4000, greyhound 3000. **939.** \$1.25 per lb. **940.** *A* by $18\frac{1}{3}$ yds. in $4\frac{3}{4}$ min. **941.** $20\frac{19}{73}$ carats. **942.** \$10. **943.** $6\frac{2}{3}$ miles per hour in the opposite direction. **944.** 6 ft., $4\frac{1}{2}$ ft., 3 ft. **945.** Twice ; $1\frac{1}{5}$ mi. and $2\frac{2}{5}$ mi. from start. **946.** 2671·9308...c. in. **947.** $18\frac{1}{3}$°. **948.** 6 in. **949.** $\frac{1}{16}$ sq. in. **950.** 30·557...ft. **951.** 180. **952.** 120 days. **953.** $5\frac{173}{180}$ grains. **954.** 35·38 oz. **955.** \$12.50. **956.** $77\frac{1}{2}$. **957.** 5 %. **958.** $2\frac{41}{64}$ gals. ; $9\frac{23}{64}$ gals. **959.** $11\frac{1}{13}$ min. **960.** 23°, – 4°. **961.** 53·6°. **962.** 564. **963.** \$234. **964.** 136 rods. **965.** \$10. **966.** $\frac{7}{110}$ ft. **967.** 8 days. **968.** $\frac{4}{5}$. **969.** 50. **970.** $24\frac{1}{5}$ °/ₒ. [It is understood that his profit of 10 °/ₒ is made at the time of sale, not in 6 months' time.] **971.** 18·43...times. **972.** $87\frac{3}{8}$. **973.** $4\frac{4}{9}$ gals. **974.** 2·76...in. **975.** \$5700. **976.** $1\frac{1}{11}$ secs. **977.** 35. **978.** \$1610. **979.** Speed of boat in still water = 3 × rate of river. **980.** $2\frac{118}{121}$ min. to 6. **981.** 62, 29. **982.** *A*, \$301.75 ; *B*, \$785.25 ; *C*, \$1087.00. **983.** $10\frac{10}{87}$. **984.** \$3078. **985.** 507 ; (radius of sphere = 19·053...ft.) **986.** 31·692...ft. high. **987.** 64000. **988.** 700 yds. **989.** \$250 ; \$200. **990.** 3·92...in. **991.** 3753·6... **992.** 224 ft. **993.** \$154.35. **994.** 12 sq. ft. 88·56 sq. in. **995.** $\$9009\frac{1}{111}$. **996.** 4·94 in. **997.** 40·6 in. **998.** \$1000. **999.** 5 °/ₒ. **1000.** 214415... **1001.** (i) $5\frac{15}{32}$ °/ₒ ; (ii) $14\frac{32}{128}$ °/ₒ.

THE END

Plane and Solid Geometry

By ARTHUR SCHULTZE, Ph.D.

AND

F. L. SEVENOAK, A.M., M.D.

Half Leather 12mo $1.10 net

PLANE GEOMETRY (Separately) 12mo Cloth 80 cents net

"I am emphatically in favor of the aim of this book. The average student, to-day, gets out of his course in geometry very little except knowledge of propositions, and this is more or less permanent according to the retentiveness of the student's memory. The acquisition of power in solving geometrical problems should be the aim of the teaching of geometry. To recognize this is a great step in advance, and I hope the present volume will contribute greatly to this end. The 'Suggestions to Teachers' are excellent." — PROFESSOR PERCY I. SMITH, *Sheffield Scientific School of Yale University, New Haven, Conn.*

"Schultze and Sevenoak's Geometry has been in use in the De Witt Clinton High School during the past year with the most gratifying results. With no other text have we been able to secure such originality in demonstration as was manifested by the class using this book. Memory work was never attempted by beginners, and the book worked equally well in classes that had previously had a part of the subject. More and better work can be accomplished in a definite time, through the use of Dr. Schultze's methods as set forth in this book, than in any other way. The book adapts itself readily to the use of large or small classes, or to the private use of the individual.

"All Geometries ordered for use this fall were Schultze and Sevenoak's. It is without question the best Geometry published. It is a teachable book. The question of the 'royal road' to Geometry has been solved. I unqualifiedly recommend its use in high or private schools." — H. B. PENHOLLOW, *De Witt Clinton High School, New York City.*

"The excellent arrangement, the careful drawing of figures, the simplicity and directness of the demonstrations, as well as the choice and progressive variety of the 'originals,' make up a most teachable text." — G. W. KENNEDY, *High School, Troy, N.Y.*

"The plan of the book is just what I have been using for some time, and it commends itself to me." — J. P. CONOVER, *St. Paul's School, Concord, N.H.*

THE MACMILLAN COMPANY

64–66 Fifth Avenue, New York

BOSTON CHICAGO SAN FRANCISCO ATLANTA

HALL AND KNIGHT'S

Algebra for Colleges and Schools

Revised for American Schools and Colleges by

FRANK L. SEVENOAK, A.M., M.D.

12mo **Half Leather** **$1.10 net**

ELEMENTS OF ALGEBRA

The "Elements of Algebra" contains the first 366 pages of the "Algebra for Colleges and Schools."

12mo **Half Leather** **90 cents net**

"I prefer it to any other elementary work on the market. Clearness of demonstration and the great educational value of its methods are the attractive features." — PROFESSOR E. W. NICHOLS, *Virginia Military Institute, Lexington, Va.*

"I should like to express my satisfaction that there is at least one good elementary algebra suited for preparation to college." — MISS GRACE B TOWNSEND, *Wellesley Hills, Mass.*

"I take pleasure in stating that for several years I taught algebra in a private institution, using, as a text, Hall and Knight's 'Algebra for Colleges and Schools,' for college preparation. Since teaching in the public schools of New York City, I have taught both beginning and advanced classes, using this same text. I have been impressed at all times with the admirable grading of the work and the natural order of exercises, and consider it a book that can be used throughout the ordinary four-year high school course. The book contains many points which are unique, and I have no hesitation in stating that, from results obtained, I consider it the best working text-book on this subject I have seen." — DR. ARTHUR SCHULTZE, *High School of Commerce, New York City.*

THE MACMILLAN COMPANY

64–66 Fifth Avenue, New York

BOSTON CHICAGO SAN FRANCISCO ATLANTA

FISHER AND SCHWATT'S

Series of Text-books on Mathematics

Rudiments of Algebra . . .	$ .60	School Algebra	$1.00
Secondary Algebra	1.08	Elements of Algebra . . .	1.10
Complete Secondary Algebra	1.35	Higher Algebra	1.50
Quadratics and Beyond . .	.90	Text-book of Algebra, Part I	1.40

The Fisher and Schwatt algebras have achieved a marked success in the short time that they have been before the public. The "Higher Algebra" is used as a regular text-book at Harvard University, and the elementary books are established in secondary schools of corresponding rank.

These text-books may be resolved into two series, the more popular of which consists of the "Rudiments" (for grammar schools) and the "Secondary" (for high schools). The "Complete Secondary" includes the material of the "Secondary" and also chapters on Continued Fractions, Summation of Series, the Exponential and Logarithmic Series, Determinants, and Theory of Equations. Thus, while the "Secondary" covers the requirements for admission to colleges, the "Complete Secondary" covers the requirements for admission to any scientific school, and is also sufficiently full for the ordinary work of the first year in college. "Quadratics and Beyond" consists of the second half of the "Complete Secondary," bound separately for the convenience of advanced and reviewing classes in secondary schools, and for college freshman work.

The second series constitutes a more difficult course and places more emphasis on the theory of mathematics. In this series the "School Algebra" corresponds roughly to the "Secondary," and the "Elements" to the "Complete Secondary."

The "Higher Algebra" is a book for college courses. The "Text-book of Algebra" is a high school book containing an unusually large number of graded exercises; it is a valuable mine of problems bearing on the regular high school work.

THE MACMILLAN COMPANY

64-66 Fifth Avenue, New York

BOSTON **CHICAGO** **SAN FRANCISCO** **ATLANTA**